Additional problems

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Elevator waiting time

Mr. Smith works on the 13th floor of a 15 floor building. The only elevator moves continuously through floors 1, 2, . . . 14, 15, 14, . . . 2, 1, 2, . . . , except that it stops on a floor on which the button has been pressed. Assume that time spent loading and unloading passengers is very small compared to the travelling time. Mr. Smith complains that at 5pm, when he wants to go home, the elevator almost always goes up when it stops on his floor. What is the explanation?

Intransitive Dice

- Real numbers are transitive, meaning that if we know that $x > y$ and $y > z$ then $x > z$.
- However, probabilities of events are not always transitive.
- This may be more aligned with real life. For example, consider three tennis players, say Alice, Bella, and Claire.
  - Alice wins Bella more frequently;
  - Bella wins Clair more frequently;
  - does Alice win Clair more frequently?

Intransitive Dice

Consider three dice with different sides numbers:

- Die A has sides 2, 2, 4, 4, 9, 9.
- Die B has sides 1, 1, 6, 6, 8, 8.
- Die C has sides 3, 3, 5, 5, 7, 7.

To play a game using dice A and B, you can chose which die to roll and your opponent rolls the other. The one who tolls a larger number wins. Which die do you want to choose? How about B v.s. C?

Bertrand's Box

There are three boxes, one contains two gold coins, one contains two silver coins, and one contains a gold coin and a silver coin. A box is selected at random and a coin is taken from that box at random. If the coin is a gold coin, what is the probability that the other coin in that box is also a gold coin.

Canadian Lottery

Canadian lottery officials learned the importance of careful counting the hard way when they decided to give back some unclaimed prize money that had accumulated. They purchased 500 automobiles as bonus prizes and programmed a computer to determine the winners by randomly selecting 500 numbers from their list of 2.4 million subscriber numbers. The officials published the unsorted list of 500 winning numbers, promising an automobile for each number listed. To their embarrassment, one individual claimed (rightly) that he had won two cars. The officials were flabbergasted – with over 2 million numbers to choose from, how could the computer have randomly chosen the same number twice? Was there a fault in their program?

Birthday Problem

Suppose that a room contains 23 people and each year has 365 days.

- What is the probability that at least two of them have a common birthday?
- What is the probability that some one in that room has the same birthday as yours?

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The probability of a common birthday is surprisingly larger. Here are numbers:

n, number of people	4	16	23	32	40	56
Pr(common birthday)	.016	.284	.507	.753	.891	.988

The probability that someone's birthday is the same as yours is quite small. We need 253 random selected people to have it to be 0.5.

n	4	16	23	32	40	56	252	253
Pr	0.011	0.043	0.061	0.084	0.104	0.142	0.499	0.500

Monty Hall problem

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?