Risk Assessment: a Forest Fire Example

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Abstract

The concern of this paper is obtaining baseline values for the number of forest fires as a function of time and location and other explanatories. A model is developed and applied to a large data set from Federal lands in the state of Oregon. To proceed the data are grouped into small spatial-temporal cells (voxels). Fires are rare so there are many of these voxels with no fires. In fact there are so many such cells that in the analyses presented a sample is taken to make the work manageable. The paper sets down a likelihood for the sampled data and fits a generalized additive model involving location, elevation and day of the year as explanatories.

Keywords. Forest fires; generalized additive model; Oregon; risk analysis; sampled data; wildfires

1 Introduction

Forest fires represent a problem of considerable societal importance. We mention the following report that appeared in the San Francisco Chronicle of 7/16/2002,

> *... Nearly two weeks ago, the Forest Service used up the entire $321 million budgeted for firefighting in 2002. It is expected to spend another $645 million by the end of the year. ...Wildfires have already burned more than 3.3 million acres this year, more than twice the yearly average over the last decade.*

The concern here is the development of a risk model for use in estimating the probability of a forest fire taking place at a particular location and time as a function of those and other explanatory variables. The work is implemented for the case of a fine grid of cells and an accompanying large data set. An analysis is carried out for a region surrounding the state of Oregon, henceforth referred to as Oregon, and employing: location, elevation and day of year as explanatories.

The elements of the approach are:

1. a spatial-temporal point process and associated covariates,
2. likelihood-based inferential methods developed for such processes,

3. approximation of the point process by a 0-1 valued process on a lattice,
4. a sampling of the 0 i.e. no-fire cells,
5. generalized additive model technology.

Oregon was chosen for this pilot study because of its approximate rectangular geographic shape and its high rate of fires. The work presented here has as its goals model development and the estimation of baseline values for future work. The model contains but a few explanatory variables. Employing a sample of the no-fire cases rather than all the no-fire cases available is an unusual aspect of the approach presented.

The sections of the paper are: Introduction, Risk assessment, Previous statistical work, Model and analysis development, The data, Results and Discussion.

Risk assessment is familiar to Terry Speed. His papers include: reviews of the procedures that have been employed in the nuclear industry [26], [29], an analysis concerning a ship following a specified sea route [28], and an analysis of risk to levees from adverse weather conditions [27]. The first mentioned contains the following wonderful exchange with the Director of Technology, U.S. National Transportation Safety Board,

> *Dear Professor Speed,*
>
> *In response to your aerogramme of April 5, 1977, the Chairman's statement concerning the chances of two jumbo jets colliding (6 million to one) has no statistical validity nor was it intended to be a rigorous or precise probability statement. The statement was made to emphasize the intuitive feeling that such an occurrence has a very remote but not impossible chance of happening.*
>
> *Thank you for your interest in this regard.*
>
> *Sincerely yours ...*

From these papers and personal observation it is clear that Terry knows a lot about taking risks. May he continue to have fun doing so for many years.

2 Risk assessment

Probabilistic risk assessment can be defined as the process of estimating, for some class, the probabilities of hazardous events taking place within a specified time period and in a specified context. Such an assessment often proceeds by reducing a particular complex system to its simpler components. This is followed by the fitting and validation of stochastic models associated with the components. Typically large doses of substantive subject matter are required in such modeling and data analysis projects.

This paper is concerned with the case of forest fires. Wildfires are a natural disturbance in virtually all the world's ecosystems and the annual losses are staggering, see the *Chronicle* report above. It seems clear that fire occurrence depends on local conditions such as: location, elevation, wind velocity, precipitation, temperature, air humidity, topography, litter type, level of suppression amongst other explanatories. In our work fire ignition will be viewed as a random phenomenon. A pertinent conceptual model is: when the temperature (or some related latent variable) at a given location exceeds a threshold, depending on the local conditions, a fire breaks out. The latent variable will depend on explanatories such as those listed above. In the work below the logit transform will be employed. It corresponds to a latent variable with a logistic distribution.

3 Previous statistical work

Often a Poisson model has been employed for the number of fires, see Dayananda [5], Poulin-Costello [19], Mandallaz and Ye [10, 11]. In other cases it is a logistic: Chou et al. [4], Poulin-Costello [19], Martell et al. [13]. In another approach McKenzie et al. [14] use multiple regression and regression trees. Markov chain models are employed in Martell [12]. Peng and Schoenberg's works, [24], [18], relate wildfire incidence to temperature, precipitation, fuel moisture and fire history for Los Angeles County. These researchers find that time expired since a location has burned previously appears important. Roads et al. [22] use a regression model for fire occurrence with 6 fire danger indices by fuel type as explanatories. There is also spatial autocorrelation. [20] provide a review.

4 Model and analysis development

4.1 The spatial-temporal conditional intensity function

To begin suppose that the space-time domain is broken up into voxels $(x, x + dx] \times (y, y+dy] \times (t, t+dt]$. Consider the spatial-temporal point process, N, with conditional intensity function assumed to exist and defined by

$$\lambda(x, y, t) \;=\; Prob\{dN(x, y, t) \;=\; 1|H_t\}/dxdydt$$

where $dN(x, y, t) \;=\; N(dx, dy, dt)$ counts the number of fires in the voxel where H_t is the history of the process N up to and including time t. Supposing that λ contains a parameter θ the log-likelihood function will be written

$$L(\theta) \;=\; \int_0^T \int_x \int_y log[\lambda(x, y, t|\theta)]dN(x, y, t) \;-\; \int_0^T \int_x \int_y \lambda(x, y, t|\theta)dxdydt \qquad (1)$$

see Fishman and Snyder [7]. Explanatories may appear in λ but are presently suppressed in the notation.

Asymptotics of maximum likelihood estimates based on point process likelihoods have been developed in Ogata [17], Sagalovsky [23], Rathburn [21], Schoenberg [25]. The last two papers focus on the spatial-temporal case.

Consider next practical approaches to using the log-likelihood (1) in practice.

4.1.1 Approach 1

The second term of (1) is the awkward one in our case because it covers such a large area in space-time. It will be approximated. One way to do this is by sampling points from an independent spatial Poisson process, $M(dx, dy, dt)$, of rate π on the space $\mathcal{R} \times [0, T]$. Then the log-likelihood (1) may be approximated by

$$\int_0^T \int_x \int_y log[\lambda(x, y, t|\theta)]dN(x, y, t) \ - \ \int_0^T \int_x \int_y \lambda(x, y, t|\theta)dM(x, y, t)/\pi \qquad (2)$$

Averaging over M, but holding the process N fixed, the expected value of (2) is (1). The expected number of points in the approximating sum of the second term is π times the volume of $\mathcal{R} \times [0, T]$.

4.1.2 Approach 2

A model that is often simpler to deal with follows. It is an approximation to Approach 1. Replace the spatial-temporal point process, $N(dx, dy, dt)$, by a 0-1 valued process $N_{x,y,t}$ on a lattice with $N_{x,y,t} = 1$ if there is a fire in the corresponding voxel and by 0 otherwise for (x, y) in $\mathcal{R}$; $t = 0, ..., T - 1$. (This idea was used to advantage in [2].) Suppose then that

$$Prob\{N_{x,y,t} = 1|H_{t-1}\} \ = \ \lambda_{x,y,t}$$

with H_t the history up to and including t. A Bernoulli approximation to the log likelihood (1) is now

$$\sum_{x,y,t} N_{x,y,t} \ log(\ \lambda_{x,y,t}) \ + \ \sum_{x,y,t} (1 - N_{x,y,t}) \ log \ (1 \ - \ \lambda_{x,y,t}) \qquad (3)$$

In the present case there are many voxels for which $N_{x,y,t}$ is 0 and so the second sum in (3) contains many terms. To deal with this, randomly select 0-voxels with probability π and include them alone in the analysis. (In what follows there will actually be two-stage sampling with $\pi \ = \ \pi_1\pi_2$. This is to possibly obtain more efficient estimates. The estimation procedure then involves two types of uncertainty; one from the fire process itself and a second from the sampling of data points. The latter component will decrease with increasing π.)

To simplify the notation for the moment, index the voxels by k rather than x, y, t. Let S denote the collection of the voxels that had a fire and the sample of those that did not. In what follows it will be assumed that the N_k are independent given the explanatories. This condition will be relaxed in future work. Using the identity $Prob\{A|B\} \;=\; Prob\{B|A\}Prob\{A\}/Prob\{B\}$ one has the conditional probability

$$Prob\{N_k = 1|H_{t-1},\ k\ in\ S\} \;=\; \gamma_k \;=\; \lambda_k/(\lambda_k + (1-\lambda_k)\pi)$$

with the complimentary probability for the event $N_k = 0$. Now by elementary algebra

$$logit\ \gamma_k \;=\; logit\ \lambda_k \;+\; log\ \pi$$

Using the indicated assumption of independence the log-likelihood based on the N_k for k in S is

$$\sum_{k\ in\ S} [N_k\ log(\ \gamma_k) \;+\; (1-N_k)\ log(1-\gamma_k)] \tag{4}$$

i.e. a Bernoulli likelihood. Appropriate uncertainty measures are often available for estimates obtained by maximizing (4), see the Appendix.

In the analyses $\eta \;=\; logit\ \lambda$ will be a "linear predictor" based on explanatories. To obtain estimates one can use a generalized linear model program, such as $glm()$ of Splus, with an offset of $log(1/\pi)$ to carry out an analysis. In fact the generalized additive model program $gam()$, [8], will be employed below.

Above an assumption of independence was made. For the present this will be made reasonable by the inclusion of explanatories. For example location is meant to handle the similarity of nearby values.

4.1.3 Approach 3

Another method begins, again, by approximating the log-likelihood (1) by one based on 0-1 variates. Further suppose once again the 0-voxels have been sampled. Let $\{\delta_{x,y,t}\}$ denote independent Bernoulli variates corresponding to the sampling with parameter π. They are to be independent of the fire process, N. Consider the approximate log-likelihood

$$\sum_{x,y,t}[\ N_{x,y,t}\ log(\ \lambda_{x,y,t}) \;+\; (1-N_{x,y,t})\ log(1 \;-\; \lambda_{x,y,t})\ \delta_{x,y,t}\ /\ \pi]$$

$$= \sum_{k\ in\ S} w_k[N_k\ log\ \lambda_k \;+\; (1-N_k)\ log(1-\lambda_k)] \tag{5}$$

The w_k are weights that equal 1 at locations with fires and equal $1/\pi$ at the sampled locations with no fire. If $\pi \;=\; 1$ this reduces to the log-likelihood (3).

4.2 Assessing fit

Suppose a particular link function has been employed, e.g. the logit, and one wishes to assess it. The fitted linear predictor values may be assigned to the cells of a histogram. For each cell some of the corresponding N's will be 0 and some will be 1. The "number of 1's" divided by "the number of 1's" plus "the number of 0's" weighted up by $1/\pi$ provides a nonparametric estimate of the function $\lambda(\eta) = Prob\{N = 1|\eta = 1\}$ with η representing the linear predictor of the generalized additive employed model. This estimate may be compared to $\lambda(\eta|\hat{\theta})$ where $\hat{\theta}$ is the maximum likelihood estimate. An example is provided below, see Figure 6.

4.3 Predictions

Quantities of substantial interest to the Forest Service managers, for planning purposes, include probabilities such as

$$\lambda_{x,y,t+1} = Prob\{N_{x,y,t+1} = 1|H_t\}$$

The fitted linear predictor and its statistics may be used to compute pertinent estimates. Provided some explanatories in the model are leading variables, and they can be reasonably forecast, one would have useful predictions.

The work presented here has in mind obtaining baseline values using models involving location, elevation and day of year only. The next stage of research will study improvements obtained when leading variables, e.g. based on meteorology, are included.

5 The data

In this pilot study fire occurrence data from Federal lands in the state of Oregon were used. The data consisted of locations and dates of every fire greater than 0.1 acre that occurred between April 26, 1989 and December 31, 1996. (There were 15,786 such fires.) The date refers to the day the fire started. The source of the fire location data was the USDA Forest Service, National Fire Occurrence Data Base [30]. Figure 1 provides an elevation map of Oregon with federal fire locations for 1990. The dark winding horizontal line at the top of the figure is the Columbia River. It provides the border of Oregon with Washington. Figure 2 shows the so-called "Federal Mask", i.e. the Federal lands in Oregon. Their area accounts for approximately 56% of the state.

Figure 3 is a time series plot of the square roots of fortnightly counts of fires over the time period of the study. A clear annual effect corresponding to the fire season may be seen. There is no serious suggestion of a trend.

Data on response variables and explanatories were selected as follows. The fires were those inside Federal lands, delineated by of Figure 2. To get the quantities for beginning data analyses a two-stage sampling procedure was employed for the voxels without a fire because of the data management problem. In the first stage of the sampling a collection of days was selected, with each day having probability $\pi_1 = 0.1$ of being picked. In the second stage a proportion $\pi_2 = 0.0012$ of cells inside the Federal mask was picked. This resulted in a total of 73880 cases of which 15,786 were

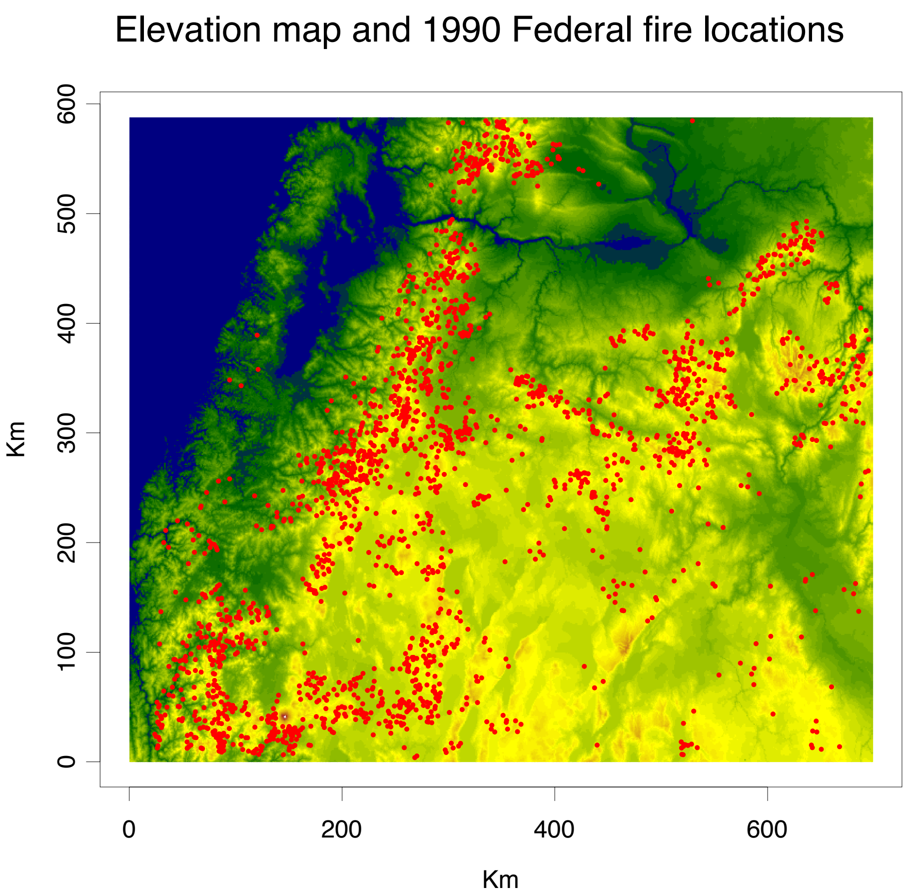

Figure 1: Federal fire locations in 1990. Dark blue corresponds to water and the red circles to fire locations.

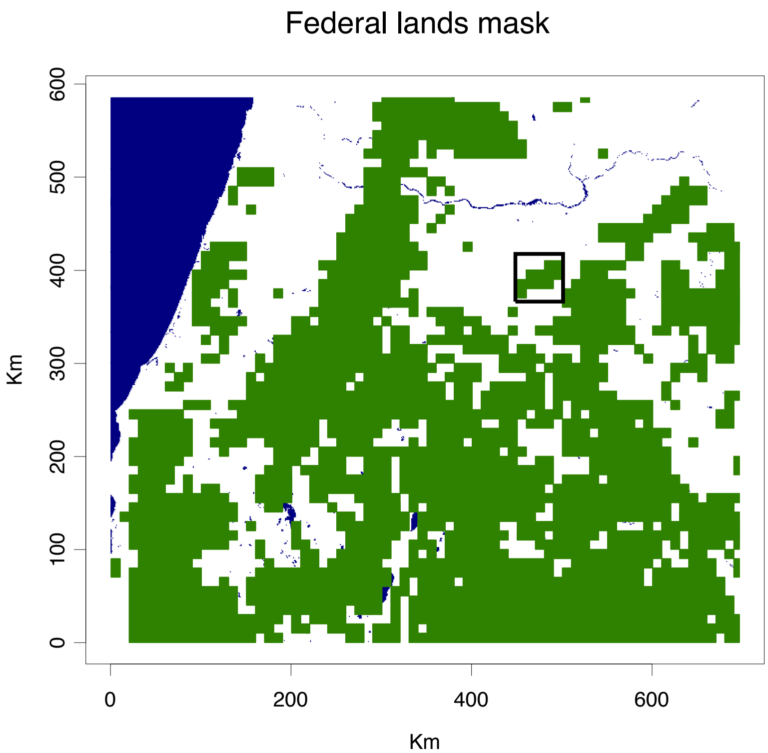

Figure 2: The Federal lands in Oregon. The box refers to the region taken for an example in Section 6. It is referred to as Region B.

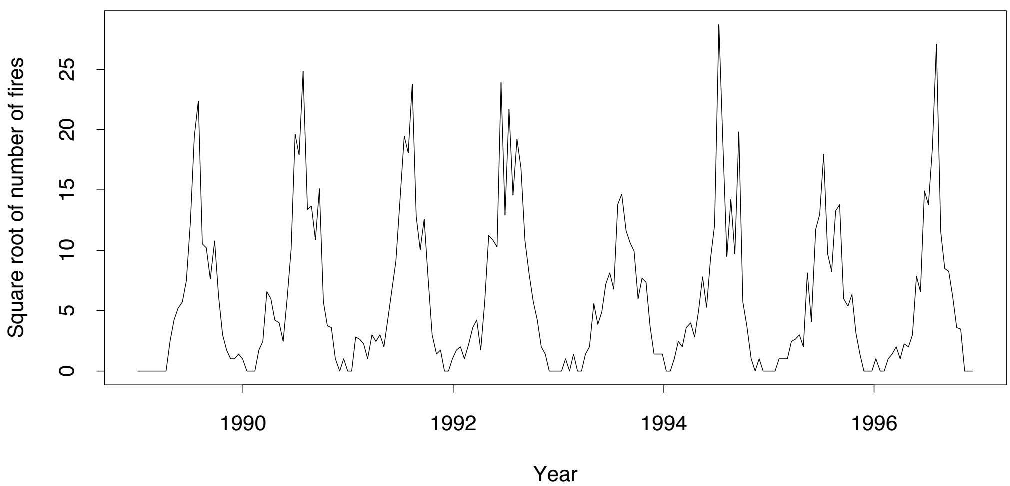

Figure 3: Square root of fortnightly count of Oregon fires in Federal lands 1989-1996.

fire occurrences N_k of (4) equals 1 and 58094 were with N_k equal 0. The overall rate of fires experienced for the period of the study was 15786/(2760*209490) = .0000273 per km^2 per day. (There were 2760 days and a region of area 209490 km^2 in the study.)

To obtain the estimates, the log-likelihood (4) was maximized. Specifically the function $gam()$ of Splus, [8], was used to fit the Bernoulli model with linear predictor

$$\eta_k = logit\ \lambda_k = g_1(x_k, y_k) + g_2(d_k) + g_3(e_k) \qquad (6a)$$

where (x_k, y_k) corresponds to the locations (in meters) coordinates of the k-th response, d_k corresponds to day in year and e_k refers to elevation at the location. The $g()$ are (nonparametric) smooth functions to be estimated. The function $g_2(.)$ has period 1 year and is based on a periodic spline [15]. The model corresponds to

$$Prob\{N = 1|\eta\} = exp\{\eta\}/(1 + exp\{\eta\}) \qquad (6b)$$

One of the interesting aspects of the work is that the data set is quite large and awkward. Maps for the state of Oregon were extracted from the CD's, converted to ASCII files and then a sample was selected in two stages. Locations of fires were recorded in units of latitude and longitude. These were converted to metric units. Even with a small fraction (π=0.00012) of the available voxels, each gam run required approximately 10 minutes to run on a 900MHz computer. This meant that it was not easy to do exploratory data analysis.

6 Results

6.1 The fit

Figure 4 provides the estimated spatial effect for model (6a,b). It was obtained by the second approach described above. The results are presented in both perspective and contour form. Some general features apparent in this map are the low intensity level in the south-eastern region of Oregon, contour levels -2 and -3. This region is mostly high desert with few forests. Regions with the historically highest level, contour level 1, of fire appear to be in the Cascade Mountains.

Figure 5 provides plots of the estimated effect of elevation and day of the year. Unsurprisingly the latter indicates amongst other things more fires in the summer as was evident in Figure 3. The elevation effect increases for those less than 2000m. The approximate 95% bounds are those output by the jackknife They are obtained by splitting the 7 years of data into 7 random segments of 365 days. (One reference to the jackknife is Chapter 11 in Efron and Tibshirani [6]. A justification for its validity in the present case is given in the Appendix. The variances estimated are overall as opposed to conditional.) Generally the bounds are small as might be expected given the large amount of data involved. The uncertainty bounds for the higher elevations are wide because there are not many locations in the data set.

They do not differ greatly from those produced by gam.

Figure 6 implements the method of Section 4.2 for assessing the appropriateness of the model (6a,b). It is a plot the empirical relative frequencies of fires as points. These are computed after grouping the data into classes based on the linear predictor. Also provided, as a solid line, are the estimated probabilities from model (6a,b) and, as dashed lines, approximate binomial confidence bounds. The fit appears reasonable, but one does notice points above the curve on the right. This departure may disappear when other explanatories are included in the model.

6.2 An example

Forest managers are interested in estimates of the number of fires likely to occur in a given region during a given fire period. Amongst other things these estimates will help them allocate resources. As an example, using the output above, estimated fire probability values were produced for the shaded region within the box of Figure 2. This is is the Heppner Ranger District of the Umatilla Forest in Oregon. It will be referred to as region B. It is taken as representative of the sort of region of interest in wildfire risk estimates for the U.S. generally.

Figure 7 and Table 1 present some of the results. The solid line in Figure 7 gives the estimate of the monthly rates obtained by fitting the model with location, elevation, and day of year as explanatories. The shaped region gives approximate 95% marginal confidence limits for the estimate. These are obtained in the same jackknife computations as produced the errors bounds of Figure 5. In this case the linear predictions are perturbed by $\pm 2s.e.$ and converted to probabilities by the transform (6b). The resulting fire rates are seen to peak in the summer season with an estimated value of 6.74 fires for the month of August.

The dots in Figure 7 refer to the naive estimate of the rates of fires obtained by dividing the total number of fires for a month by the number of years of observation. The vertical lines give approximate 95% limits employing a Poisson approximation. One notes differences, but it may be remarked that the only physical characteristics of Region B being used in the present estimation are its elevations.

To be specific managers are interested in things like

$$Prob\{i\ or\ more\ fires\ in\ July\},\ i\ =\ 1,\ 2,\ ,...$$

Table 1 provides estimates of such together with approximate 95% confidence limits for the estimates. The count is based on the sum of Bernoulli variables. When the probabilities of the Bernoullis differ the distribution is called the Poisson-Binomial,

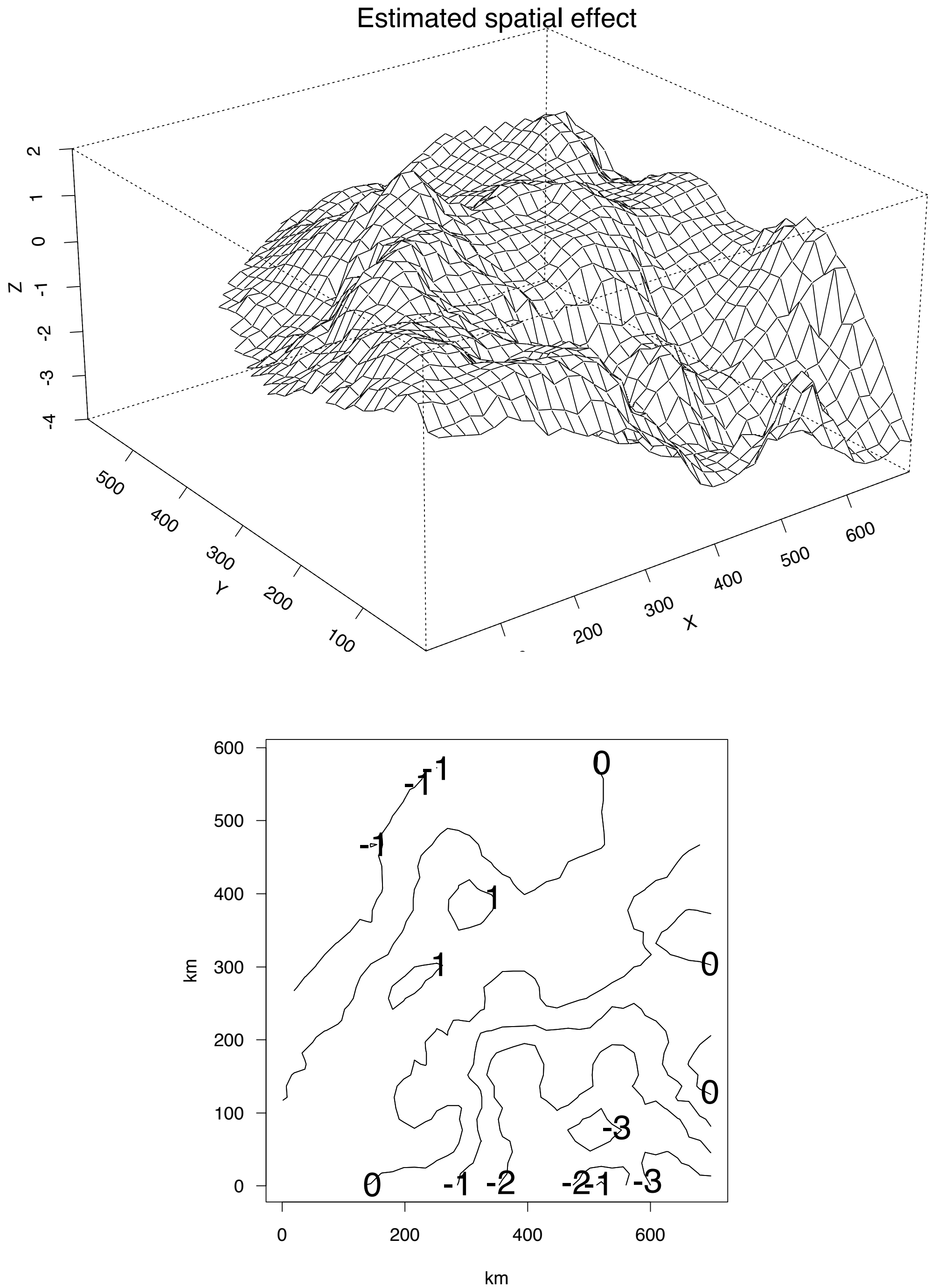

Figure 4: Estimated spatial effect, $\hat{g}_1$, for model (6a,b).

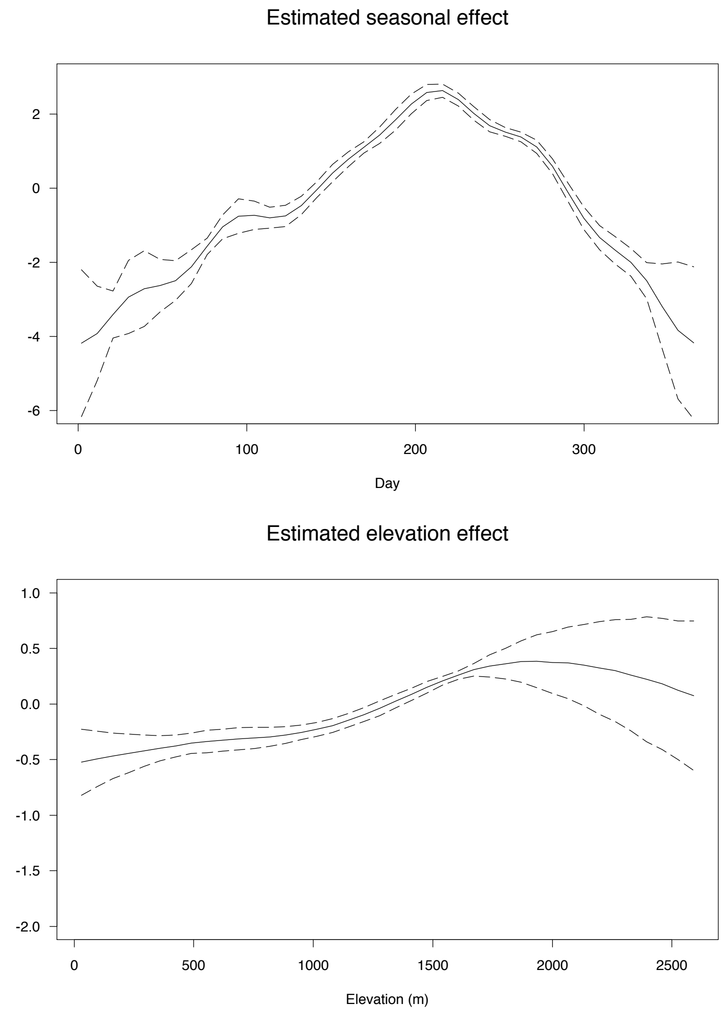

Figure 5: Estimated effects $\hat{g}_2$ of day in year and $\hat{g}_3$ of elevation for the model (6a,b). The dashed lines provide approximate marginal 95% bounds computed by a jackknife procedure.

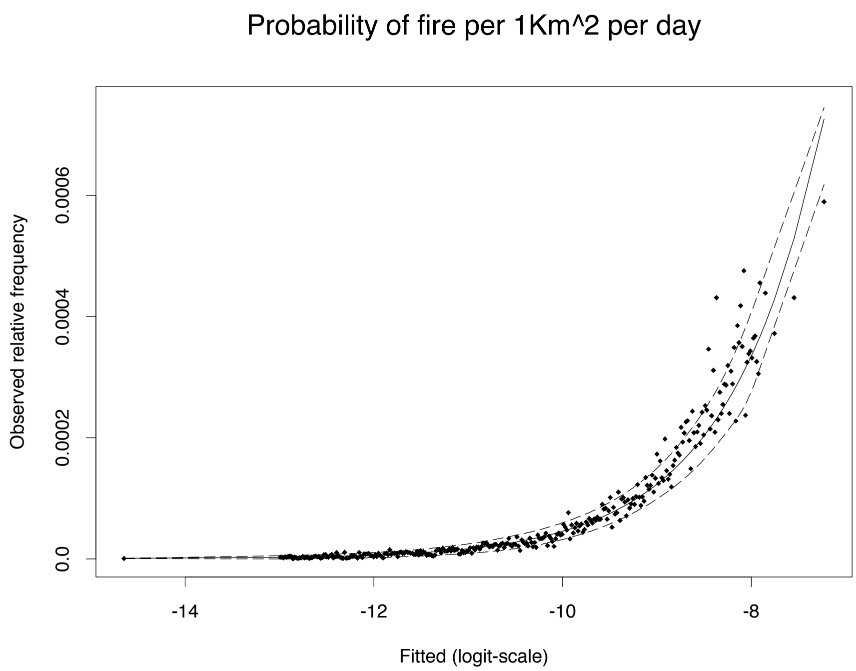

Figure 6: Observed relative frequencies of fire, after grouping the data into classes based on the fitted linear predictor, $\hat{\eta}$. The solid curve is the fitted logistic curve. The dashed lines are smoothed approximate 95% limits obtained via a binomial approximation.

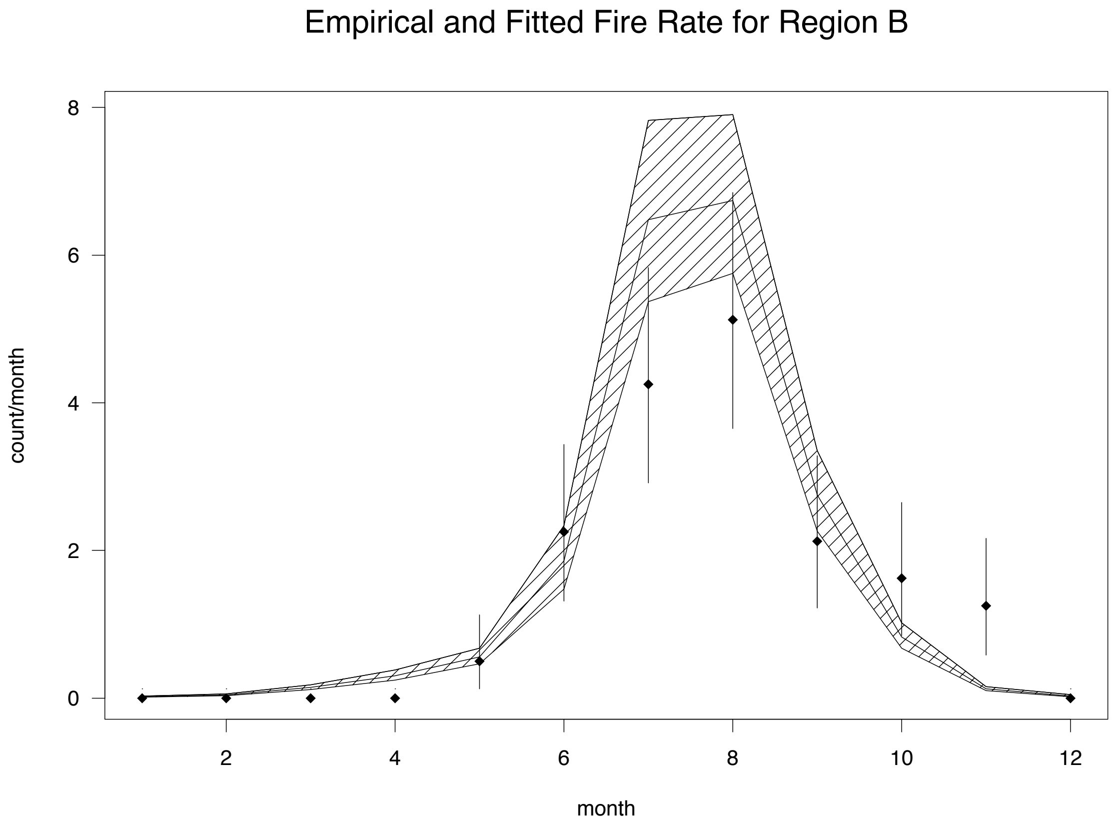

Figure 7: The solid central line gives the fitted rate of fires by month. The shaded region gives +- 2 s.e. limits. The points are the monthly empirical rates of fires. Vertical lines are +- 2 s.e. limits for the points.

[9]. These authors show that this distribution is well approximated by a Poisson in the case that the largest of the individual pixel probabilities is small. This seems to be the case in the present situation where the risk of fire is low, particularly since the pixels are small. For example the estimated probability of 7 or more fires in July for region B is .324 with an approximate confidence interval of (.176,.520).

Here the only serious explanatory employed beyond location is elevation. The goal of future work is prediction using leading explanatories. It will be interesting to see the extent to which the confidence interval shrinks as these are brought into the model.

i	*probability*	*confidence interval*
1	.989	(.967,.996)
2	.956	(.895,.983)
3	.887	(.771,.948)
4	.774	(.610,.883)
5	.628	(.441,.784)
6	.470	(.291,.658)
7	.324	(.176,.520)
8	.206	(.097,.386)
9	.121	(.049,.268)
10	.066	(.023,.174)
11	.033	(.010,.106)
12	.016	(.004,.060)
13	.007	(.001,.032)
14	.003	(.001,.016)
15	.003	(.000,.007)
16	.001	(.000,.003)
17	.000	(.000,.001)
18	.000	(.000,.001)
19	.000	(.000,.000)
20	.000	(.000,.000)

Table 1: Estimated probability of i or more fires and approximate 95% confidence limits for the month of July and region B.

7 Discussion

The model, (6a,b), for the breaking out of wildfires has been set down and fit. The model was motivated by threshold considerations and is of nonparametric character. The assumptions include smooth dependence of the risk probability on location, elevation, day of the year. A logistic link function is employed.

There are many sources of variability present in addition to the random nature of wildfires. The simplest is that arising from using a sample of the no-fire voxels. The sampling was two-stage without replacement. Finite population correction factors were ignored in computing uncertainty estimates, but they are negligible. The sampling fraction, π, will be increased in future studies.

The principal source of the variability that shows itself in Figure 7 and Table 1 is possibly that arising from missing explanatories. To the extent possible this will be dealt with in future studies. Additional data sources to be included then are weather data from the Weather Information Management System [14], fire danger indices from NIFMID [16] and more topographic data, e.g., aspect and distance to nearest population center.

This has been a pilot study with the purpose of obtaining baseline values. These will be used as standards for later forecasting models.

One difficulty needs to be mentioned. In using $gam()$ it was found that the results of prediction runs depended on which other predictions were carried out at the same time. This may be due to the accumulation of roundoff error. Studies are continuing. The results presented in this paper are for runs involving all the predictions being carried out at the same time. The reason for this choice is that the computing time required was less than, say, running one prediction at a time.

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Appendix

The use of the jackknife in Section 6.1 may based on the assumption that K is large, that the values

$$(N_k, x_k, y_k, d_k, e_k),\ k = 1, ..., K$$

may be treated as an i.i.d. sample from some distribution and that the statistics computed are M-estimates in some sense. The argument follows, see the Appendix of [1].

Given sample values $\mathbf{X}_1, ..., \mathbf{X}_n$ an M-estimate is defined as a statistic, θ, providing the minimum value of a penalty function

$$M(\mathbf{X}_1, \theta)\ +\ ...\ +\ M(\mathbf{X}_n, \theta)$$

Maximum likelihood estimates are an example.

If $\mathbf{m}(\mathbf{X}, \theta)$ denotes the derivative of $M(\mathbf{X}, \mathbf{A})$ with respect to θ, then in many cases the M-estimate is the solution of the equation

$$\mathbf{m}(\mathbf{X}_1, \hat{\theta})\ +\ ...\ +\ \mathbf{m}(\mathbf{X}_n, \hat{\theta})\ =\ \mathbf{0}$$

Suppose that θ_0 is the "true value" of θ satisfying $E\{\mathbf{m}(\mathbf{X}, \theta_0)\}\ =\ 0$. Suppose $\mathbf{m}'$ denotes the derivative of $\mathbf{m}$ with respect to θ. Let $\mathbf{a}'$ denote $E\{\mathbf{m}'(\mathbf{X}, \theta_0)\}$. Then under regularity conditions Chibisov [3] develops the asymptotic expansion

$$\sqrt{n}(\hat{\theta}\ -\ \theta_0)\ =\ -\ \mathbf{a}'^{-1} \sum_{i=1}^{n} \mathbf{m}(\mathbf{X}_i, \theta_0)/\sqrt{n}\ +\ \rho_n/n$$

giving a uniform bound for $P_{\theta_0}\{||\rho_n|| > v_n\},\ v_n > 0$. The estimate may thus be represented as a standardized sample mean and a smaller term. As Tukey and others showed the jackknife works for regular functions of means. One thus has a form of asymptotic justification of the use of the jackknife in the present context. The formal justification will need to handle the case in which $dim(\hat{\theta}_n)\ \to \infty$ at some rate.