AMERICAN INTERNATIONAL UNIVERSITY BANGLADESH
DEPARTMENT OF EEE/CoE, FACULTY OF ENGINEERING
CONTROL SYSTEM
FINAL ASSIGNMENT

Antenna azimuth position control system:

The antenna azimuth control system currently available on the market is described as a servo controlled antenna through the use of gears and feedback potentiometers. The current design lacks any sort of compensator controller that would provide stability control. Our team must analyze the current configuration and determine the stability.

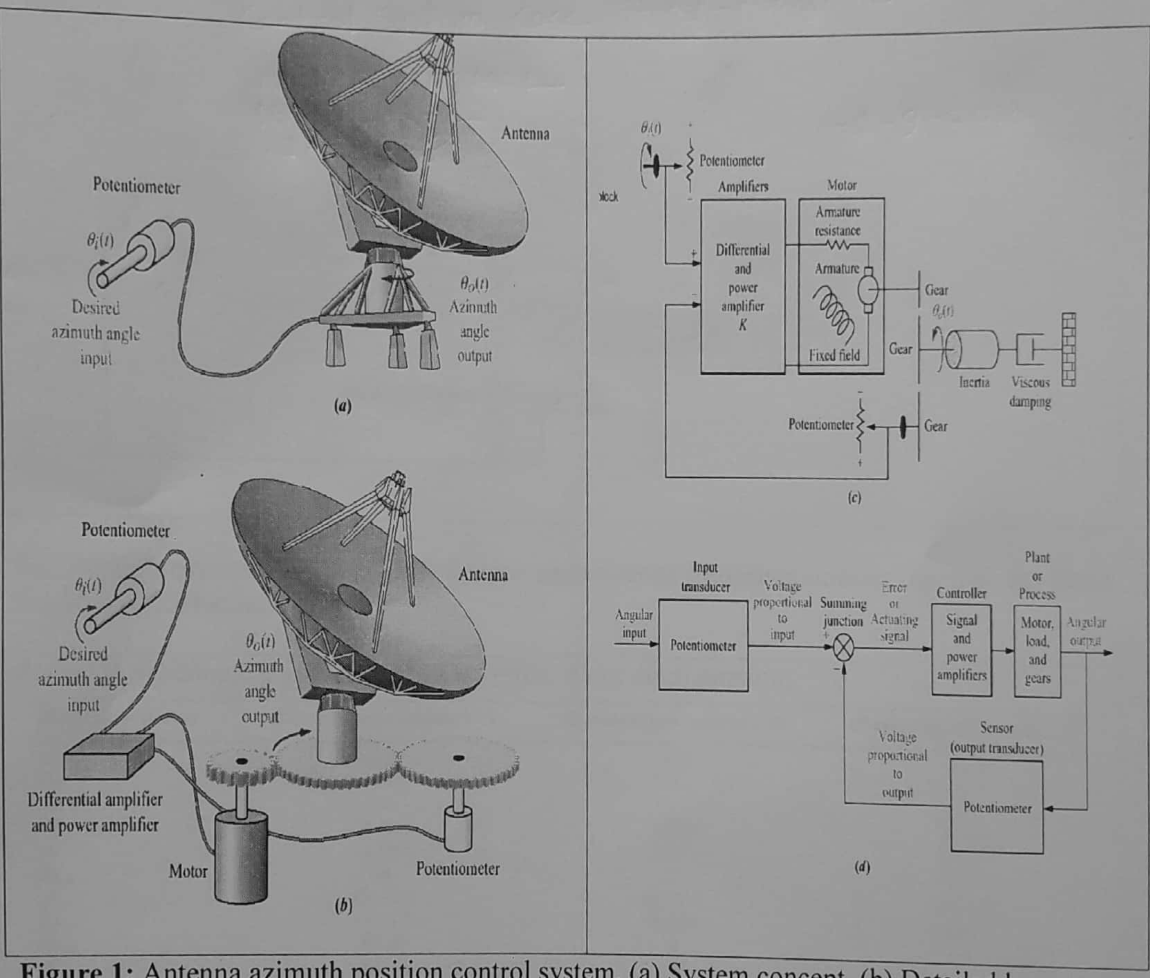


Figure 1: Antenna azimuth position control system. (a) System concept. (b) Detailed layout. (c) Schematic block diagram. (d) Functional block diagram.

Antenna azimuth position control system Control Systems Final Assignment
Assignment # 01

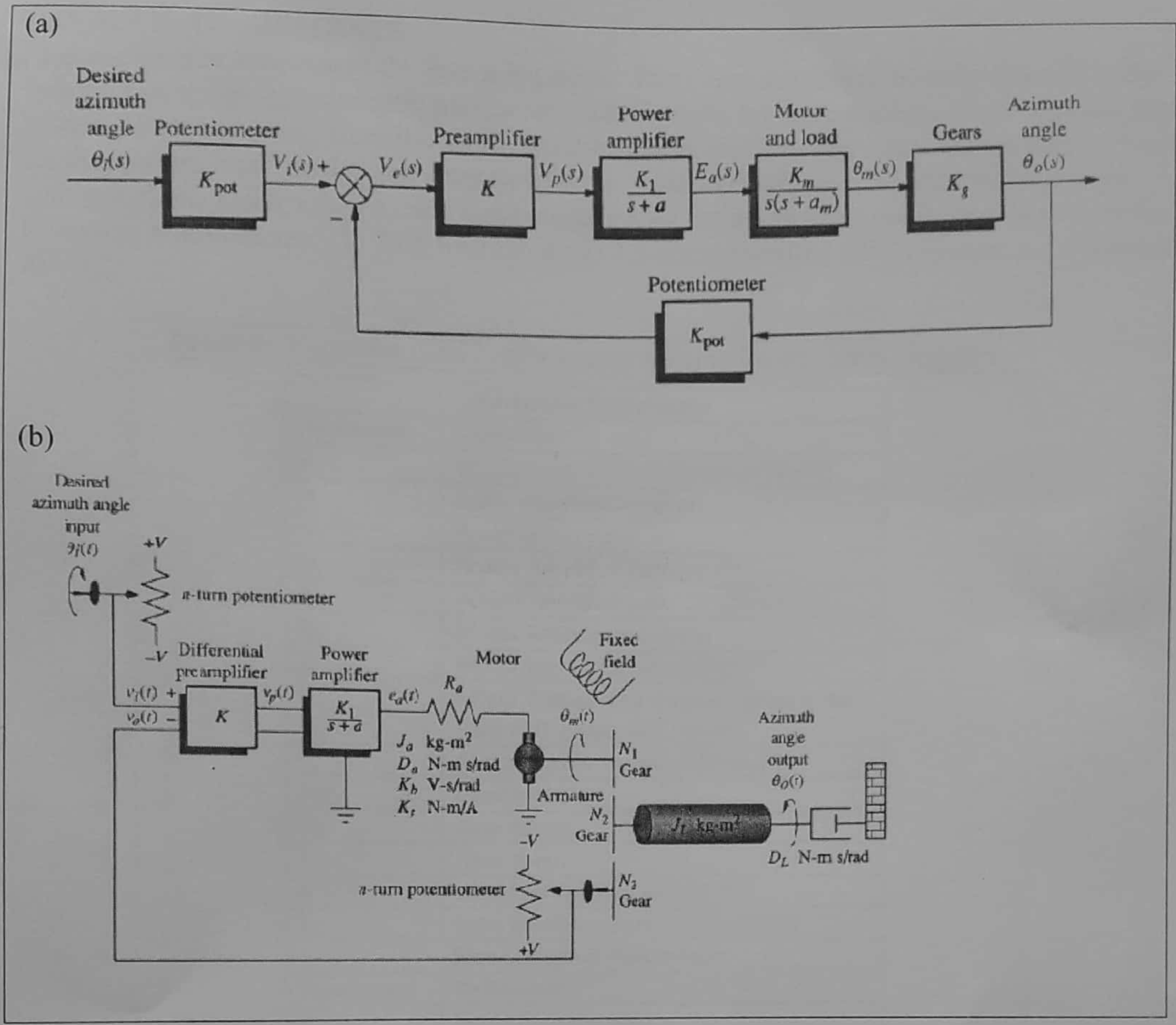


Figure 2: subsystems of the overall system, each with its associated transfer function. (a) Block diagram. (b)Schematic diagram.

Table 1: Schematic parameters for the system (three configuration).

Parameter	Configuration 1	Configuration 2	Configuration 3
V	10	10	10
n	10	1	1
K	—	—	—
K_1	100	150	100
a	100	150	100
R_a	8	5	5
J_a	0.02	0.05	0.05
D_a	0.01	0.01	0.01
K_b	0.5	1	1
K_t	0.5	1	1
N_1	25	50	50
N_2	250	250	250
N_3	250	250	250
J_L	1	5	5
D_L	1	3	3

Antenna azimuth position control system Control Systems Final Assignment
Assignment # 01

DESIGN REQUIREMENTS:

Find the transfer function of the five subsystems. Then, find the overall transfer function of the system. Use the given schematic parameters (See Table 1) for three configurations to model the system. Find the system stability, steady-state response, and time response. Sketch the root locus for the system. Now, design the antenna azimuth position control system for the above three configurations. Design the cascade compensations for the above three configuration to meet the following requirements: (1) 20% overshoot, (2) 1.5-second settling time, (3) static error constant, Kv= 12.

Table 2: Variables used in this system schematics and block diagram.

Schematic Parameters	
Parameter	**Definition**
V	Voltage across Potentiometer [Volts]
N	Turns of potentiometer
K	Preamplifier gain
K_1	Power Amplifier Gain
a	Power Amplifier pole
R_a	Motor Resistance [ohms]
J_a	Motor Inertial constant [kg-m^2]
D_a K_b	Motor Dampening constant [N-m s/rad] Back EMF constant [V-s/rad]
K_t	Motor Torque constant [N-m/A]
N_1	Gear teeth
N_2	Gear teeth
N_3	Gear teeth
J_L	Load inertial constant [kg-m^2]
D_L	Load inertial constant [N-m s/rad]
Block Diagram Parameters	
Parameter	**Definition**
K_{pot}	Potentiometer gain
K	Preamplifier gain
K_1	Power Amplifier gain
a	Power Amplifier pole
K_m	Motor and load gain
a_m	Motor and load pole
K_g	Gear ratio

Input Potentiometer; Output Potentiometer

$$\frac{V_i(s)}{\theta_i(s)} = K_{pot} = \frac{V}{n\pi}$$

For Con. 1, $\frac{10}{10 \times \pi} = 0.318$

For Con. 2, $\frac{10}{1 \times \pi} = 3.183$

For Con. 3, $\frac{10}{1 \times \pi} = 3.183$

Pre-amplifier

$$\frac{V_p(s)}{V_e(s)} = K$$

Power Amplifier

$$\frac{E_a(s)}{V_p(s)} = \frac{K_1}{s+a}$$

For con 1. $\frac{100}{s+100}$

For con 2. $\frac{150}{s+150}$

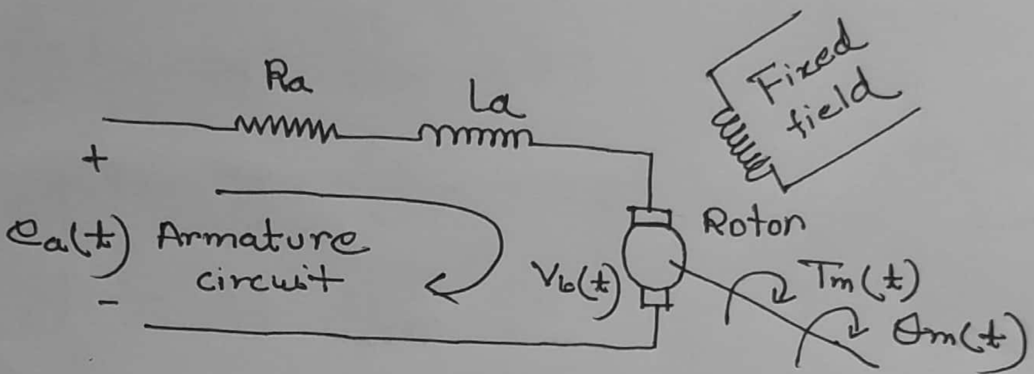
For Con 3. $\frac{100}{s+100}$

Motor and load:

→ Transfer function of motor from armature voltage to armature displacement

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_m}{s(s+a_m)}$$

The power Amplifier is the motor, attached to the gears and Load, which in this case is an antena



Applying KVL,

$$R_a I_a(s) + L_a s I_a(s) + V_b(s) = e_a(s) \quad \text{----} (I)$$

$$T_m(s) = K_t I_a(s) \quad \therefore I_a(s) = \frac{T_m(s)}{K_t} \quad \text{---} (II)$$

$$V_b(s) = K_b s \phi_m(s) \quad \text{----} (III)$$

$$T_m(s) = (Js^2 + D_m s)\, \phi_m(s) \quad \text{---} (IV)$$

Thus replacing the corresponding variables with their equivalents into eq (I) and simplifying creates equation

$$\frac{(Js^2 + D_m s)(R_a + L_a s)\,\phi_m(s)}{K_t} + K_b s \phi_m(s) = E_a(s)$$

$$\left[\frac{(Js^2+D_m s)(R_a+L_a s)+K_b K_t s}{K_t}\right]\phi_m(s)=E_a(s)$$

$$\frac{\phi_m(s)}{E_a(s)}=\frac{\frac{K_t}{JR_a}}{s\left(s+\frac{D_m R_a+K_b K_t}{JR_a}\right)}$$

$$K_g=\frac{N_1}{N_2}$$

Config 1, $K_g=\frac{25}{250}=0.1$,

Config 2, $K_g=\frac{50}{250}=0.2$

Config 3, $K_g=\frac{50}{250}=0.2$

$$J=J_a+J_L(K_g)^2=$$

Config 1, $J=0.02+1\times0.1^2=0.03$

Config 2, $J=0.05+5\times0.2^2=0.25$

Config 3, $J=0.05+5\times0.2^2=0.25$

$$D_m=D_a+D_L(K_g)^2$$

Config 1, $D_m=0.01+1\times0.1^2=0.02$

Config 2, $D_m=0.01+3\times0.2^2=0.13$

Config 3, $D_m=0.01+3\times0.2^2=0.13$

Scanned by CamScanner

$$K_m = \frac{K_t}{J R_a}$$

Config 1, $K_m = \frac{0.5}{0.03 \times 8} = 2.08$

Config 2, $K_m = \frac{1}{0.25 \times 5} = 0.8$

Config 3, $K_m = \frac{1}{0.25 \times 5} = 0.8$

$$a_m = \frac{D_m R_a + K_b K_a}{J R_a}$$

Config 1, $a_m = \frac{0.02 \times 8 + 0.5 \times 0.5}{0.03 \times 8} = 1.71$

Config 2, $a_m = \frac{0.13 \times 5 + 1 * 1}{.25 * 5} = 1.32$

Config 3, $a_m = \frac{0.13 \times 5 + 1 * 1}{.25 * 5} = 1.32$

Block Diagram Parameters

Parameter	Configuration1	config 2	config 3
K_{pot}	0.318	3.183	3.183
K	-	-	-
K_1	100	150	100
a	100	150	100
K_m	2.08	0.8	0.8
a_m	1.71	1.32	1.32
K_g	0.1	0.2	0.2

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_m}{s(s+a_m)}$$

For Config 1 $= \dfrac{2.08}{s(s+1.71)}$

For config 2 $= \dfrac{0.8}{s(s+1.32)}$

For config 3 $= \dfrac{0.8}{s(s+1.32)}$

→ Tranfer function relating load displacement to armature voltage, Motor, load & gear

$$\frac{\theta_o(s)}{E_a(s)} = K_g * \frac{\theta_m(s)}{E_a(s)}$$

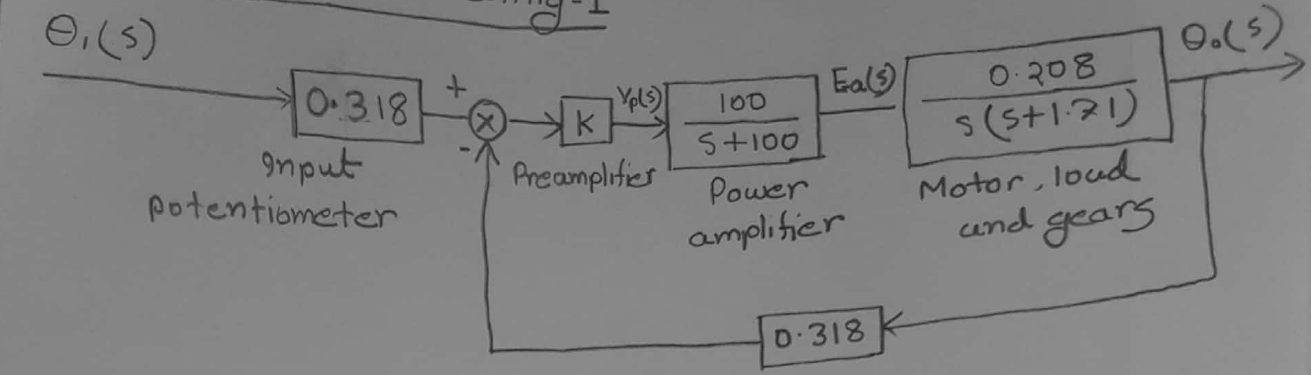
For congfig 1 $= 0.1 * \dfrac{2.08}{s(s+1.71)} = \dfrac{0.208}{s(s+1.71)}$

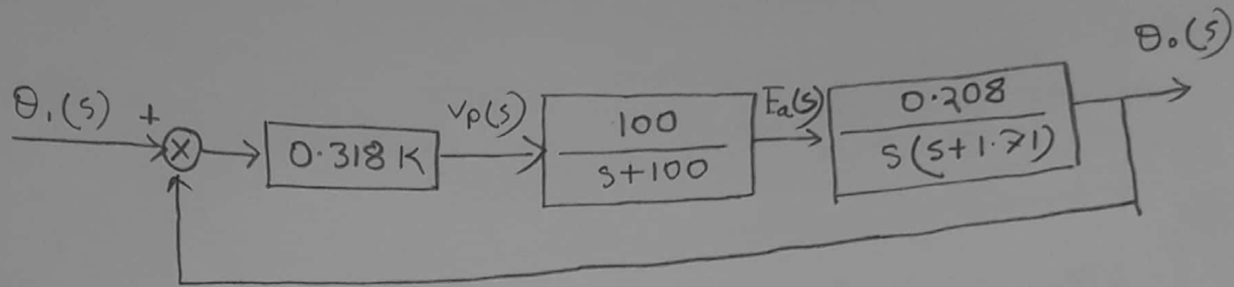
For config 2 $= 0.2 \times \dfrac{0.8}{s(s+1.32)} = \dfrac{0.16}{s(s+1.32)}$

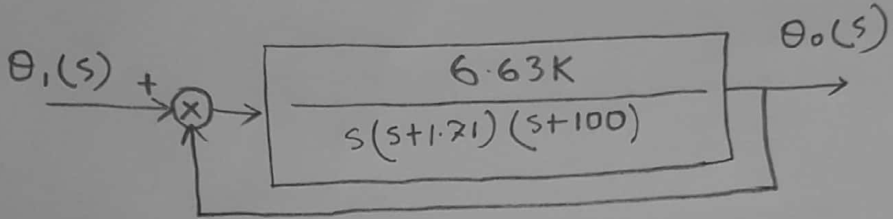
For config 3 $= \dfrac{0.16}{s(s+1.32)}$

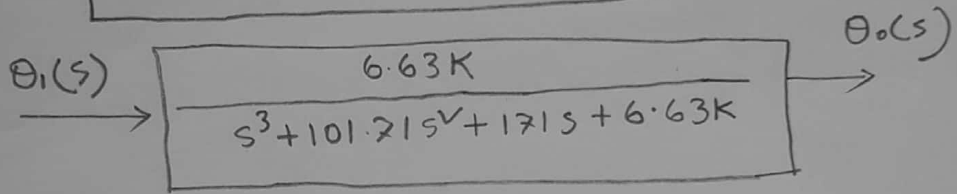
~~Power Amplifiers~~

Transfer function: For config-1

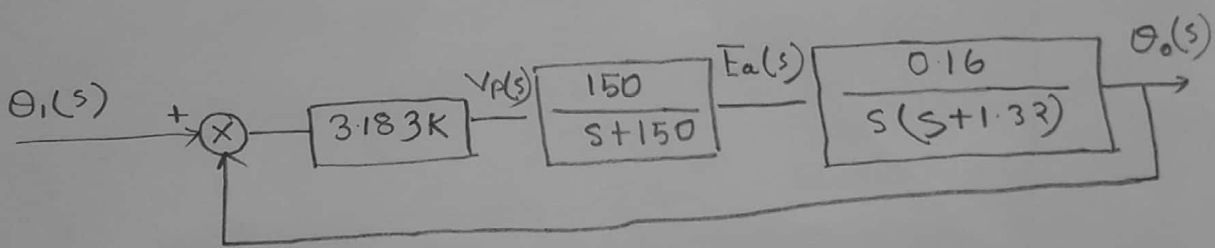


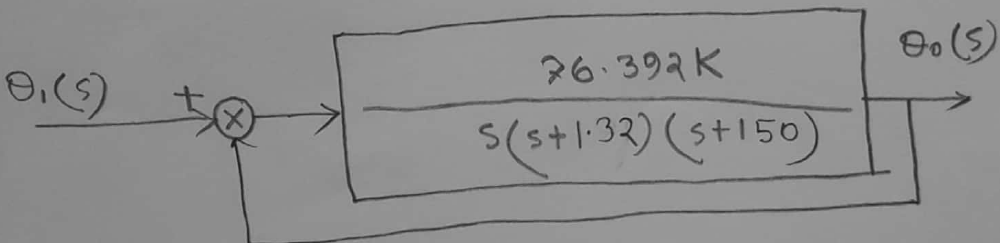






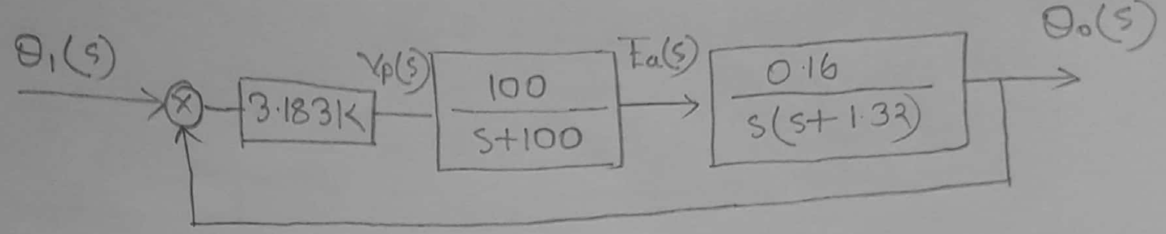
Transfer function: For config-2





$$\theta_i(s) \longrightarrow \boxed{\dfrac{76.392K}{s^3 + 151.32s^2 + 198s + 76.392K}} \longrightarrow \theta_o(s)$$

Transfer function for configuration 3:



$$\theta_i(s) \longrightarrow \boxed{\dfrac{50.928K}{s^3 + 101.32s^2 + 132s + 50.928K}} \longrightarrow \theta_o(s)$$

Stable for config 3:

$$T(s) = \frac{50.928K}{s^3 + 101.32s^2 + 132s + 50.928K}$$

s^3	1	132
s^2	101.32	50.928K
s^1	13374.24 − 50.928K	0
s^0	50.928K	

s^1 $13374.24 > 50.928K$

$\Rightarrow K < 262.611$

s^0 $50.928K > 0$

$\Rightarrow K > 0$

$\therefore \; 0 < K < 262.61$

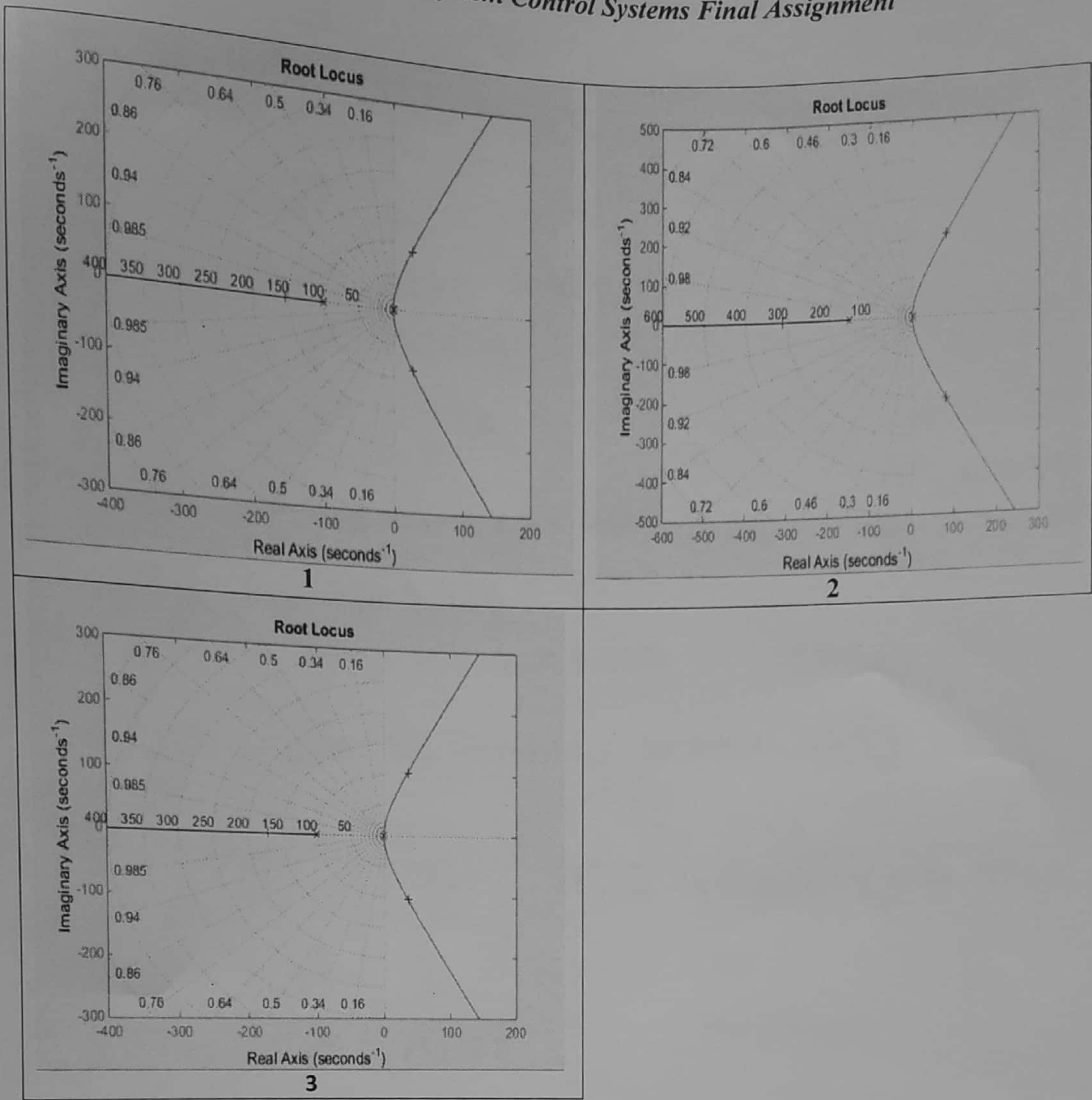


Figure 3. Root locus for configuration 1, 2 and 3.

System stability:

For config 1. $T(s) = \dfrac{6.63K}{s^3 + 101.71s^2 + 171s + 6.63K}$

$$
\begin{array}{lll}
s^3 & 1 & 171 \\
s^2 & 101.71 & 6.63K \\
s^1 & 17392.41 - 6.63K & 0 \\
s^0 & 6.63K &
\end{array}
$$

s^0 $6.63K > 0$

$\Rightarrow K > 0$

s^1 $17392.41 - 6.63K > 0$

$\Rightarrow 6.63K < 17392.41$

$\Rightarrow K < 2623.299$

$\therefore$ System will be stable $0 < K < 2623$

For config 2, $T(s) = \dfrac{76.392K}{s^3 + 151.32s^2 + 198s + 76.392K}$

$$
\begin{array}{lll}
s^3 & 1 & 198 \\
s^2 & 151.32 & 76.392K \\
s^1 & 29961.36 - 76.392K & 0 \\
s^0 & 76.392K &
\end{array}
$$

s^0 $76.392K > 0$

$\Rightarrow K > 0$

s^1 $29961.36 - 76.392K > 0$

$\Rightarrow K < 392.205$

$\therefore$ System will be stable $0 < K < 392.205$

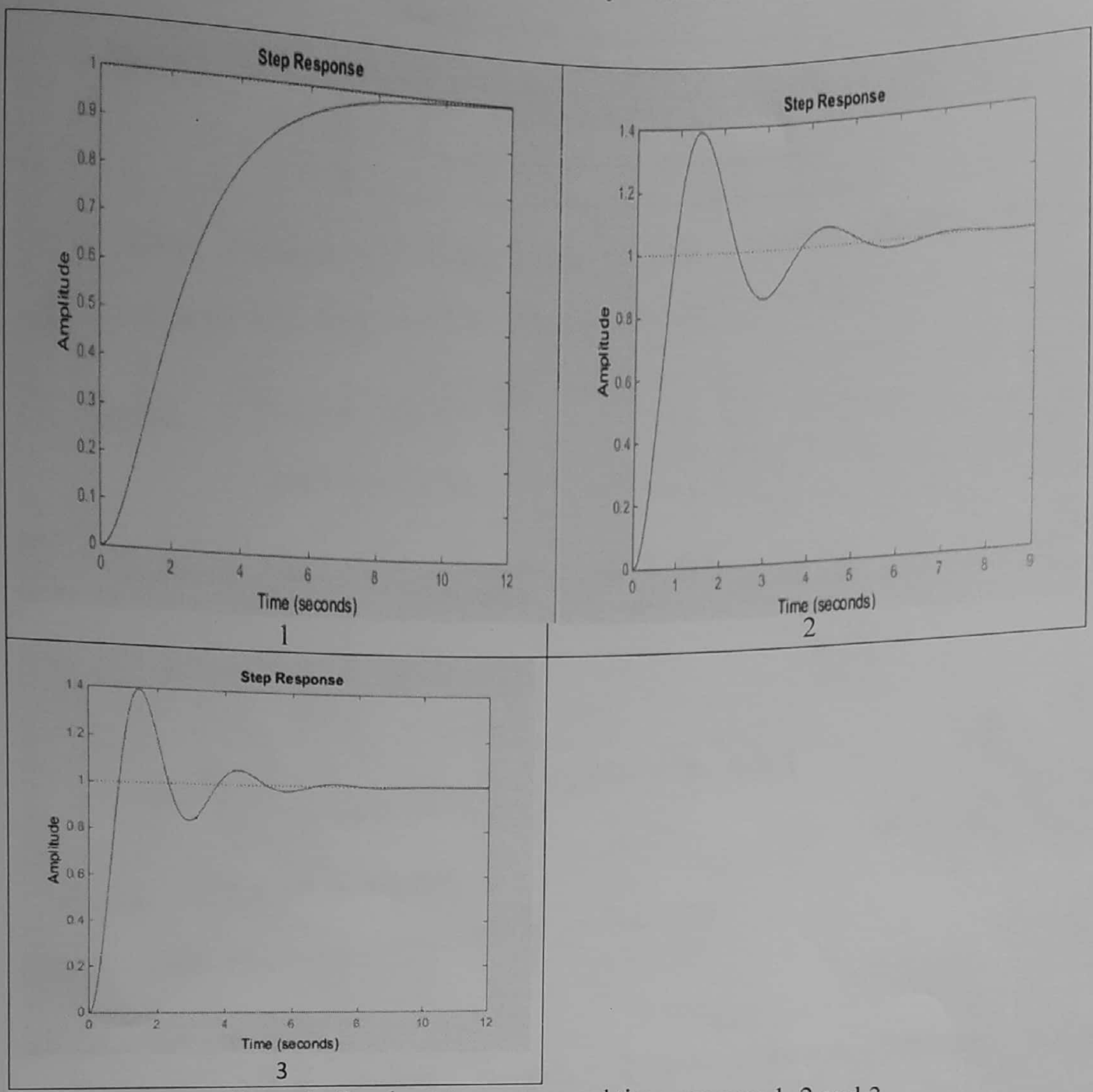
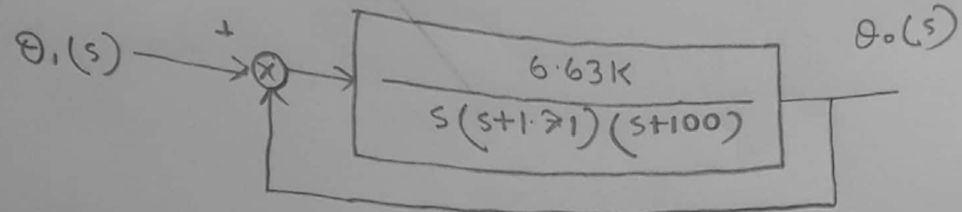


Figure 4 : Steady-state response and time response 1, 2 and 3.

For config 1



At 20% overshoot line $\zeta = 0.407$ and dominant poles at $-0.651 + 1.46j$ with a gain of 4.03.

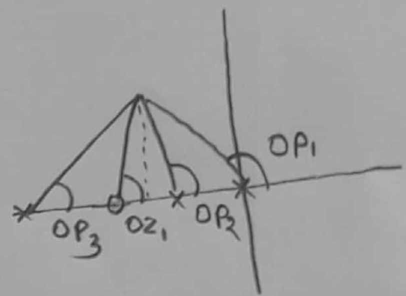
$$S_{d\,old} = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$$

$$= -0.651 + j1.46$$

$$T_{s\,old} = \frac{4}{\zeta\omega_n} = \frac{4}{0.651} = 6.144$$

$$T_{s\,new} = \frac{4}{\zeta\omega_n}$$

$$\Rightarrow 1.5 = \frac{4}{0.407 \times \omega_n} \Rightarrow \omega_n = 6.552$$



$$S_{d\,new} = -2.667 + j5.98$$

Now arbitrarily selecting $-z_c = -5$

Angle criterion,

Σ open loop poles angle $-$ Σ open loop zeros angle $= 180°$

$$\Rightarrow OP_1 + OP_2 + OP_3 + OP_4 - OZ_1 = 180°$$

$$\Rightarrow 180° - \tan^{-1}\left(\frac{5.98}{2.667}\right) + 180° - \tan^{-1}\left(\frac{5.98}{2.667-1.71}\right) + \tan^{-1}\left(\frac{5.98}{100-2.667}\right)$$
$$+ OP_4 - \tan^{-1}\left(\frac{5.98}{5-2.667}\right) = 180°$$

$$\Rightarrow OP_4 = 32.04$$

$$\tan\theta_P = \frac{\omega_d}{P_c - \sigma_d}$$

$$\Rightarrow \tan(32.04) = \frac{5.98}{P_c - 2.667}$$

$$\Rightarrow P_c = 12.222$$

Finally Lead $G_{Lead} = \dfrac{s+5}{s+12.222}$

Again,

Old system $K_{v_o} = \lim_{s \to 0} \dfrac{6.63 \times 4.03}{(s+1.71)(s+100)}$

$$= \frac{6.63 \times 4.03}{1.71 \times 100}$$

$$= 0.156$$

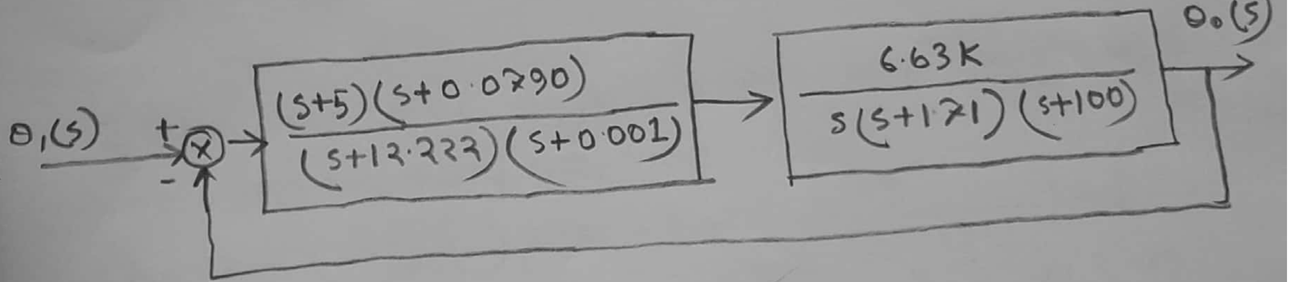
$$\frac{K_{vn}}{K_{vo}} = \frac{z_c}{p_c}$$ [as $-p_c = -0.001$]

$$\Rightarrow \frac{12}{0.152} = \frac{z_c}{0.001}$$

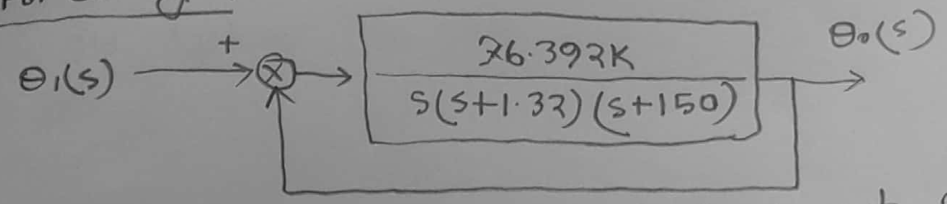
$$\Rightarrow z_c = 0.0790$$

Finally lag, $G_{lag} = \dfrac{s+0.0790}{s+0.001}$

$$G_{LLC}(s) = \frac{(s+5)(s+0.0790)}{(s+12.222)(s+0.001)}$$



For config 2:

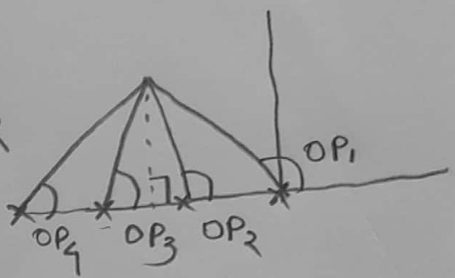


At 20% overshoot $\zeta = 0.563$ and dominant poles at $-0.793 + j2.75$ with a gain of 262.

$$S_{d\,old} = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$$
$$= -0.793 + j2.75$$

$$T_{s\,old} = \frac{4}{\zeta\omega_n} = \frac{4}{0.793} = 5.0445$$

$$T_{s\,new} = \frac{4}{\zeta\omega_n}$$
$$\Rightarrow 1.5 = \frac{4}{0.793 \times \omega_n} \Rightarrow \omega_n = 3.362$$



$$S_{d\,new} = -1.893 + j2.779$$

Now arbitrarily selecting $-z_c = -5$

Angle criterion

$\sum$ open loop poles angle $- \sum$ open loop zeros angle $= 180$

$$\Rightarrow OP_1 + OP_2 + OP_3 + OP_4 - OZ_1 = 180$$
$$\Rightarrow 180 - \tan^{-1}\left(\frac{2.779}{1.893}\right) + 180 - \tan^{-1}\left(\frac{2.779}{1.893-1.32}\right)$$
$$+ OP_4 + \tan^{-1}\left(\frac{2.779}{150-1.893}\right) - \tan^{-1}\left(\frac{2.779}{5-1.893}\right) = 180$$

$$\Rightarrow OP_4 = -83.44$$

$$\tan\theta_p = \frac{\omega_d}{P_c - \sigma_d} \Rightarrow \tan(-83.44) = \frac{2.779}{P_c - 1.893}$$

$$\therefore -P_c = -1.573$$

Finally Lead $G_{Lead} = \frac{s+5}{s+1.573}$

Again

Old system $K_{vo} = \lim_{s \to 0} \dfrac{76.392 \times}{s(s+1.32)(s+150)}$

$$= \frac{76.392 \times 262}{1.32 \times 150}$$

$$= 101.084$$

$$\frac{K_{vN}}{K_{vo}} = \frac{z_c}{p_c} \qquad [\text{If } -p_c = -0.1]$$

$$\Rightarrow \frac{12}{101.084} = \frac{z_c}{0.1}$$

$$\therefore -z_c = -0.0119$$

Finally lag, $G_{lag} = \dfrac{s+0.0119}{s+0.1}$

$$G_{LLC}(s) = \frac{(s+5)(s+0.0119)}{(s+1.573)(s+0.1)}$$