Homework 1: Optics and Image Sensing

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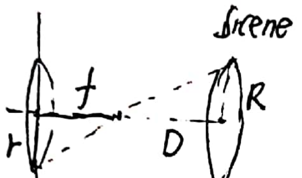
1 Problem I

a) Circle, disk

b) the area of the image: ~~4mm~~^{2} $0.25mm^2$

Proof:



Set R: the radius of the scene circle disk

D: the distance from the pinhole to the disk

f: focal length: the image to the pinhole

According to the content of problem:

$\pi r^2 = 1mm^2$

Similar triangle: and when $D \Rightarrow 2D$

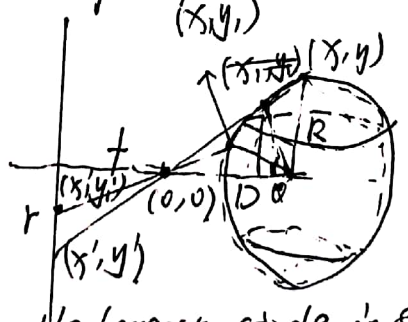
$\frac{f}{D} = \frac{r}{R} \Rightarrow \frac{f}{2D} = \frac{r'}{R} = \frac{r}{2R} \Leftrightarrow r' = \frac{1}{2}r$

$\therefore$ the new area of the image: $\pi r'^2 = \frac{1}{4}\pi r^2 = \frac{1}{4} \times 1 = 0.25\, mm^2$

c) ~~circle~~ circular disk

Set the top of point: (x, y) and counterpart points (x', y')

Set ~~the~~ one point (x_1, y_1) and its counterpart (x_1', y_1')



$\therefore$ set R is the radius of the sphere

D: distant from the center of the sphere to pinhole (0,0)

f: focal length

r: the radius of the image

Similar triangle:

$\frac{y}{y'} = \frac{x}{x'} \Leftrightarrow \frac{R}{\text{points}} = \frac{D}{f}$

$\Rightarrow$ the counterpart of (x, y) which in the largest circle in sphere ~~is also~~ constitutes a circle whose radius: $\frac{Rf}{D}$

- other points ~~not~~ in the circle whose radius less than R

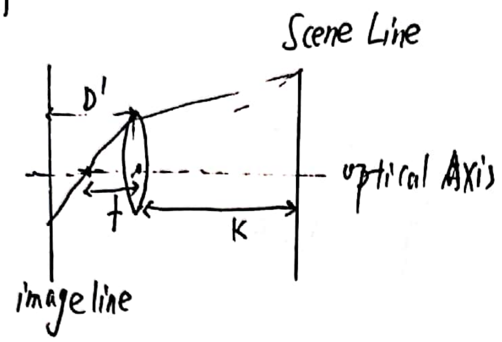
$\frac{R\sin\theta}{r'} = \frac{D - \cos\theta}{f} \quad \therefore r' = \frac{R\sin\theta \cdot f}{D - \cos\theta}$

$\therefore$ these points also constitute a circle

$\therefore$ the ~~image~~ shape of image will be a circular disk, which could be seen as composed of lots of circle.

2 Problem 2

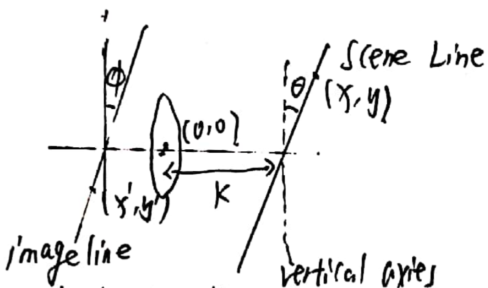
a) the answer: $\dfrac{Df}{D-f}$



∵ Thin lens formula

$$\frac{1}{D'}+\frac{1}{D}=\frac{1}{f} \Leftrightarrow D'=\frac{Df}{D-f}$$

b)



claim: image line of the scene line is still a line, but one that is tilted

proof: Set the center of lens: (0,0)
set one point in scene line: (x, y)
Set one point in image line: (x', y'): (x, y)'s counterpart
Set the equation of scene Line: $y=kx+b$ where $k=\cot\theta$, $b=-k\cdot K$

by similar triangle and thin lens formula

$$\begin{cases} \dfrac{x}{x'}=\dfrac{y}{y'} \\ \dfrac{1}{x}+\dfrac{1}{-x'}=\dfrac{1}{f} \\ y=kx+b \end{cases} \Rightarrow \begin{cases} \dfrac{x}{x'}=\dfrac{kx+b}{y'} \\ \dfrac{1}{x}=\dfrac{1}{f}+\dfrac{1}{x'} \end{cases} \Rightarrow \begin{cases} x=\dfrac{x'b}{y'-kx'} \\ x=\dfrac{x'f}{x'+f} \end{cases}$$

$$\Rightarrow \frac{x'b}{y'-kx'}=\frac{x'f}{x'+f} \Rightarrow y'=\left(k+\frac{b}{f}\right)x'+b$$

∴ $\cancel{y'=k(1-\frac{k}{f})x'+b}$

∵ $\cancel{k \neq f}$

∴ ~~(x', y')~~ the image line is a line and is titled.

c) According to the content above: prove $\tan(\phi) = \frac{f}{k-f}\tan(\theta)$

$$y' = \left(k + \frac{b}{f}\right)x' + b$$

$$\cot(\phi) = k + \frac{b}{f} = \frac{-k \cdot K}{f} + k = k\left(1 - \frac{K}{f}\right)$$

$$= k\left(\frac{f-k}{f}\right)$$

$$= \cot\theta \cdot \frac{f-k}{f}$$

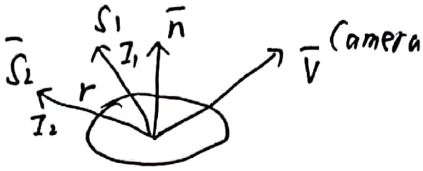
$\because \cot(\phi) = \frac{1}{\tan(\phi)}$

$\cot(\theta) = \frac{1}{\tan\theta}$

$\Rightarrow \tan(\phi) = \frac{k}{k-f} \cancel{\cot\theta} \cdot \tan\theta$

3 problem3

a)



set distance : r

set equal Intensity: I

∵ Lambertian BRDF

$f(\theta_i,\phi_i,\theta_r,\phi_r)=\frac{\rho_d}{\pi} \quad 0\le\rho_d\le 1$

$= \frac{L(\theta_r,\phi_r)}{E(\theta_i,\phi_i)}$

$$\Rightarrow \begin{cases} L=\frac{\rho_d}{\pi}E \\ E=\frac{I\cos\theta_i}{r^2}=\frac{I(\bar n\cdot\bar s)}{r^2}\end{cases}$$

$\Rightarrow L=\frac{\rho_d}{\pi}\frac{I}{r^2}(\bar n\cdot\bar s)$

set single direction S_3, Intensity I_3

$L_3=\frac{\rho_d}{\pi}\frac{I_3}{r_3^2}(\bar n\cdot\bar S_3)$

equally with $L_1+L_2=\frac{\rho_d}{\pi}\frac{I_2}{r_2^2}\cdot(\bar n\cdot\bar S_2)+\frac{\rho_d}{\pi}\frac{I_1}{r_1^2}(\bar n\cdot\bar S_1)$

∵ $r_1^2=r_2^2=r$ & $I_1=I_2=I$

$\Rightarrow L_3=L_1+L_2$

$\Rightarrow \frac{\rho_d}{\pi}\frac{I_3}{r_3^2}(\bar n\cdot\bar S_3)=\frac{\rho_d}{\pi}\left(\frac{I}{r^2}\cdot(\bar n\bar S_2+\bar n\bar S_1)\right)$

$\Rightarrow \frac{I_3}{r_3^2}(\bar n\cdot\bar S_3)=\frac{I}{r^2}(\bar n\bar S_2+\bar n\bar S_1)$

$\Rightarrow \bar S_3=\frac{I r_3^2}{r^2 I_3}(\bar S_2+\bar S_1)$, $I_3=\frac{I r_3^2\bar n(\bar S_2+\bar S_1)}{r^2\bar n\bar S_3}$

b) According to above content ~~based~~

$L_3=L_1+L_2$

$\frac{I_3}{r_3^2}(\bar n\cdot\bar S_3)=\frac{I_1}{r_1^2}\bar n\cdot\bar S_1+\frac{I_2}{r_2^2}\bar n\cdot\bar S_2$ ∵ $r_1=r_2=r$

$\bar S_3=\frac{(I_1\bar S_1+I_2\bar S_2)\cdot r_3}{r^2\cdot I_3}$, $I_3=\frac{r_3^2\cdot(I_1\bar n\bar S_1+I_2\bar n\bar S_2)}{r^2\bar n\bar S_3}$