Applications of Secure Multiparty Computation: Robotics as a Case Study

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by

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CERTIFICATE

It is certified that the work contained in this thesis, titled "Applications of Secure Multiparty Computation: Robotics as a Case Study" by Sarat Chandra Addepalli, has been carried out under my supervision and is not submitted elsewhere for a degree.

Date	Advisor: Dr. K. Srinathan

To My Parents and Brother

Acknowledgments

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Abstract

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Chapter 1

Introduction

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1.1 First Section

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1.2 Second Section

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Few suggestions

Method	Frobnability
Theirs	Frumpy
Yours	Frobbly
Ours	Makes one's heart Frob

Table 1.1: Results. Ours is better.

1.2.1 Mathematics

Please number all of your sections and displayed equations. It is important for readers to be able to refer to any particular equation. Just because you didn't refer to it in the text doesn't mean some future reader might not need to refer to it. It is cumbersome to have to use circumlocutions like "the equation second from the top of page 3 column 1". (Note that the ruler will not be present in the final copy, so is not an alternative to equation numbers). All authors will benefit from reading Mermin's description of how to write mathematics (see math.pdf).

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List and number all bibliographical references in 9-point Times, single-spaced, at the end of your paper. When referenced in the text, enclose the citation number in square brackets, for example [2]. Where appropriate, include the name(s) of editors of referenced books.

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Referring to [1], we state that so and so.

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[1] This is what a footnote looks like. It often distracts the reader from the main flow of the argument.

Chapter 2

Secure Multiparty Computation and its Primitives

2.1 SMPC Primitives

2.1.1 Oblivious Transfer

Oblivious transfer is a type of protocol in which a sender sends a potential subset of messages to the receiver but is oblivious as to whether which ones (if any) were received.

Michael Rabin [4] introduced the first kind of oblivious transfer protocol, in which the sender sends a message with probability $\frac{1}{2}$, but is oblivious whether the receiver received it or not. A more useful form of this protocol called the *1-2 Oblivious Transfer* was developed by Shimon Even, Oded Goldreich and Abraham Lempel [3]. This protocol addresses the following problem: the sender has two messages m_0 and m_1, and the receiver wants one of the messages m_b, but the sender needs to remain oblivious about b, and the reciever needs to be oblivious about the value of $m_{\bar{b}}$.

In the algorithm **??**, we show one possible implementation of *1-2 Oblivious Transfer* using the RSA cryptosystem [5], but in general, the oblivious transfer protocol can be implemented using any "trap-door" functions, such as one-way-functions and public key cryptosystems, or even Shamir's secret sharing [7].

2.1.2 Shamir's Secret Sharing

This SMPC primitive addresses the following problem: suppose a group of treasure hunters would like to lock a safe in such a way that it can't be opened unless there are atleast five (say) of them present at any given time. How many locks and keys would be required for this?

In [6], Shamir proposes a way of sharing a secret among n players, such that any k or more players can reconstruct the secret, but no set of $k-1$ or less players can do so. This is called a (k, n) *secret sharing scheme*, and is achieved by using $k-1$ degree polynomials as described follows:

The player who wishes to share a secret first chooses a $k-1$ degree secret random polynomial (by choosing the $k-1$ coefficients r_1 to r_k), say $f(x)$, and sets the constant term to the value of the secret. He then calculates the value of the "share" to be sent to each player i, as $f(i)$. With this, it is ensured

Require: A has two messages, m_0, m_1, and wants to send exactly one of them to B, but does not want to know which B receives.

A generates a RSA key pair, comprising the modulus N, the public exponent e and the private exponent d

A also generates two random values, x_0, x_1 and sends them to B along with the public modulus and exponent.

B picks b to be either 0 or 1, and selects either the first or second x_b.

B generates a random value k and blinds x_b by computing $v = (x_b + k^e) \mod N$, which he sends to A.

A doesn't know which of x_0 and x_1 B chose, so she attempts to unblind with both of her random messages and comes up with two possible values for k: $k_0 = (v - x_0)^d \mod N$ and $k_1 = (v - x_1)^d \mod N$. One of these will be equal to k since it will correctly decrypt, while the other will produce another random value that does not reveal any information about k.

A blinds the two secret messages with each of the possible keys, $m'_0 = m_0 + k_0$ and $m'_1 = m_1 + k_1$, and sends them both to B.

B knows which of the two messages can be unblinded with k, so he is able to compute exactly one of the messages $m_b = m'_b - k$

Ensure: each player i has a share v_i of the secret v

Algorithm 1: On 1-2 oblivious transfer

Require: A player has a secret value v which he has to share

select a random number r

$f(x) = v + r_1 x + r_2 x^2 + \ldots + r_{k-1} x^{k-1}$

for all players i **do**

send the value $v_i = f(i) = v + r_1 i + r_2 i^2 + \ldots + r_{k-1} i^{k-1}$ to player i

end for

Ensure: each player i has a share v_i of the secret v

Algorithm 2: On sharing a secret

that each player has a "share" of the secret, which he may reconstruct if and only if atleast $k-1$ other players are willing to do so.

Notice, that a $k-1$ degree polynomial's equation can be reconstructed with the knowledge of any k points on the curve (as in the case of any k players colluding), but any set of $k-1$ or less points will yield no information about the equation of the curve (which means that any set of $k-1$ players or less will not be able to reconstruct the secret!), and thus the objective is achieved.

2.1.2.1 Secret Addition

Armed with the subroutines for secret sharing and reconstruction, we can do additional things with it. Notice that the "share"s are but numerical values, infact plot points on a polynomial curve. As with any polynomial, we can add, multiply, and do other arithmetic operations, but what does adding polynomials mean, in this sense?

What we mean by doing arithmetic operations on the polynomial is the following: Supposing two secrets a and b were shared by two different players according to their choice of random private polynomials $f_a(x)$ and $f_b(x)$. Let the shares of the i th player be a_i and b_i respectively, of the two secrets. Now if the each of the individual players were to add their own shares, this would result in each of them creating c_i, say, where $c_i = a_i + b_i$. But these individual shares would be plot points on the polynomial curve $f_c(x) = f_a(x) + f_b(x)$, which means that if the reconstruction subroutine were to be run on these new shares, it would result in the reconstruction of the value $a + b$! And similar to the case of sharing a secret, even when the shares are added, the privacy of the secret still remains intact, that is, all the players (except those two who shared a and b) still have no information about the value of the individual secrets.

Require: n is the number of players, each with private input s_i.
  {Phase 1: Sharing}
  for each player i **do**
    Share secret s_i
    for each player j **do**
      send j's share s_{ij} to player j
    end for
  end for
  for each player j **do**
    $sum_j = \sum_{i=1}^{n} s_{ij}$
  end for
Ensure: each player is left with sum_j, the share of the sum of the private inputs s_i.

Algorithm 3: Computing Secure Addition

This process is illustrated in the algorithm 3, where each of the individual players share their private secrets, and add the individual shares. This results in the addition of each of the secrets, so that at the end of this routine, the group of individual players as a whole are each left with a share of the total sum of the private inputs of all of the players.

2.1.2.2 Secret Multiplication

2.1.3 Privacy Preserving Union

Chapter 3

Robotics and its Primitives

3.1 Introduction to Robotics

3.2 Problems in Robotics

3.3 Localization

3.4 Global Localization

Chapter 4

A Framework for Secure Localization

Chapter 4 goes here ...

Chapter 5

Conclusions

Conclusion goes here ....

Related Publications

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