An Energy Approach to Angular Acceleration in a Flywheel-Mass System

Abstract:

A fundamental law of physics, the conservation of energy, is utilized in this experiment to predict the angular acceleration of a rotating disc. Theoretical angular acceleration calculated through power and rate of change in kinetic energy is compared with measured values recorded through video analysis. Measured angular acceleration values had adhered to expectations, with slight deviation within uncertainty range, validifying the theoretical calculation approach. The major sources of error in this experiment were systematic error due to friction, which was mathematically removed, and random error due to instrumental uncertainty, which was included in uncertainty calculations. As measured values after consideration of friction were consistent with values predicted and there was a strong correlation in the observed data, the effect of other errors is likely minimal and would not invalidate our conclusion.

Purpose:

The primary objective of this experiment is to predict and compare the theoretical and measured angular acceleration of a rotating mass. The theoretical acceleration was calculated through power and rate of change of kinetic energy. The measured angular acceleration was derived through curve fit on video analysis.

Hypothesis:

Based on the principle of conservation of energy, the total energy between gravitational potential energy, kinetic energy, and frictional energy lost should be constant, with kinetic energy change rate of total system power. The predicted angular acceleration is calculated to be $[-19.58 rad/s^2 \pm 0.3221 rad/s^2]$ with supplemental acceleration loss due to friction.

Method:

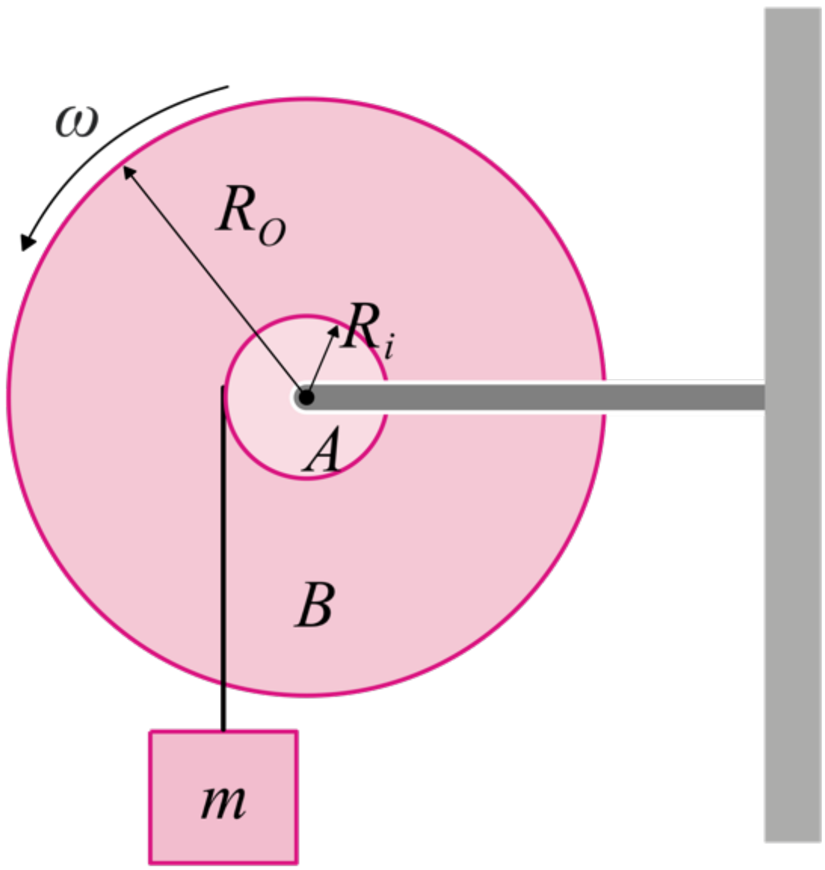

Fig.1 [Outer disc of mass ***B*** ($1.860 \times 10^{-2} kg \pm 5 \times 10^{-5} kg$), radius $\boldsymbol{R_O}$ ($1.270 \times 10^{-1} m \pm 5 \times 10^{-5} m$) is attached to an axle of mass ***A*** $7.400 \times 10^{-3} kg \pm 5 \times 10^{-5} kg$), radius $\boldsymbol{R_i}$ ($3.000 \times 10^{-3} m \pm 5 \times 10^{-5} m$), which has a rope wrapped around it with mass ***m*** attached($1.004 \times 10^{-1} kg \pm 5 \times 10^{-5} kg$)(4 sigfigs)]

Experiment was set up as shown in *Fig.1*, with a camera placed in such position to eliminate parallax error and accurately record rotational motion. Mass ***m*** was released, resulting in combined rotation between mass A and B of ω with angular acceleration of α or $\dot{\omega}$. The motion of the disc during and after the rope unwinds around R_i was recorded in tracker and analyzed through custom SciPy numerical analysis.

Results:

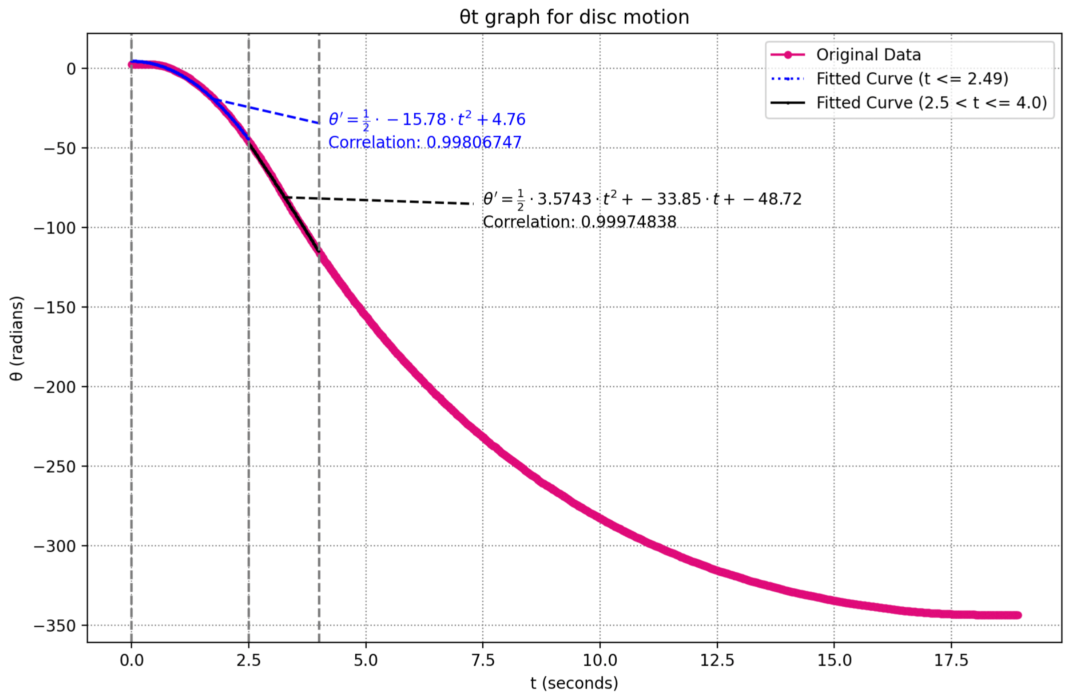

Fig.2 (Total rotational angle of disc graphed against time analyzed in matplotlib and SciPy; time segments for during and after mass fall are $t \leq 2.49s$ and $2.5s < t \leq 4.0s$ respectively; angular acceleration for mass fall and deceleration of friction are $-15.78 rad/s^2$ and $3.574 rad/s^2$ respectively; correlation values of curve-fit are $r = 0.9981$ and $r = 0.9997$ respectively)

Discussion:

As depicted in *Fig.2*, disc is accelerating counter clockwise during mass fall with value of $-15.78\ rad/s^2$ and decelerating after the mass has fully left the object due to friction, of value $3.574 rad/s^2$. Taking into consideration of friction's effect, true measured angular acceleration due to mass is $-19.35 rad/s^2$. This value is only a $0.2219 rad/s^2$ deviation from the predicted value of $-19.58 rad/s^2$. The uncertainty of the predicted value was calculated to be $\pm 0.3221 rad/s^2$ due to the random error in measuring the axle radius (R_i). Taking into consideration of this uncertainty shows that the measured angular acceleration falls within the uncertainty range, suggesting that experimental values highly align with our predictions. In our analyzation done through SciPy, coefficient for curve fits were

$r = 0.9981$ and $r = 0.9997$ (*Fig.2*), both of which are extremely close to 1, validating the accuracy of the graphically derived acceleration.

The most significant source of error was acceleration loss due to friction. However, this error was eliminated mathematically by finding its precise impact in $2.5 < t \leq 4.0$ (*Fig. 2*), resulting in an adjustment of the predicted value. This adjustment reconciled for this systematic error and allowed for experimental values to correctly align with theoretical values. Other sources of error may include: random error due to measurement uncertainty and video analysis inaccuracies, and systematic errors due to inconsistency in disc and rod density. However, due to the minimal difference between the predicted and measured acceleration and strong correlation values, the effect of these errors is likely insignificant, and should not compromise the validity of our conclusion.

Conclusion:

The measured angular acceleration adhered to expectations, with a deviation within uncertainty ranges. The strong correlation between the experimental formula that provided us with the acceleration and the measured points further supported the validity of the measured acceleration, which strongly supports the theoretical values calculated. Further experiments could seek to explore how changing the moment of inertia but maintaining the mass affects the experiment or attempt to calculate the instantaneous acceleration for a non-uniform object.