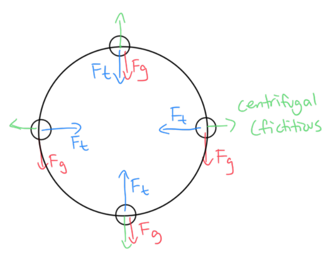
Remember Radians

forces

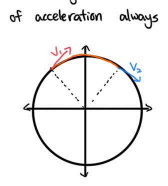
Centripetal Motion



Coriolis force & centrifugal force are both fictitious forces

it should also be known that $F_{net} = m a_{cm}$ (centre of mass)

first things first, centripetal acceleration. this is a type of acceleration always directed at the center of a circle.



is calculated by len $= 2\pi r\left(\frac{\theta}{360}\right)$ × circumference $(2\pi r)$

$a = \frac{v^2}{R}$

$\vec{a}_c = r\omega^2$ (radians)

$V = r\omega$

$\omega = \theta/s$ or $\dot{\theta}$ (w/ in radians)

$\omega = \omega_0 + \alpha t$

$\dot{\omega} = \omega_0 + \alpha t$

$\dot{\omega} = \alpha$

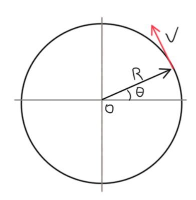
$\ddot{\theta} = \alpha$

$\vec{a} = \frac{4\pi^2 r}{T^2}$ T = time taken for one rotation

$\vec{a}$ is always towards the center

$F_{net} = m\vec{a} = m\frac{v^2}{r}$

one full rotation is $\frac{2\pi}{\omega}$



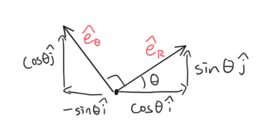
$\vec{r} = R\cos(\omega t)\hat{i} + R\sin(\omega t)\hat{j}$

$\vec{r} = \begin{bmatrix} R\cos(\omega t) \\ R\sin(\omega t) \end{bmatrix} = R\begin{bmatrix} \cos(\omega t) \\ \sin(\omega t) \end{bmatrix}$

$\hat{i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\hat{j} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

now for

$\hat{e}_R$, or $\hat{r}$ $\quad\hat{e}_\theta$, or $\hat{\theta}$



the position vector $\vec{R} = R\hat{e}_R$ the unit vector can be thought of as the direction, thus $\underbrace{R}_{\text{magnitude}}\underbrace{\hat{R}}_{\text{direction}}$

So, remember $V = r\omega$? this is scalar and we need the direction

thus, $\vec{V} = R\dot{\theta}\hat{e}_\theta$ or $\vec{V} = r\omega\hat{\theta}$

$\dot{\theta}^2 = \dot{\theta}_0^2 + 2\ddot{\theta}\theta$

$\omega^2 = \omega_0^2 + 2\alpha\theta$

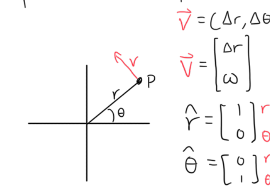
$\vec{a}_c = -\frac{V^2}{R}\hat{e}_R + \dot{V}\hat{e}_\theta$

btw, α commonly means proportionate, which is why we use $\ddot{\theta}$

$\vec{a}_c = \begin{bmatrix} -\frac{V^2}{R} \\ \dot{V} \end{bmatrix}$ force $\hat{\theta}$

also kinda funny, all of big five work here, just as a side note

polar



$P = (r, \theta)$

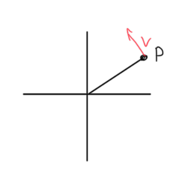
$\vec{V} = (\Delta r, \Delta\theta)$

$\vec{V} = \begin{bmatrix} \Delta r \\ \omega \end{bmatrix}$

$\hat{r} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{matrix} r \\ \theta \end{matrix}$

$\hat{\theta} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{matrix} r \\ \theta \end{matrix}$

regular

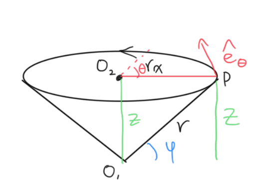


$P = (x, y)$

$\vec{V} = \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$

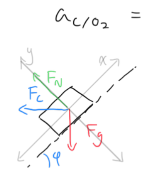
$\hat{i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{matrix} x \\ y \end{matrix}$

$\hat{j} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{matrix} x \\ y \end{matrix}$



$a_{c/O_2} = \frac{(V\hat{e}_\theta)^2}{r_x}$ for object on point P

applying a translation factor of φ



if friction existed, it would have a size of

$F_g\cos(\varphi)\mu$

$F_{gx} = F_g\sin(\varphi)$

$F_{cx} = F_c\cos(\varphi)$

Significant formulas / tricks 🙈

$F_{net} = m\vec{a}$ Actually this is more common mistakes

→ Both $\vec{F}$ and $\vec{a}$ have angles to it as they are both vectors.

action reaction forces never act on the same object.

For circular motion, there are two types of references

→ Inertial reference, where law of inertia holds true.

the camera's ΣF is the same

→ non-inertial reference, where the camera's position is relative to the object in question.

```
object Camera(){
  Vector2 position
  Vector2 velocity
  object reference
  Vector2 acceleration
}
```

the fictitious force is only apparent, or taken into consideration for certain reference points. { important }

F_2, should be labelled as non-apparent to current frame of reference.

There is no special formatting for the subscript here.

Relevant information & Basic guide

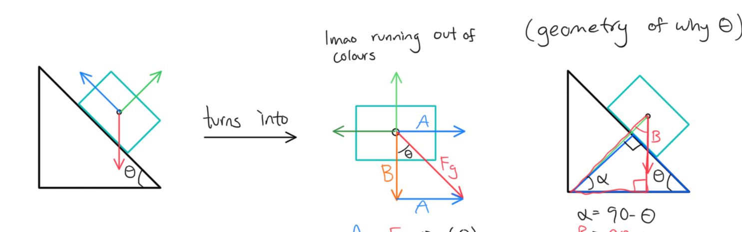
Starting from scratch, the most important thing to know is

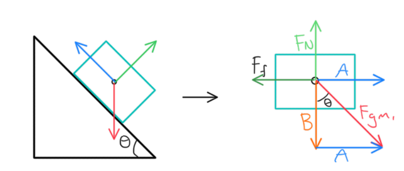
$F_{net} = m\vec{a}$ $\quad F_{net}$ means the sum of all forces involved in the system. $F_{net} = \sum\vec{F} = F_1\begin{bmatrix} F_{1x} \\ F_{1y} \end{bmatrix} + F_2\begin{bmatrix} F_{2x} \\ F_{2y} \end{bmatrix} \ldots + F_n\begin{bmatrix} F_{nx} \\ F_{ny} \end{bmatrix}$

$F_{net} = \begin{bmatrix} \Sigma F_x \\ \Sigma F_y \end{bmatrix}$

On right formulas are provided, so knowing what each character means and how to apply them is more important.

k, to the basics. We start with a free body diagram. {FBD}





$F_{friction}$, or F_f for short

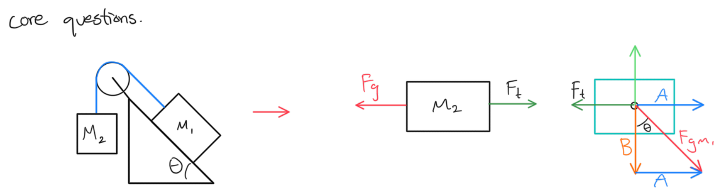
$= F_N\mu_f$

$= \cos(\theta)(M_1 g)\mu_f$

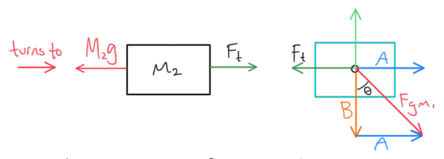
$F_{net} = ma$, $a = \frac{\sin(\theta)(M_1 g) - \cos(\theta)(M_1 g)\mu_f}{M} = g(\sin(\theta) - \cos(\theta)\mu_f)$

This question has made me realize there isn't enough space on a FBD for all the values. Thus, I will label them Alphabetically.

After reviewing the example problems there isn't many difficult AF questions here, and they can mostly be simplified to a few core questions.

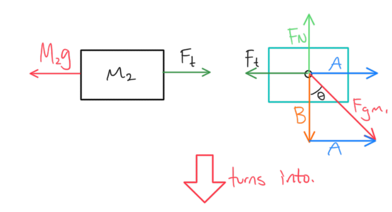


$F_{net} = m\vec{a}$ $\quad F_{net} = \sum F \xrightarrow{\text{turns to}}$

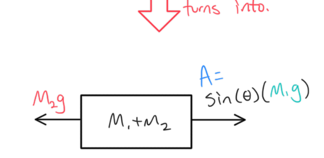


Now! Very important on where to place the friction force, because you don't know if the force from M_1 or M_2's gravity is strong enough, and in which direction.

first step is to find the force without friction, and which direction in a frictionless system:



F_N and B cancel each other here



$F_{friction}$, or F_f for short

$= F_N\mu_f$

$= \cos(\theta)(M_1 g)\mu_f$

if $|M_2g - \sin(\theta)(M_1g)| > \cos(\theta)(M_1g)\mu_f$ then the system actually moves. M_2g and $\sin(\theta)(M_1g)$ determine which direction the system is moving in.

Assuming M_2g is bigger: $F_{net} = M_2g - \underbrace{(\sin(\theta)(M_1g) + \cos(\theta)(M_1g)\mu_f)}_{\text{A, but with friction}}$

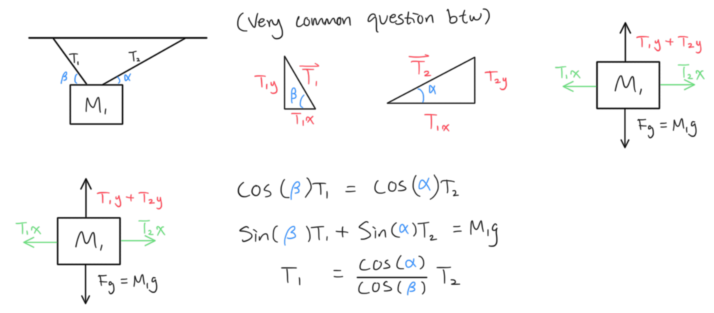
$F_{net} = ma$, $a = \frac{M_2g - \overbrace{(\sin(\theta)(M_1g) + \cos(\theta)(M_1g)\mu_f)}^{\text{A, but with friction}}}{M_1 + M_2}$

Moving on...

Although force of tension has always been a difficult concept for me to understand.

We understand tension through a question.

(Very common question btw)



$\cos(\beta)T_1 = \cos(\alpha)T_2$

$\sin(\beta)T_1 + \sin(\alpha)T_2 = M_1g$

$T_1 = \frac{\cos(\alpha)}{\cos(\beta)}T_2$

$\frac{\sin(\beta)\cos(\alpha)}{\cos(\beta)}T_2 + \sin(\alpha)T_2 = M_1g$

$T_2(\cos(\alpha)\tan(\beta) + \sin(\alpha)) = M_1g$

grade 11 review.

Newton's laws in 2D!

Just as a reminder, the laws are

an object will stay at rest or uniform motion unless acted upon by an unbalanced external force

$F_{net} = m\vec{a}$ the sum of all forces on an object is mass × acceleration. In uniform motion, $F_{net} = 0$

For every action force there is an equal and opposite reaction force.

So for the normal force and friction, the normal force is the force you get when an object is pressed into a surface. also known as 支撐力; perpendicular to surface

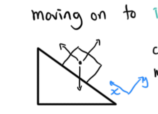
for friction, there is static friction vs kinetic friction. the coefficient of friction is the ratio between the normal force and the force of friction.

μ_s → static $\quad\mu_k$ → in motion

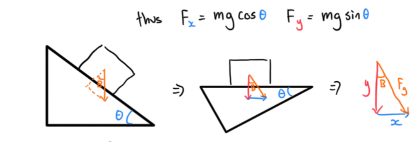
the coefficient of friction, f_r is calculated by $\frac{F_r}{F_N}$

and, friction can be calculated from F_{net} $\frac{\text{friction}}{\text{Normal}}$

moving on to inclined planes, this question is rather common. this can be solved with a special method, which is rotating the coordinate system.

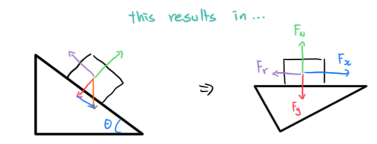


thus $F_x = mg\cos\theta$ $\quad F_y = mg\sin\theta$



B = θ

this results in ...

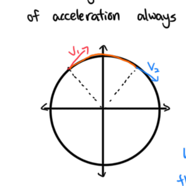


Pulleys! damn it. In order to stay sane in this course, pulleys are always frictionless, have no mass, and are infinetely strong. uh string might break if told otherwise.

too tired to do this rn, save for later

Uniform circular motion. lol gravity better not exist. by definition, this is just going around in circles.

first things first, centripetal acceleration. this is a type of acceleration always directed at the center of a circle.



is calculated by

len $= 2\pi r\left(\frac{\theta}{360}\right)$

with a bunch of work, we get the formula of ↓

$\vec{a} = \frac{-v^2}{R}\hat{j}$

Screw the proof, it is known the accel is always at the center of the circle. This look familiar? Yes, its a frequency wave. Ah the wonders of math aml right.

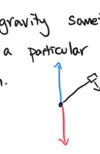
$\vec{a} = \frac{4\pi^2 r}{T^2}$ T = time taken for one rotation

$\vec{a}$ is always towards the center

Now, centripetal force. $F_{net} = m\vec{a} = \frac{mv^2}{r}$

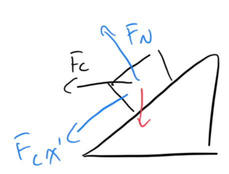
btw, V is scalar. Funnily enough, this is kind of it conceptually for centripetal force.

However, gravity sometimes comes into play here, so when you spin a particular weight around like this, the force is uneven.



this is because gravity is acting on it T-T.

now, centripetal acceleration. This isn't particularly difficult, it just takes some thinking. After finding the magnitude of the centripetal acceleration we can find the direction. tbc



ΣF_x = friction$_x$

Max speed ↙fric

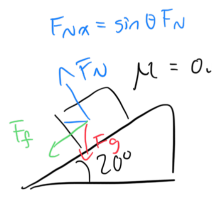
Min speed ↗fric

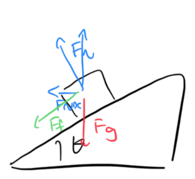
$\frac{Mv^2}{r}$

$F_{cx'} = F_{gx'}$ − friction

$F_{Nx} = \sin\theta F_N$

$\mu = 0.6$





$\cos\theta F_N - \sin\theta\mu F_N - F_g = 0$

$\frac{Mg}{\cos\theta - \sin\theta\mu} = F_N$

$\Sigma F_y = 0$ $\quad\cos\theta F_N - \sin\theta\mu F_N - F_g = 0$

$\Sigma F_x = F_c = \frac{Mv^2}{r} = \cos\theta\mu F_N + \sin\theta F_N$

$\frac{Mv^2}{r} = \cos\theta\mu F_N + \sin\theta F_N$

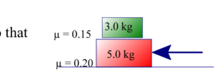
$= F_N(\cos\theta\mu + \sin\theta)$

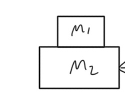
$\frac{Mg(\cos\theta\mu + \sin\theta)}{\cos\theta - \sin\theta\mu}$

$V = \sqrt{\frac{rg(\cos\theta\mu + \sin\theta)}{\cos\theta - \sin\theta\mu}}$

Practice

8. What is the maximum force you can push the 5-kg block so that the 3-kg block does not slip?





$F_{friction\,M_1} = M_1 g\mu_1$

$\Sigma F_{M_1} = M_1 a$

$a_{M_1} = \frac{\cancel{M_1}g\mu_1}{\cancel{M_1}}$

$g\mu_1$ is maximum acceleration before slip