$\cos\frac{\pi}{4} \cdot \sin\left(\frac{\pi}{2}\right)^2$

Remember chart.
Open-book
Unit circle

relative motion exemplar

$5\,m/s$, ΣM

$P = P_2 = 525$

$\frac{P_2 - P_{moon2}}{\Sigma M} = V_{canoe}$

$\frac{525-(-9.75)}{105} = 11.4\,m/s$

Something challenging! (actually its very useful)

So we know $E_k = \frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2$ right?

But! look at this! $\frac{1}{2}M_cV_c^2 + \frac{1}{2}\mu u^2$

this is momentum, but we will get back to it! its super good!

Breaking it down gives us something like this:

$$E_k = \frac{1}{2}(M_1+M_2)\left(\frac{M_1V_1+M_2V_2}{M_1+M_2}\right)^2 + \frac{1}{2}\left(\frac{M_1M_2}{M_1+M_2}\right)\underbrace{(V_1-V_2)^2}_{u}$$

$M_c = \Sigma M$ — this is a constant, won't change

this is relative velocity.

this energy could be transformed into other places

$P = \frac{W}{t} = \frac{Fd}{t} = FV$

So we have moved on to collision! fun. There are a few types of collision. We will try and represent these collisions.

Momentum of each object will be represented by the sign P_n

$\sum_0^n P_n = P_1 + P_2$ $\quad \vec{P}_n = M\vec{V}_n$

Impulse $= \Delta P$ which is $P_f - P_i = m(V_f - V_i)$
$= m(at)$
$= ma\,t$
$= F \cdot t$

Impulse, $\Delta P = F \cdot t$

so what is V_{1f} and V_{2f} after?

well $\Sigma P = M_1V_1 + M_2V_2$

After collision, $P_{1f} = \Sigma P - P_2$
$P_{2f} = \Sigma P - P_1$

Momentum can be seperated into ↑ ↂ

before

$\frac{1}{2}AV_1^2 + \frac{1}{2}BV_2^2 = \frac{1}{2}AV_{1f}^2 + \frac{1}{2}BV_{2f}^2$

$AV_1 + BV_2 = AV_{1f} + BV_{2f}$

$\cancel{\frac{1}{2}}AV_1^2 + \cancel{\frac{1}{2}}BV_2^2 = \cancel{\frac{1}{2}}AV_{1f}^2 + \cancel{\frac{1}{2}}BV_{2f}^2$

$A(V_1^2 - V_{1f}^2) = B(V_{2f}^2 - V_2^2)$

$A(V_1 - V_{1f})(V_1 + V_{1f}) = B(V_{2f} - V_2)(V_{2f} + V_2)$

$AV_1 + BV_2 = AV_{1f} + BV_{2f}$

$A(V_1 - V_{1f}) = B(V_{2f} - V_2)$

$V_1 + V_{1f} = V_2 + V_{2f}$

$V_1 - V_2 = V_{2f} - V_{1f}$

$V_{B} = 2.0808$

$M_AV_A + M_BV_B = M_AV_A' + M_BV_B'$
$= (M_A + M_B)\Sigma V$

$\Sigma V = 0.3904$

$V_A = -1.4192$

Concepts. (with spring)

So in conclusion, it is very important to remember the conservation of momentum, which is perfectly true as long as the word elastic appears.

lets attempt the test "worthy" question he had provided

13. A 1000 kg plane is trying to make a forced landing on the deck of a 2000 kg barge at rest on the surface of a calm sea. The only frictional force to consider is between the plane's wheels and the deck, and this braking force is constant and equal to one-quarter of the plane's weight. What must the minimum length of the barge be, in order that the plane can stop safely on deck, if the plane touches down just at the rear end of the deck with a velocity of 50 m/s towards the front of the barge?

50 m/s; M_1; M_2; 0 m/s

M_1; M_2; combined V_c

In a question like this, it is important to realize the conservation of overall momentum. We use $M_1V_1 + M_2V_2 = M_1V_1' + M_2V_2'$

in fact, through english analysis we get $M_1V_1 = (M_1+M_2)V_c$ (as $V_2 = 0$)

thus, $V_c = \frac{M_1V_1}{M_1+M_2}$ Using the energy formula here, energy is lost to friction alone.

thus, $\frac{1}{2}MV_1^2 + \cancel{\frac{1}{2}MV_2^2} = \frac{1}{2}\Sigma MV_c^2 + F_fd$. $\left(F_f = \frac{M_1 \cdot g}{4}\right)$

$\frac{\frac{1}{2}(M_1V_1^2 - \Sigma MV_c^2)}{F_f} = d = 340.136\,m.$

this question essentially reminds us of the importance of conservation of momentum. We just so happen to be doing our experiment on this tommorrow. A guideline to remember:

momentum is conserved if: No external forces; No mass change

lab:
1D collisions, graphed video analyzing collision inelastic

better question!

10. A 70 kg boy sits in a 30 kg canoe at rest on the water. He holds two cannonballs, each of mass 10 kg. He picks them up and throws both together over the stern of his canoe. The two balls leave his hands with a velocity of 5.0 m/s relative to the canoe. A 50 kg girl sits in a 50 kg canoe, also at rest on the water. She also holds two 10 kg cannonballs. However, she throws them over the stern of her canoe one at a time, each ball leaving her hands with a velocity of 5.0 m/s relative to the canoe. Assuming negligible friction between the water and the canoe (a poor assumption), calculate the final velocity for each canoe.

for now we are only concerned with the little girl.

$P_{B/O} = P_A$

but! think of the 3 m/s as after the throw, which results in

$P_o = P_A$
$M_o V_o = 10(V_o + 5)$
$M_o' V_o' = 10(V_o' + 5)$
$110\,V_o = 10(V_o + 5)$
$120\,V_o = 50$
$100\,V_o' = 10(V_o' + 5)$
$110\,V_o' = 50$
$\Sigma V = \frac{5}{12} + \frac{5}{11} = 0.8712\,m/s$

$MA = 0.78\,kg$
$MB = 0.835\,kg$

$\left(\frac{2M_A}{M_A+M_B}\right)V_A + \left(\frac{M_B-M_A}{M_A+M_B}\right)V_B = V_{Bf}.$

$M_1 \rightarrow M_2$

$M_1 \rightarrow M_2 \rightarrow$

final depends on type of collisions.
→ perfectly elastic
→ perfectly inelastic.

Explosion ≡ Momentum.

After explosion momentum is the same.

elastic → conservation of ke
inelastic → ke → consumed.

☆ Determine contact time.

Test concept Review

Considering his tests, reviewing energy is a good idea

test concepts:
- Impulse $= \Delta P = M \cdot \Delta V = Ma \cdot \Delta t = \vec{F} \cdot \Delta t$
- graphs
- Conservation of momentum
- elastic collisions
- inelastic collisions
- relative collisions
- 2D stuff
- Converting to 2D

Great! Concept stuff first. We start with the important formulas and their derivation.

$P = MV$

$P_1 + P_2 = P_1' + P_2'$ → when conservation of momentum holds true.

how about collisions in 1D?

for elastic collisions, I'm sure you have heard of the formula of:

$$V_1' = \left(\frac{M_1 - M_2}{M_1 + M_2}\right)V_1 + 2\left(\frac{M_2}{M_1+M_2}\right)V_2$$

Momentum compression

if you look at the graph you realize that the collision is symmetrical

ok draw Vt graph here

Doesn't this look interesting?

$V_A' - V_1 = V_c - V_A$

So, if we want to find V_A'

$V_A' = 2V_c - V_A$

which means $V_A' = 2\left(\frac{M_AV_A + M_BV_B}{M_A + M_B}\right) - V_A$

this formula gives the same answer as

$$V_1' = \left(\frac{M_1 - M_2}{M_1 + M_2}\right)V_1 + 2\left(\frac{M_2}{M_1+M_2}\right)V_2$$

but thats just expanded weirdly for no reason

Relative motion strategy! Uh...

#1, read the question

A 70 kg boy sits in a 30 kg canoe at rest on the water. He holds two cannonballs, each of mass 10 kg. He picks them up and throws both together over the stern of his canoe. The two balls leave his hands with a velocity of 5.0 m/s relative to the canoe. A 50 kg girl sits in a 50 kg canoe, also at rest on the water. She also holds two 10 kg cannonballs. However, she throws them over the stern of her canoe one at a time, each ball leaving her hands with a velocity of 5.0 m/s relative to the canoe. Assuming negligible friction between the water and the canoe (a poor assumption), calculate the final velocity for each canoe.

M_1 — Super important — M_2

the 5 m/s is very important because it is relative to the canoe

this means, taking the floor as a pov

thus, $V_c 100 + 20(5 + V_c) = 0$
$120\,V_c = -100$

So why is V_B 5+Vc and not 5−Vc?

well, Vc here is in canoe formation, so it is actually a negative.

Also, we might get asked the contact time.... this is not particularly difficult as long as we have a general idea of whats happening, or graphical analysis.

Oh btw, its not a line, it looks more like this, but for simplicity's sake we will keep it as a line

Oh so for an inelastic it looks like this.

the proper graph look more like this.

the mid air collision graph for two objects looks like this

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

elastic — inelastic

$d_1 = d_0 + V_1t + \frac{1}{2}at^2$
$d_2 = d_0 + V_1t + \frac{1}{2}at^2$

$d_B + V_{2t} + \frac{1}{2}at^2 = d_0 + V_1t + \frac{1}{2}at^2$

$t = \frac{m}{s}$

$d_B = 6 + -2t + \cancel{\frac{1}{2} \cdot 9.8t^2}$
$d_1 = 2 + 10t + \cancel{\frac{1}{2} \cdot 9.8t^2}$
$4 = 12t$
$t = 0.33\,s$

$h = 4.789$
$V_f = V_i + at$
$V_A' = 10 + -9.8 \cdot 0.33$
$= 6.733\,m/s$
$V_B' = -2 + -9.8 \cdot 0.33$
$= -5.266\,m/s$

$V_1' = 2V_c - V_1$

$V_c = \frac{2 \cdot -5.266\,m/s + 1 \cdot 6.733\,m/s}{3}$
$= -1.2662$

$-9.8\,m/s \cdot t$

$t = 0.5$

$h' = h + V_ct + \frac{1}{2}at^2$
$h' = 4.789 + -1.2663 \cdot 0.5 + \frac{1}{2} \cdot -9.8 \cdot 0.5^2$
$= 2.931\,m$

$V_B' = V_B + at$ $\quad V_B' = 2.7334 + -9.8 \cdot 0.5$
$= -2.166$
$V_A' = V_A + at$ $\quad V_A' = -9.2656 + -9.8 \cdot 0.5$
$= -14.1656\,m/s$

$-2.931 = -14.1656t + \frac{1}{2} \cdot -9.8t^2$

$d_B = -2.166t + \frac{1}{2} \cdot -9.8t^2$

2D elastic example

- In order to make the shot shown in the diagram, you need the final speed of ball B to be 0.42 m/s at an angle of 30.0° with the vertical. If ball A has a final speed of 0.52 m/s after the collision, and ball B makes it into the pocket, at what velocity do you need to hit ball A? Assume the balls have equal mass and friction is negligible.

→ the angle

$\cancel{\frac{1}{2}}\cancel{M}V_A^2 = \cancel{\frac{1}{2}}\cancel{M}V_A'^2 + \cancel{\frac{1}{2}}\cancel{M}V_B'^2$

$V_A = \sqrt{V_A'^2 + V_B'^2}$

$\begin{bmatrix} 0 \\ V_A \end{bmatrix} = V_A' + V_B'$

$\begin{bmatrix} 0 \\ V_A \end{bmatrix} = V_A'\hat{A} + \begin{bmatrix} 0.21 \\ 0.36 \end{bmatrix}$

$\begin{bmatrix} 0 \\ V_A \end{bmatrix} = \begin{bmatrix} V_A' \sin(\theta) \\ V_A' \cos(\theta) \end{bmatrix} + \begin{bmatrix} 0.21 \\ 0.36 \end{bmatrix}$

$-0.52 \sin(\theta) = 0.21$
$\theta = 23.82$
$V_A = 0.52 \cdot \cos(23.8) + 0.36$
$= 0.84$

- Two identical curling stones slide towards each other. If stone A starts at 0.55 m/s [S 35° W] before the collision, and stone B starts at 0.71 m/s [N] and ends at 0.41 m/s [N 11 ° W], what is the final velocity of stone A?

$P_i = P_f$
$P_A + P_B = P_A' + P_B'$

$\begin{bmatrix} 0.45 \\ -0.45 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.71 \end{bmatrix} = \begin{bmatrix} P_A' \\ P_B' \end{bmatrix} = \begin{bmatrix} 0.078 \\ 0.40 \end{bmatrix}$

$\begin{bmatrix} 0.45 - 0.078 \\ 0.71 - 0.45 - 0.4 \end{bmatrix} = \begin{bmatrix} 0.237 \\ -0.14 \end{bmatrix}$

$= 0.275,\ -30.57°$

- Two identical billiard balls are at rest on a level, frictionless surface, just touching each other at one common point, as shown. A third, identical ball, the cue ball, is approaching along the common tangent with a constant speed of 20 m/s, as shown. Assuming a completely elastic collision, with no spin on any of the balls, and making (reasonable) assumptions about symmetry, calculate the velocity of the cue ball after the collision.

20 m/s

$V_A = V_A' + 2\cos(30)V_B$

$\cancel{\frac{1}{2}}\cancel{M}V_A^2 = \cancel{\frac{1}{2}}\cancel{M}V_A'^2 + \cancel{M}V_B^2$

$V_A^2 = V_A'^2 + 2V_B^2$

$20 = V_A' + \sqrt{3}V_B$

$20^2 = V_A'^2 + 2V_B^2$

$\frac{20 - V_A'}{\sqrt{3}} = V_B$

$20^2 = V_A'^2 + 2\left(\frac{20 - V_A'}{\sqrt{3}}\right)^2$

$20^2 = V_A'^2 + 2\left(\frac{20^2 - 40V_A' + V_A'^2}{3}\right)$

$\cancel{3} \cdot 20^2 = 3V_A'^2 + \cancel{2} \cdot 20^2 - 80V_A' + 2V_A'^2$

$20^2 = 5V_A'^2 - 80V_A'$
$80 = V_A'^2 - 16V_A'$
$0 = V_A'^2 - 16V_A' - 80$
a b c
$= 20,\ \text{or} -4$
$0\,m/s = -4$

$\frac{\sqrt{3}}{2}$

1. A loaded railway car of mass 6000 kg is rolling to the right at 2.0 m/s when it collides and couples with an empty freight car of mass 3000 kg, rolling to the left on the same track at 3.0 m/s. What is the speed and direction of the pair after the collision?

2. A 1.0 kg ball moving with a velocity of 2.0 m/s to the right collides straight on with a stationary 2.0 kg ball. After the collision, the 2.0 kg ball moves off to the right with a velocity of 1.2 m/s. What is the velocity of the 1.0 kg ball after the collision?

3. An experimental rocket sled on a level, frictionless track has a mass of 1.4×10^4 kg. For propulsion, it expels gases from its rocket engines at a rate of 10 kg/s and at an exhaust speed of 2.5×10^3 m/s relative to the rocket. For how many seconds must the engines burn in order that the sled acquire a velocity of 50 m/s starting from rest? You may ignore the small decrease in mass of the sled and the small speed of the rocket compared to the exhaust gas. (The velocities are relative to Earth.)

4. A bomb initially at rest on a smooth, horizontal surface is exploded into three pieces. Two pieces fly off horizontally at 60° angle to each other, a 2.0 kg piece at 20 m/s and a 3.0 kg piece at 12 m/s. The third piece flies off horizontally at 50 m/s.
   a. Determine the direction of motion of the third piece.
   b. What is its mass?

5. A 300 g ball is struck by a bat with an impact that lasts 0.020 s. If the ball moves through the air towards the bat at 50 m/s and leaves at 100 m/s in the opposite direction, calculate the average force exerted by the bat on the ball.

6. A 24 g bullet is fired horizontally, embedding itself in a 10 kg block initially at rest on a horizontal ice surface. The block slides 15 m before coming to rest. Assuming the block slides with no change in its original position. Assuming that the frictional force stopping the block was constant, calculate the velocity of the bullet.

7. A 2000 kg car travelling east at 24 m/s enters an icy intersection and collides with a 3000 kg truck travelling south at 10 m/s. If they become coupled together in the collision, what is their velocity immediately after impact?

8. A nucleus, initially at rest, decays radioactively. In the process, it emits an electron horizontally to the east, with momentum 9.0×10^{-21} kg·m/s and a neutrino horizontally to the south, with momentum 4.8×10^{-21} kg·m/s.
   a. In what direction does the residual nucleus move?
   b. What is the magnitude of its momentum?
   c. If the mass of the residual nucleus is 3.6×10^{-27} kg, what is its recoil velocity?

A(100kg) and B(110kg) are in separate boats, 20m apart. The race starts with B at 5m/s and A at 4m/s. 1 second after the race starts, A throws a cannonball with mass of 10kg at B at 2m/s relative to A's boat. The cannonball takes 2 seconds to travel, then hitting B, staying in B's boat for another 2 seconds. The cannonball then proceeds to roll off B's boat at 1m/s relative to B's boat. B throws another cannonball also mass of 10kg at A at 0.15m/s relative to B's boat, but misses. Does B catch up to A? If so, in how long?

$t = 0$: A 4m/s, C 10, B 5m/s, 20m

$t = 1$: $4.45 + \left[V_A \cdot C \quad V_c = V_A' - 2\right]$, B 5m/s, 19, 100

$V_A' \cdot 90 + 10 \cdot (V_A' - 2) = 0$
$100V_A' = 20$
$V_A' = 0.2$
$V_A = 4.2$ $\quad V_c = 2.2$

$t = 3$: A 8.2, C 2.2, B, 17.4

$t = 3$: A 8.2, [C, B], 17.4

$\Sigma V = \frac{2.2 \times 10 + 5 \times 110}{120} = 4.766$

$t = 5$: A, C, B, 16.266

$t = 5$: A, 16.266, [B, C] + 4.766

$V_B' \quad V_c = V_B' - 1$

$110\,V_B' + 10(V_B' - 1) = 0$
$120V_B' = 10$
$V_B' = 0.08333$

$V_A = 4.2$

$\Delta V = 0.64493$ $\quad V_B = 4.84493$

$\frac{16.266}{0.64493} = 25.0525\,s$

Ans $= 30.05\,s$
$= 30\,s$

$\cos\left(\frac{\pi}{2}\right) \cdot \sin\left(\frac{\pi}{2}\right)^2$

ΣM

$\frac{P_2 - P_{moon2}}{\Sigma M} = V_{canoe}$

$\frac{525 - (-9.75)}{105} = 11.4\,m/s$

before

$\Sigma P_i = \Sigma P_f$

ΣM

$E_k = \frac{1}{2}(M_1+M_2)\left(\frac{M_1V_1 + M_2V_2}{M_1+M_2}\right)^2 + \frac{1}{2}\left(\frac{M_1M_2}{M_1+M_2}\right)(V_1 - V_2)^2$

$E_k = \cancel{\frac{1}{2}}(\Sigma M)\left(\frac{P_i}{\Sigma M}\right)^2 + \cancel{\frac{1}{2}}\left(\frac{M_1M_2}{\Sigma M}\right)V_1^2$

$E_k = \cancel{\frac{1}{2}}M_1V_{1f}^2 + \cancel{\frac{1}{2}}M_2V_{2f}^2$

$M_1V_{1f}^2 + M_2V_{2f}^2 = (\Sigma M)\left(\frac{P_i}{\Sigma M}\right)^2 + \left(\frac{M_1M_2}{\Sigma M}\right)V_1^2$

$P_{1f}V_{1f} + P_{2f}V_{2f} = P_1\frac{P_i}{\Sigma M} + \frac{M_1M_2}{\Sigma M}V_1^2$

$4\,m/s$; $-6\,m/s$; $-1.5\,m/s$; $-4\,m/s$

1.8 kg; 2.2 kg; P° 4.4

total momentum $= P_1 + P_2 = M_1V_1 + M_2V_2$

$\Sigma KE = \frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2$

$\Sigma E_2 = \frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2 + \frac{1}{2}k\Delta^2$

$a = -5.5$
$b = -4.5$

$a = \frac{M_1V_1 + M_2V_2}{M_1 + M_2}$
$= -1.5\,m/s$

$6 - 4 = 2$
$4 - \frac{-2}{9.5} \cdot 5.5$
$= \frac{14}{9}$ $\quad 1.555$

(another way of saying their P decrease is the same)

$\cancel{\frac{1}{2}}2.2 \cdot 6^2 + \frac{1}{2}1.8 \cdot 4^2$
$= \frac{1}{2}2.2 \cdot 1.5^2 + \frac{1}{2}1.8\left(\frac{1.8}{4.5}\right)^2 + \frac{1}{2}k\Delta^2$

$\Delta = \sqrt{\frac{2.2(6-1.5)^2 + 1.8(4^2 - \frac{1.8}{4.5})}{k}}$
$= 0.02924\,m$

Fig. 1

0.055 kg; 0.2 kg; ΣM

35 m/s

$P_A = 1.925$

$P_A = \Sigma M \cdot \Sigma V$

$\Sigma V = 7.549$

$mgh = \frac{1}{2}MV^2$

$h = \frac{1}{2}\frac{V^2}{g}$
$= 2.908\,m$

2D collisions.

$\Sigma P = 0$

Σd

$\Sigma P = 0 = M_1\vec{V}_1 + M_2\vec{V}_2$
$M_1V_1 = M_2V_2$
$V_1 = V_2\frac{M_2}{M_1}$

$t_1 = t_2$

$\frac{d_1}{V_1} = \frac{d_2}{V_2} \Rightarrow \frac{V_1}{d_1} = \frac{V_2}{d_2}$

$\frac{V_1}{V_2} = \frac{d_1}{d_2} \Rightarrow \left(\frac{V_2}{1} \cdot \frac{M_2}{M_1}\right) \cdot \frac{1}{V_2} = \frac{d_1}{d_2}$

$\frac{M_2}{M_1} = \frac{d_1}{d_2}$

$d_1 = \frac{M_2}{M_1}d_2$

$d_1 = \frac{M_2}{M_1}(B - d_1)$

$d_1 = \frac{M_2}{M_1}B - \frac{M_2}{M_1}d_1$

$d_1 + d_1\frac{M_2}{M_1} = \frac{M_2}{M_1}B$

$d_1\left(1 + \frac{M_2}{M_1}\right) = \frac{M_2}{M_1}B$

$d_1 = \frac{BM_2}{M_1 + M_2}$

$A = \frac{M_1(\Sigma d - 2w)}{M_1 + M_2} + W$

$W = 17\,cm$
$= 0.14\,m$
$\Sigma d = 1.13\,m$

$M_A = 100g + m_a$
$495.7\,g$

$M_B = 300g + m_a$
$499.5\,g$

100g ±

Rachels solution