Methods in Psycholinguistics

— Mixed effects logistic regression —

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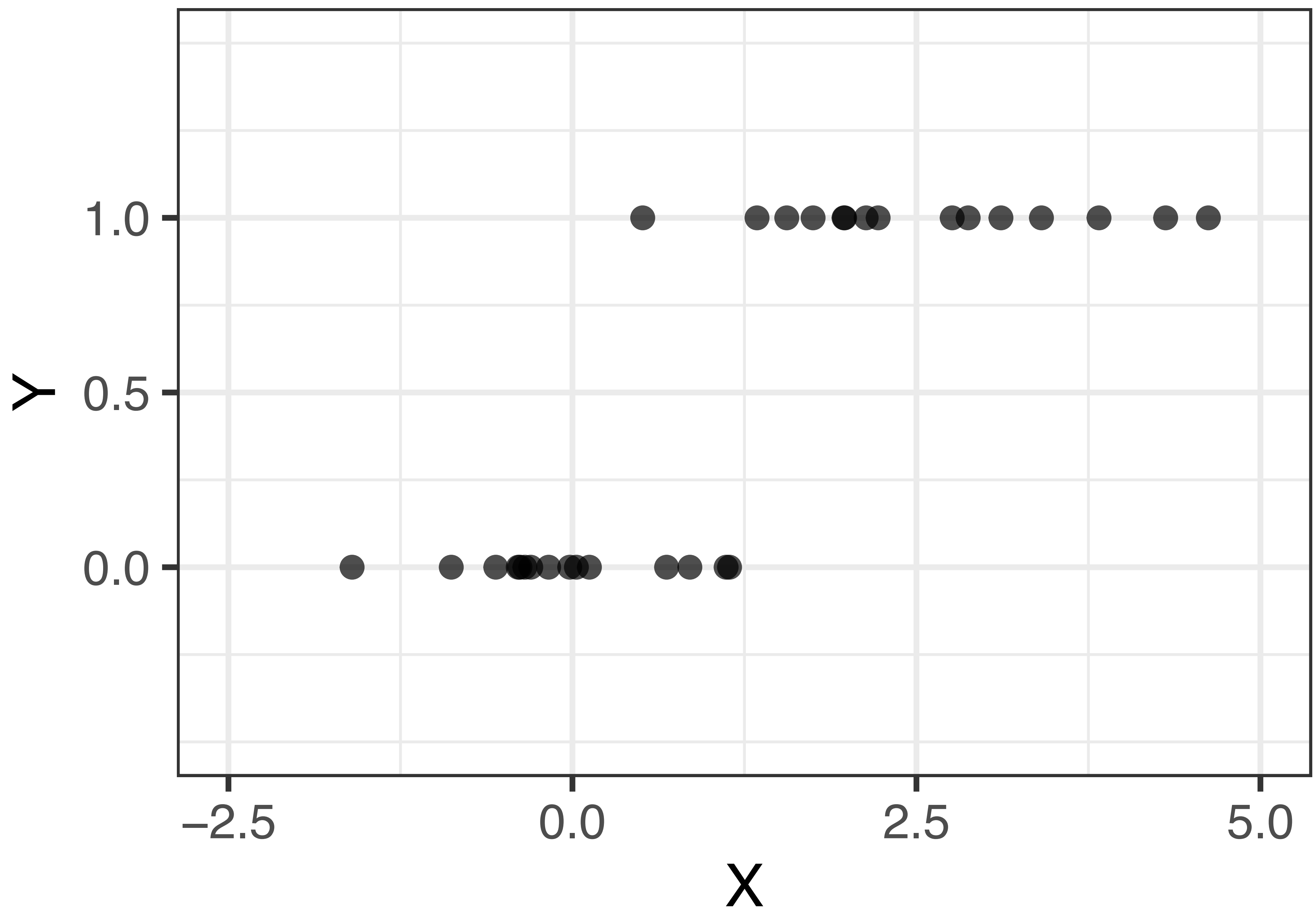
Logistic regression

- for binary (categorical) instead of continuous outcomes
- instead of predicting the mean of an outcome, we’re predicting the log odds of an event occurring
- also called “logit model”

What kind of data?

- grammaticality (binary)
- syntactic variation (e.g., dative alternation)
- phonological variation (e.g., t-deletion)
- experimental forced choice (eg., truth-value judgments)
- eye-tracking data (e.g., look to target)

Why not use linear regression for categorical outcomes?



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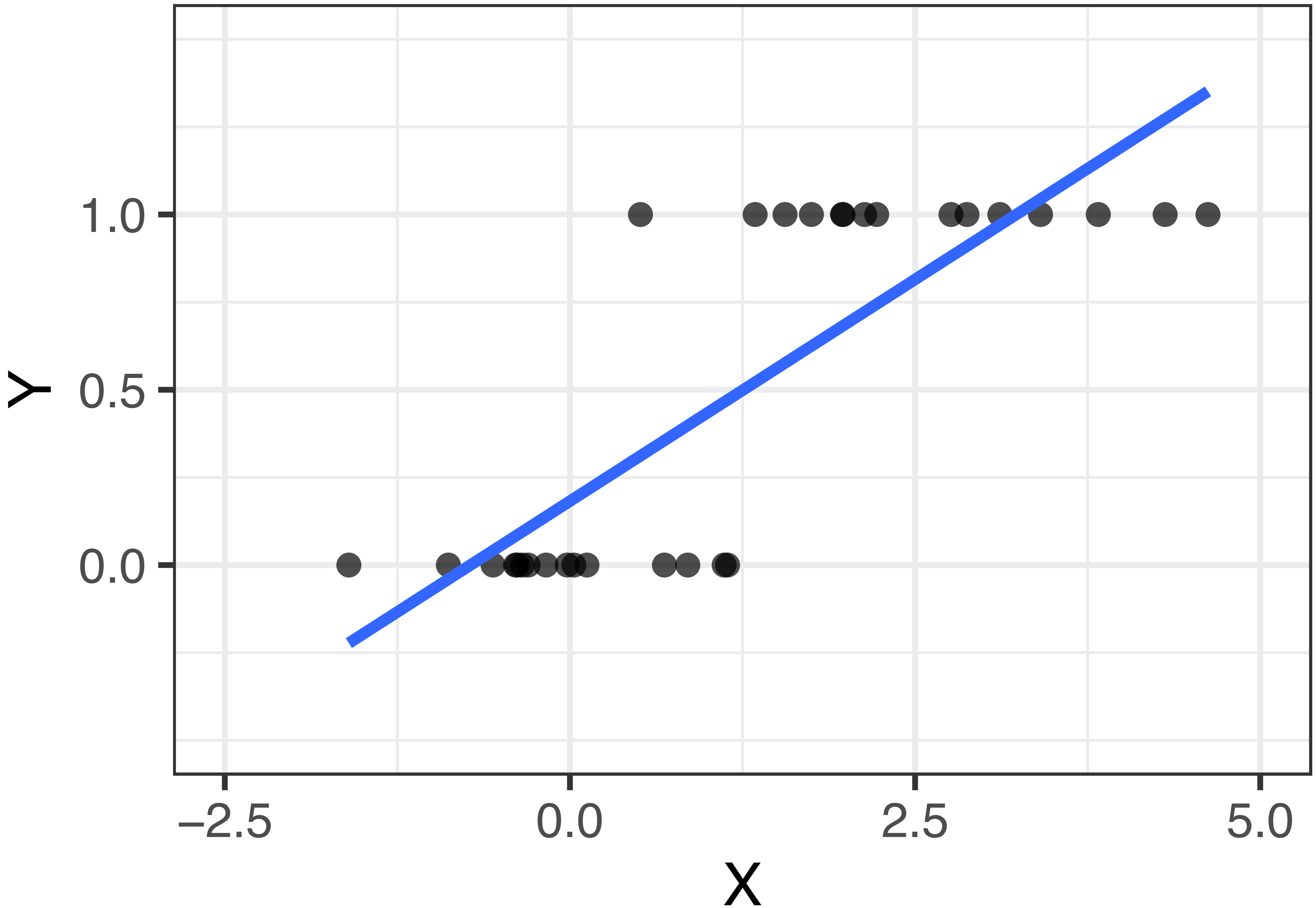

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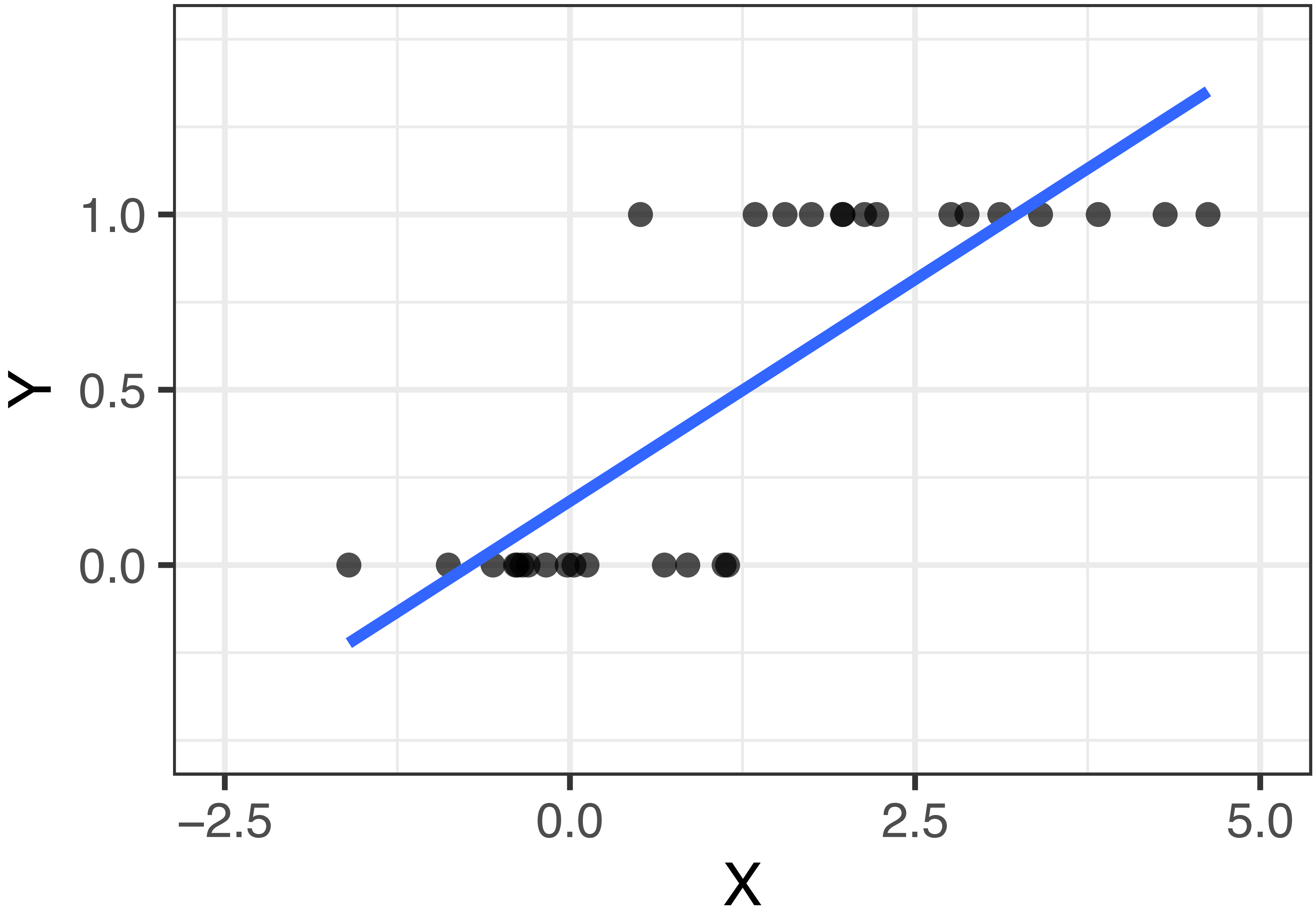

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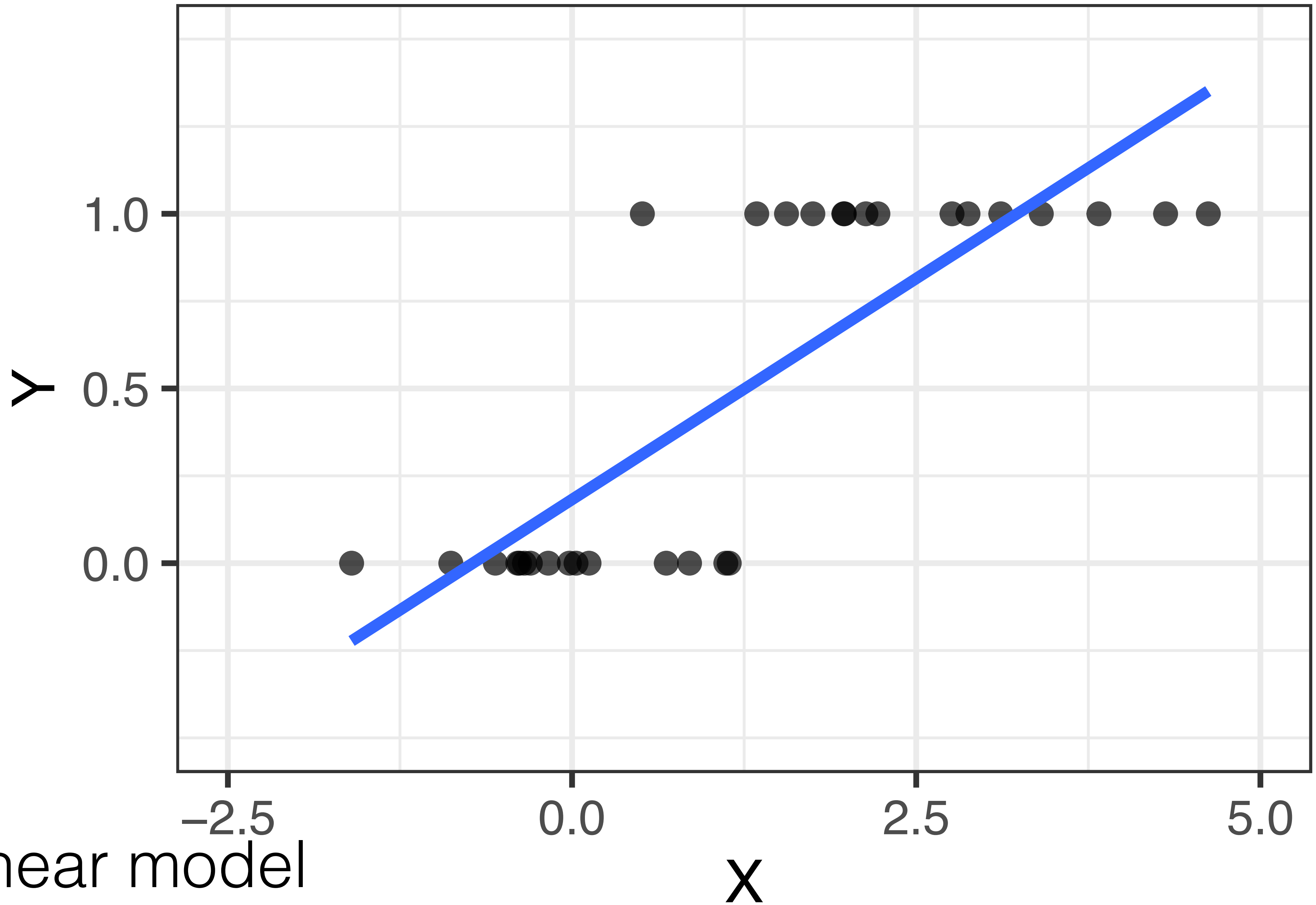
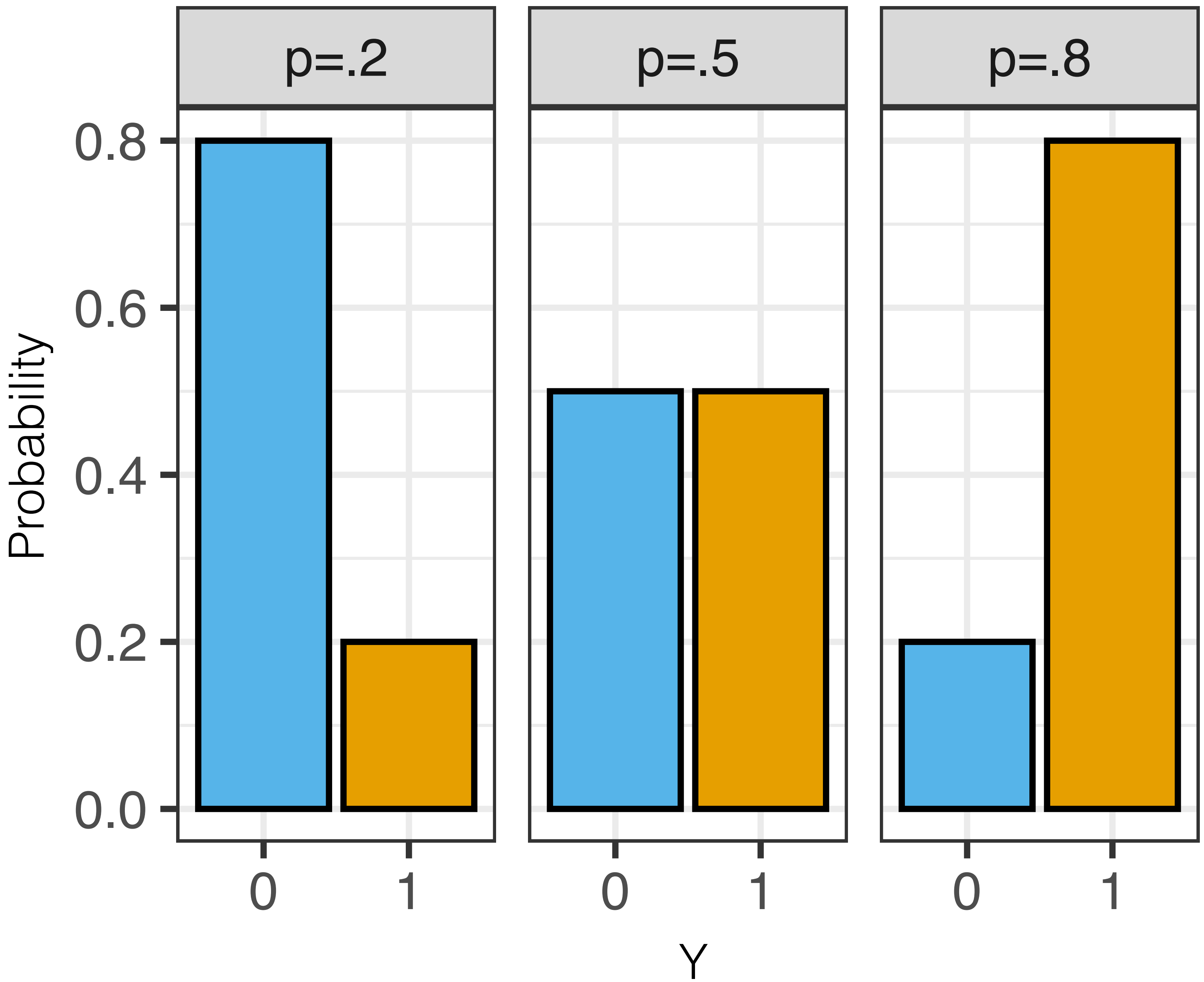


The linear model

- makes impossible predictions (values of Y > 1 or Y < 0)
- is meaningless if its assumptions are violated

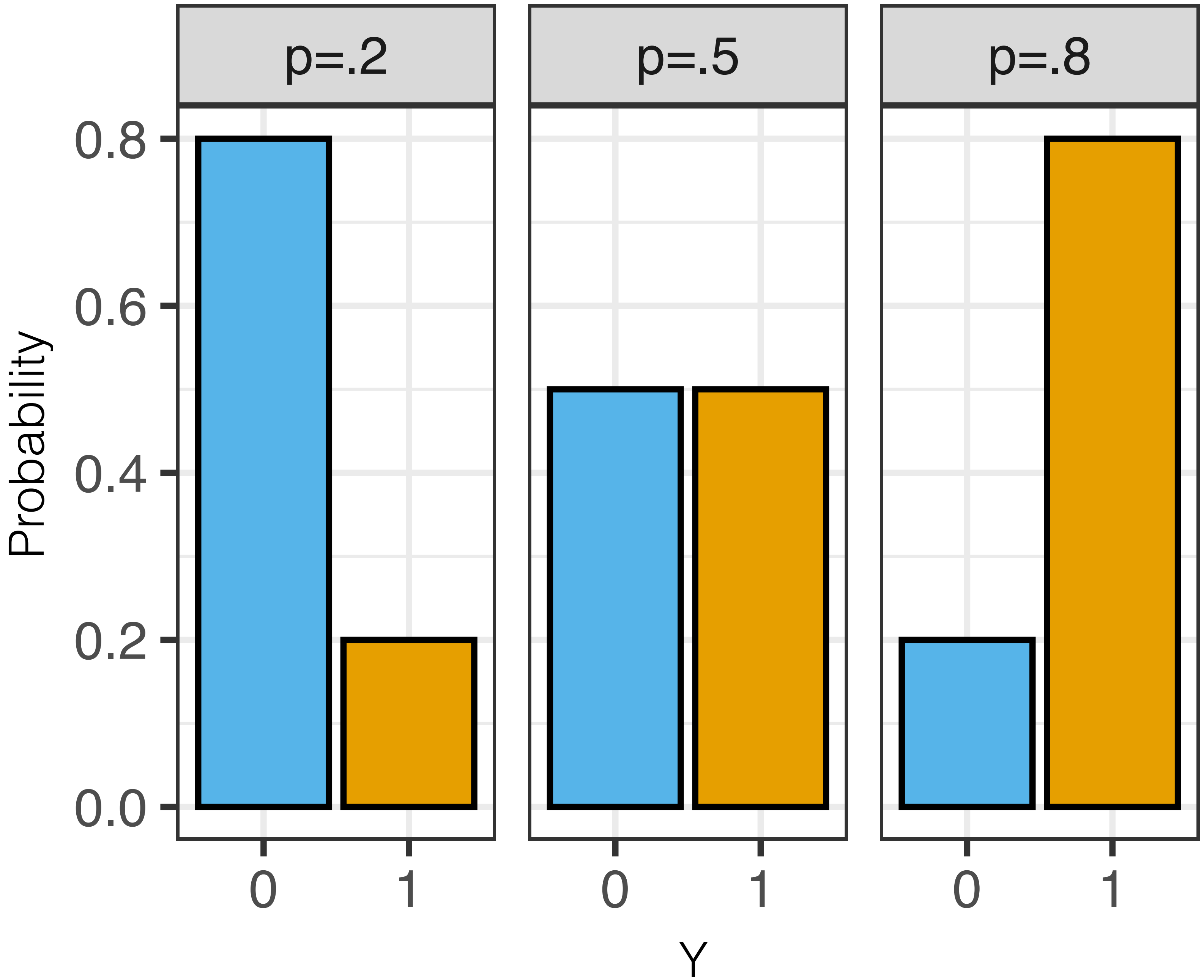
Logistic regression



Assume outcome is Bernoulli-distributed (special case of binomial distribution)

$$y_i \sim \text{Bernoulli}(p_i)$$

Logistic regression



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$$p_i = \beta_0 + \beta_1 x_i$$

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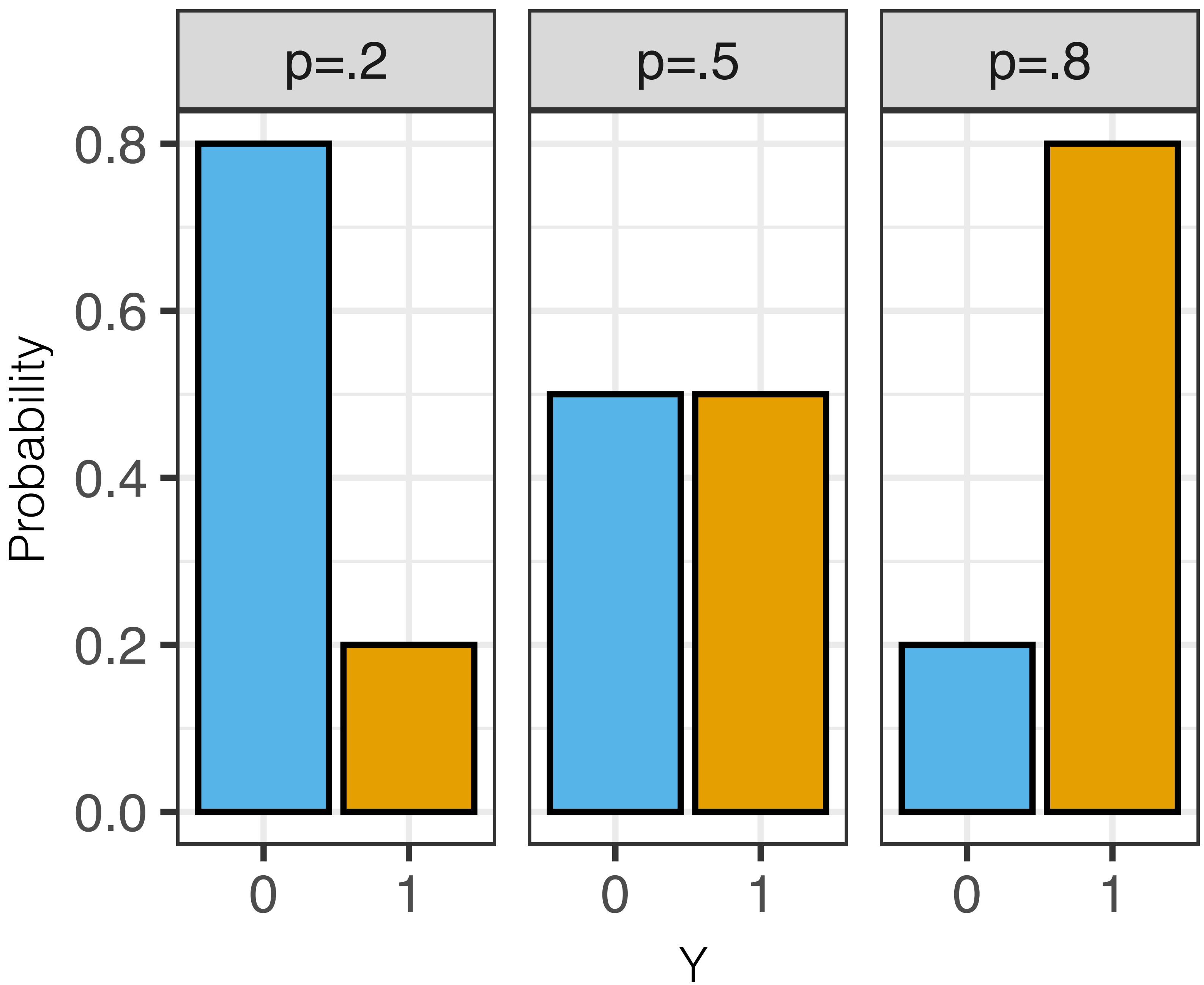

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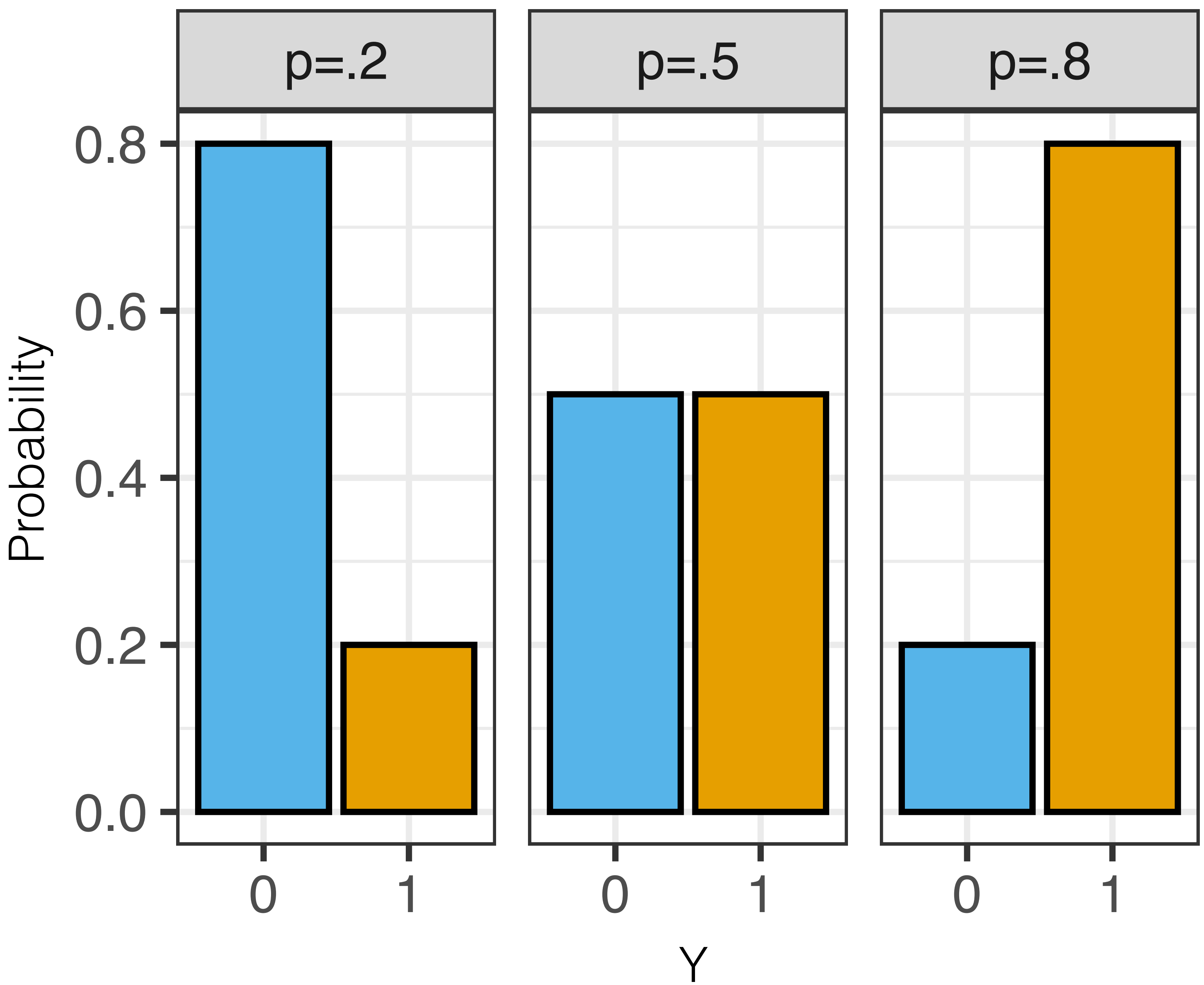

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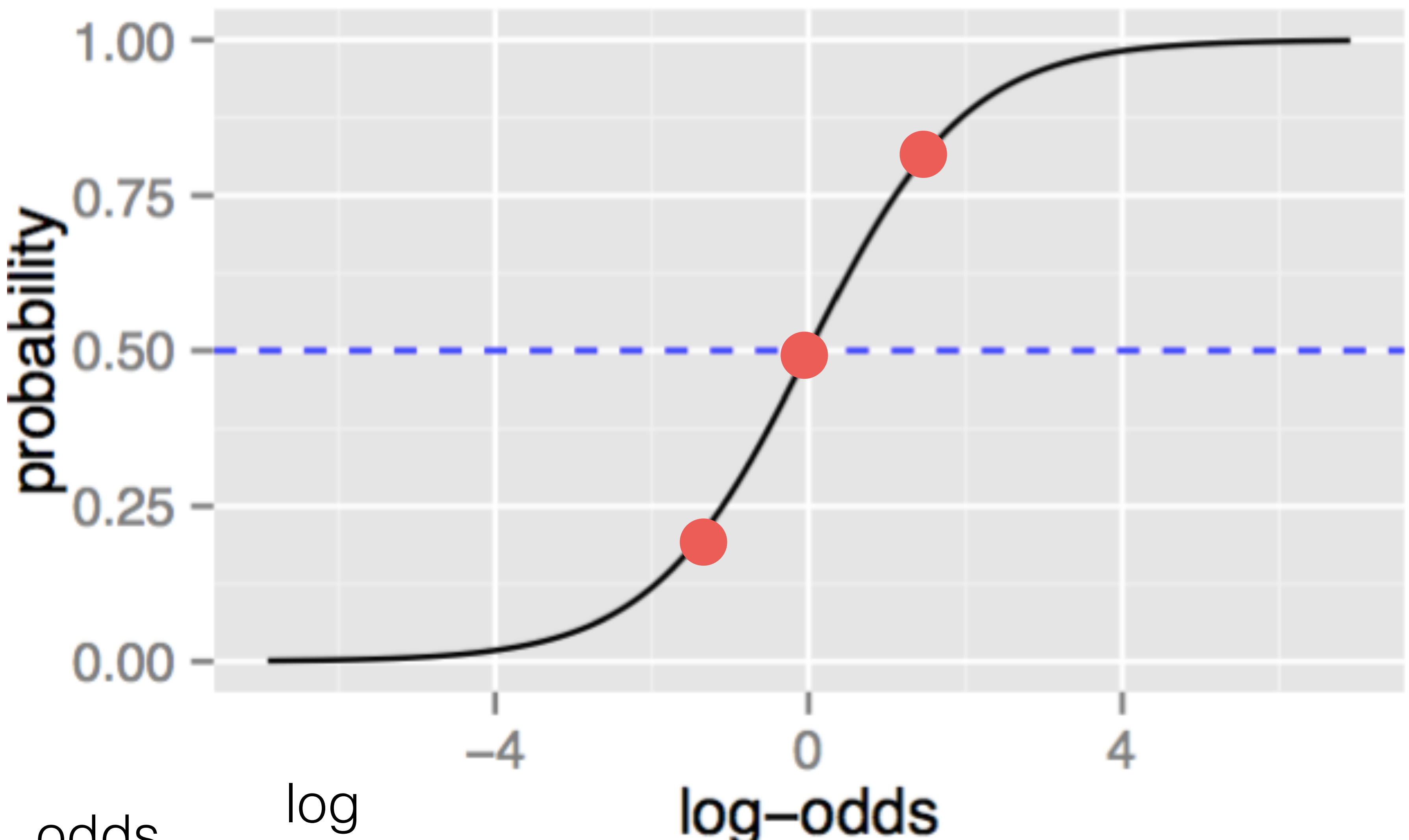
Probabilities are bounded by [0,1]

Log odds and probability



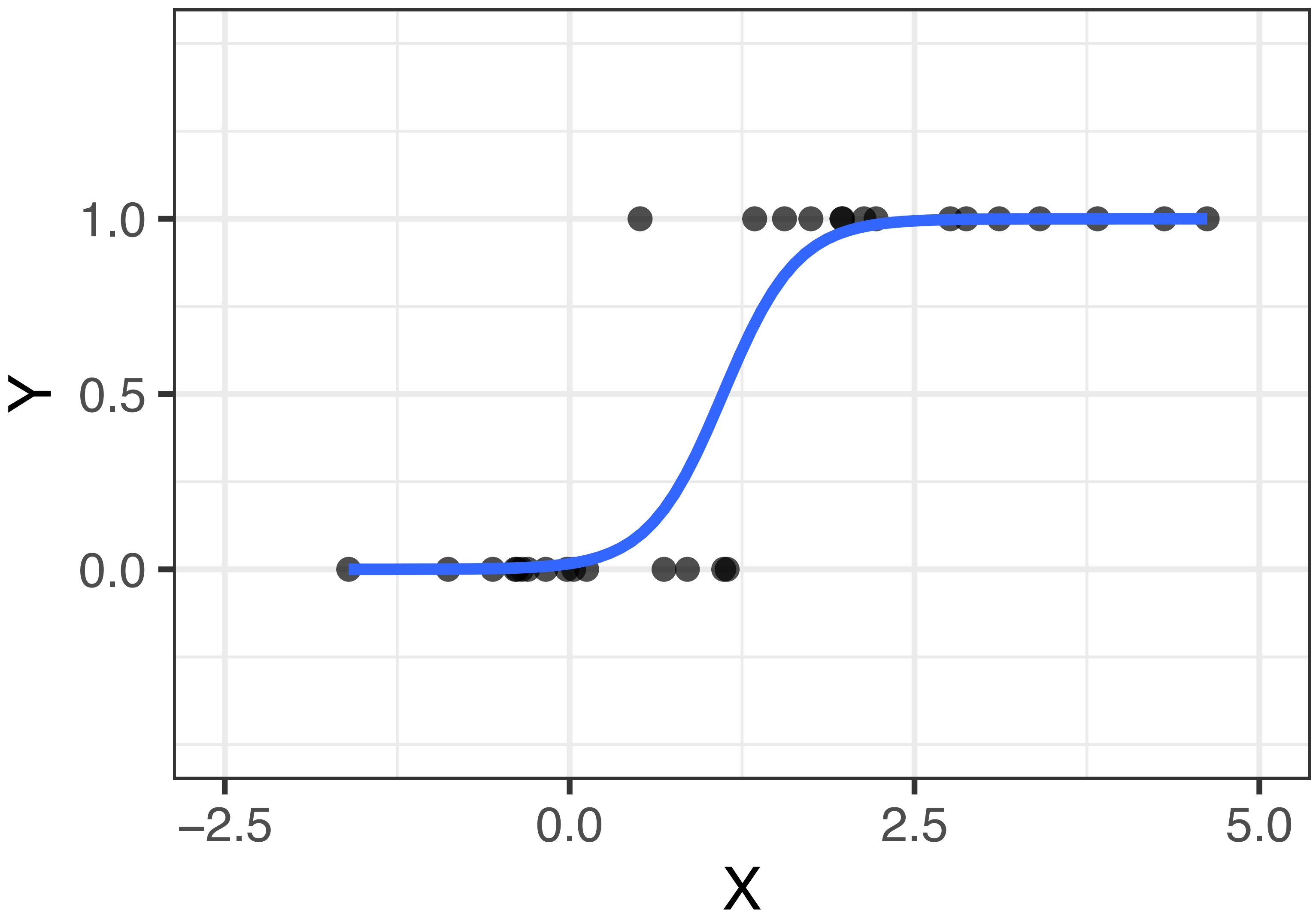
p	odds	log odds
0.2	.25 to 1	-1.39
0.5	1 to 1	0
0.8	4 to 1	1.39

log odds:

$$\ln \frac{p}{1-p}$$

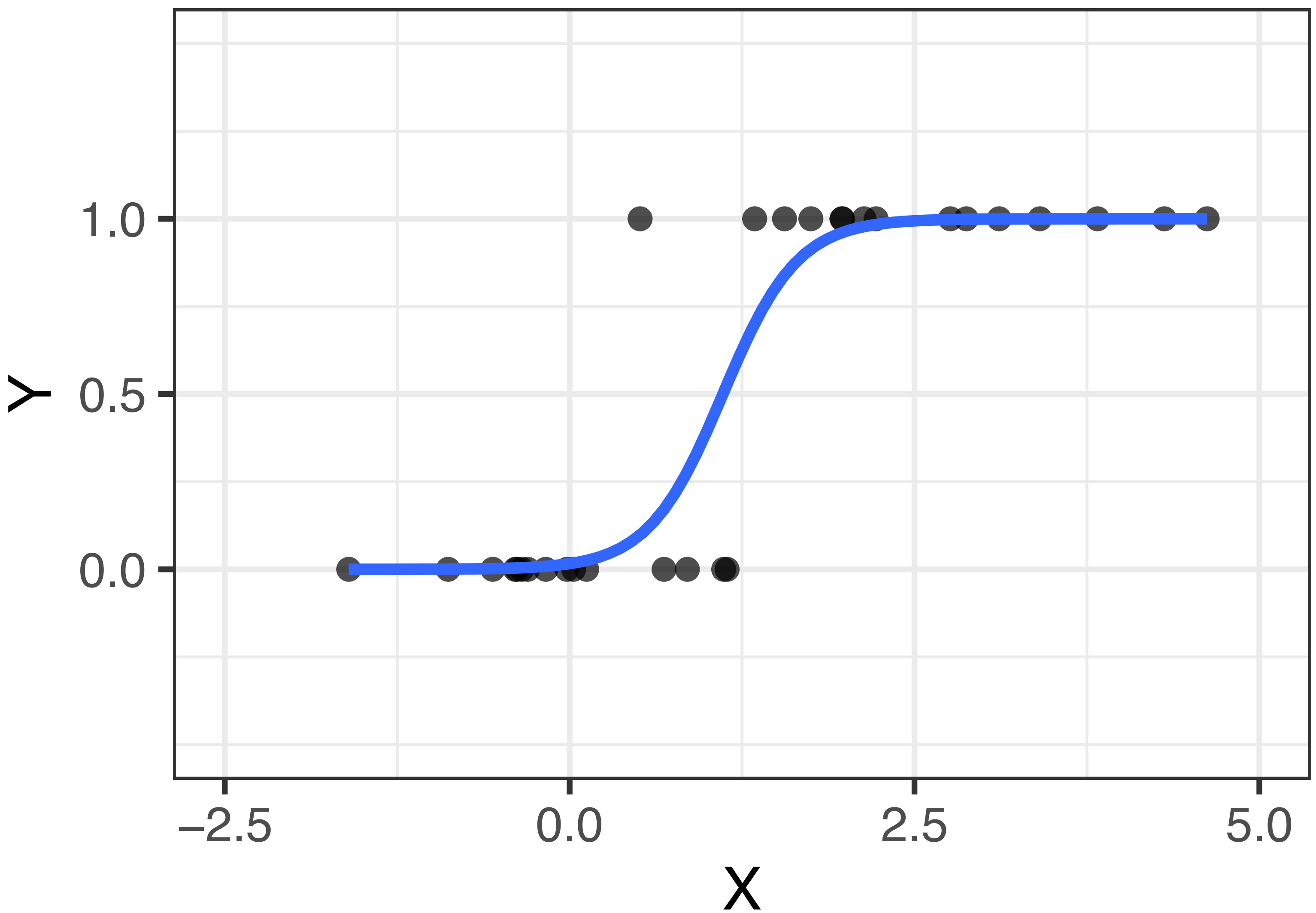
log odds range from -Inf to +Inf

Logistic regression



The logistic function compresses values into [0,1]

Logistic regression



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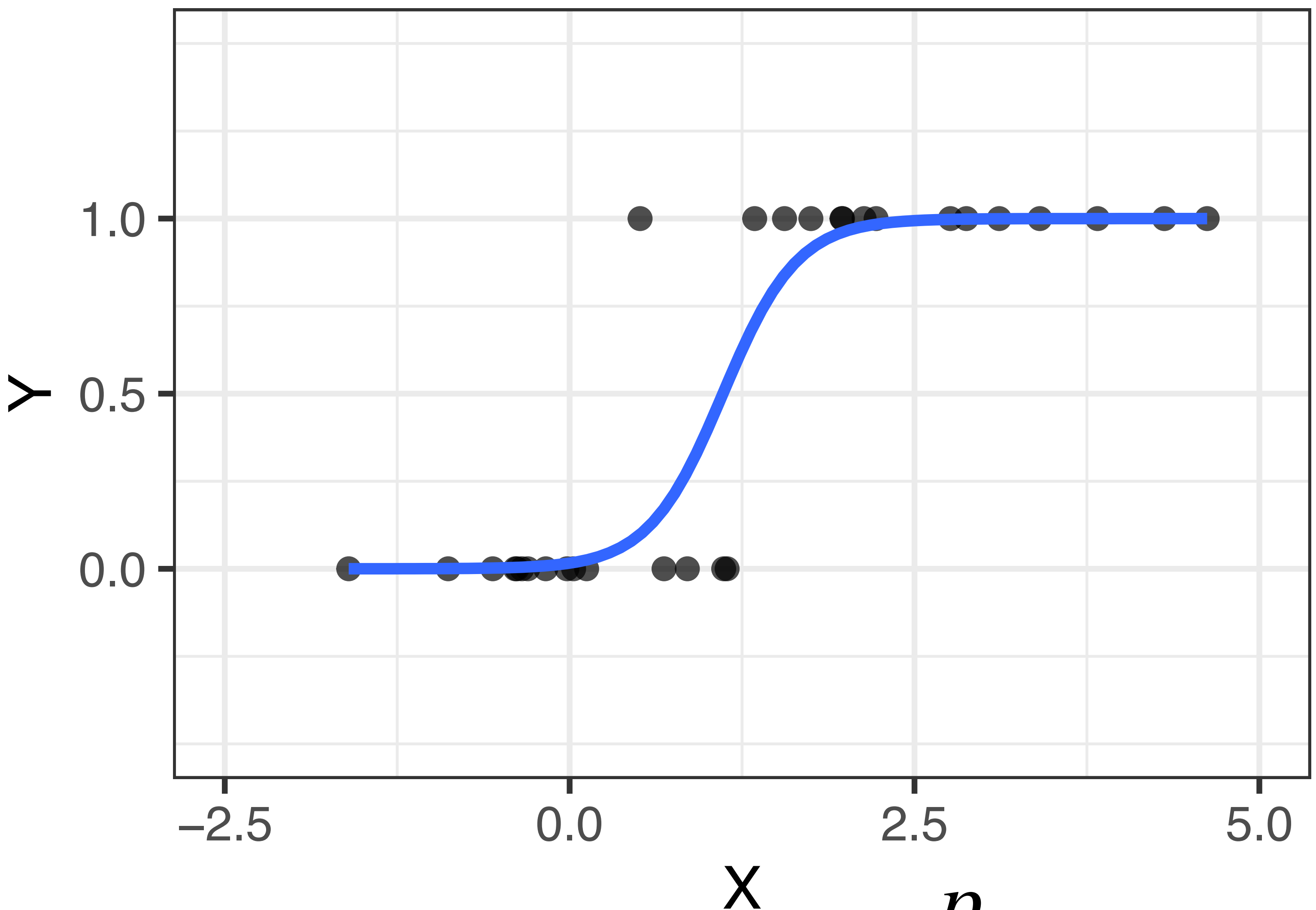
Logistic regression is a kind of GLM (with binomial link):

$$\eta = \beta_0 + \beta_1 x_1 + \ldots + \beta_n x_n$$

Logistic regression



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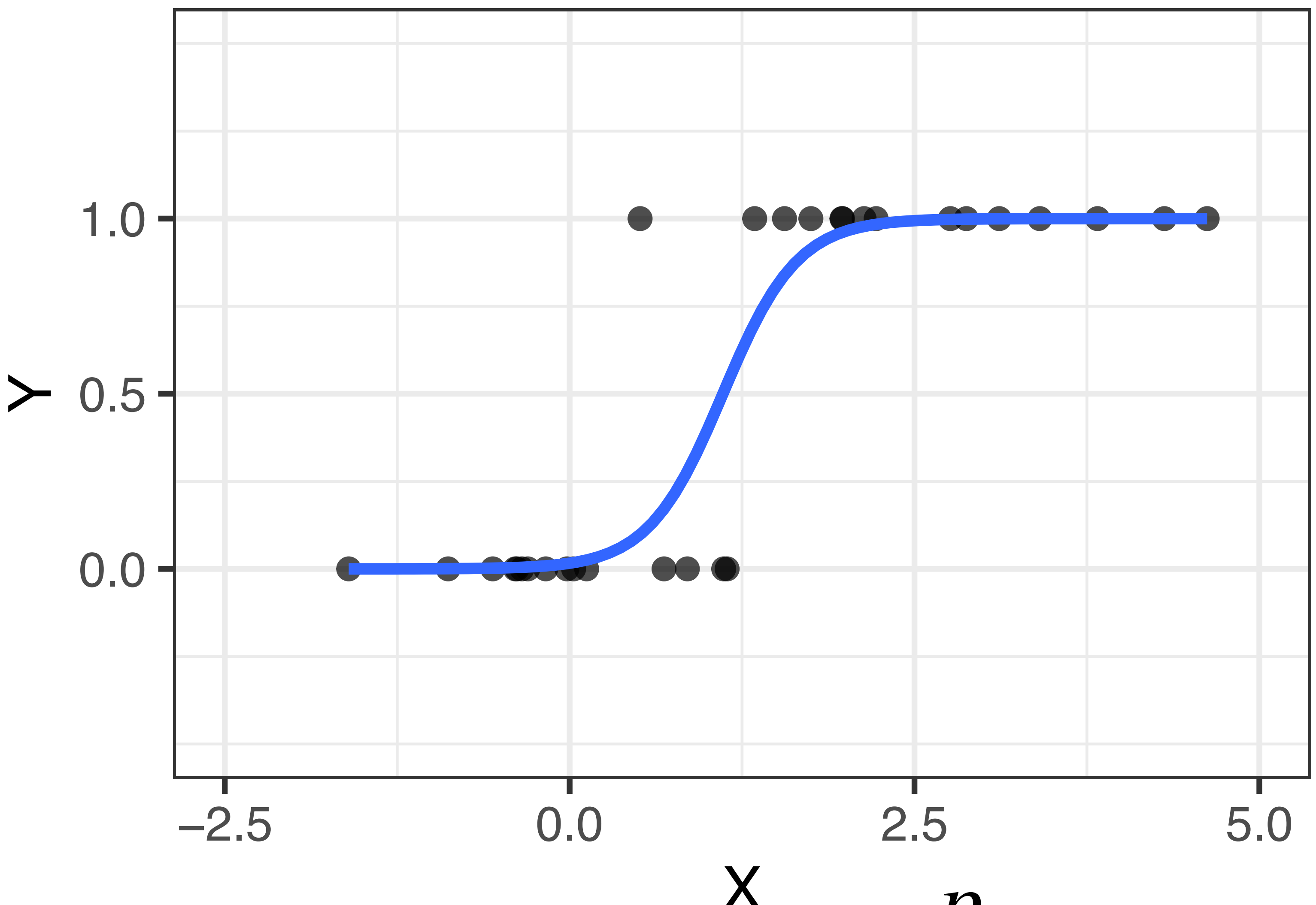

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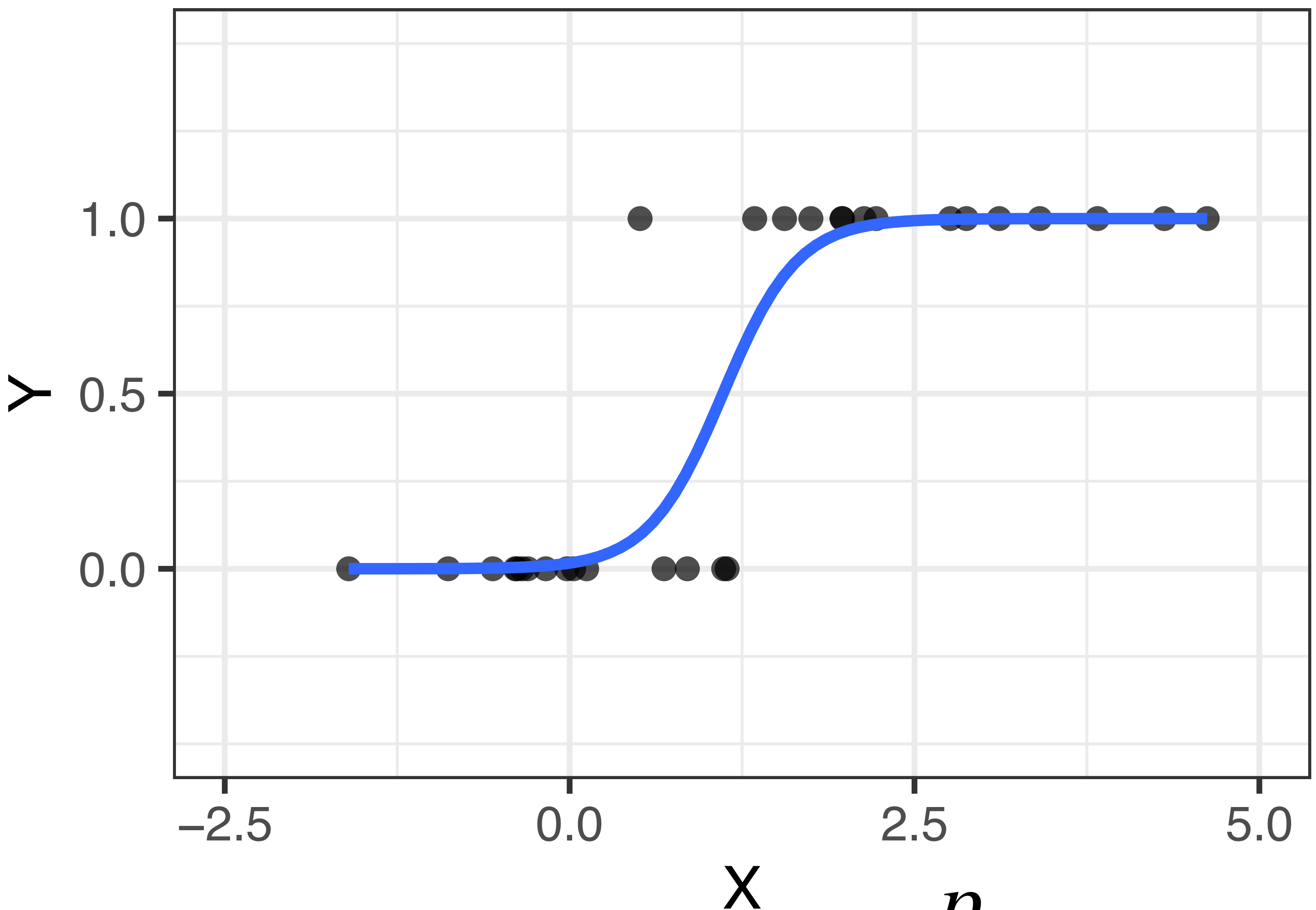

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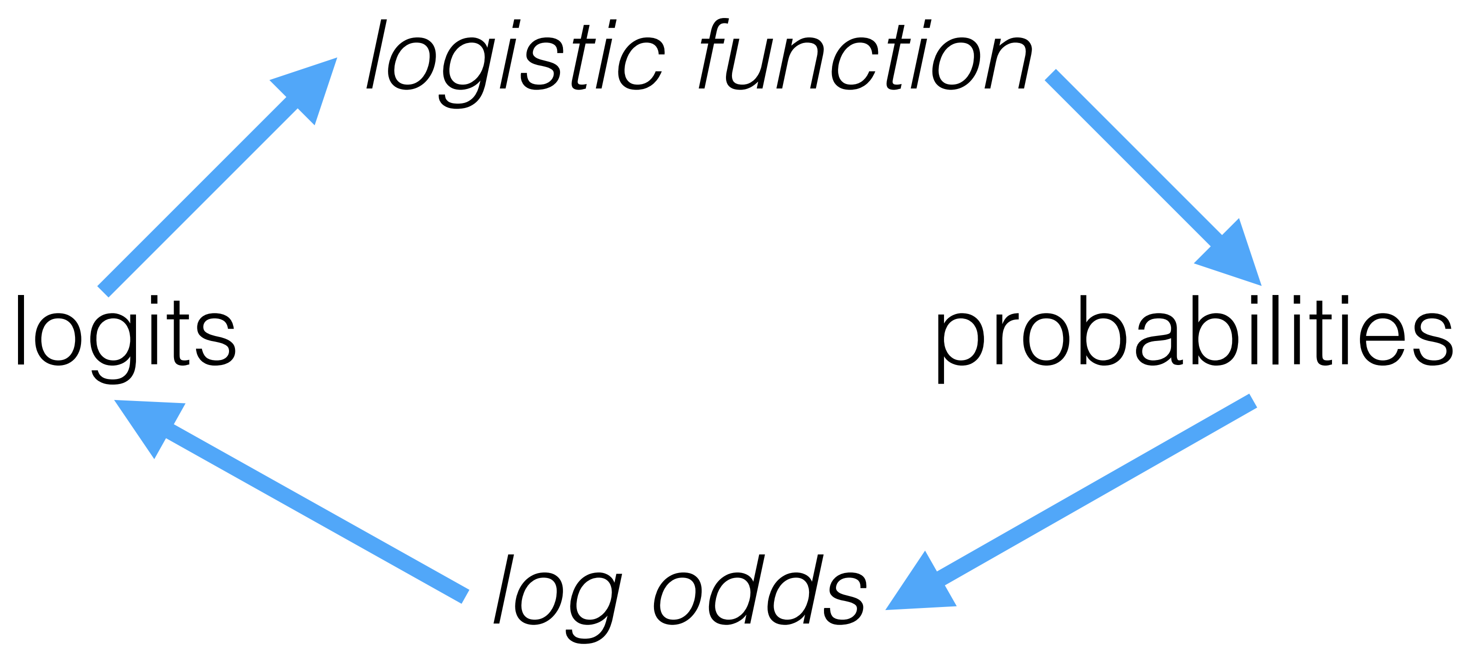
To retrieve p, apply the logistic function (inverse of log odds):

$$p = \frac{1}{1 + \exp^{-X\beta}}$$

Logistic regression



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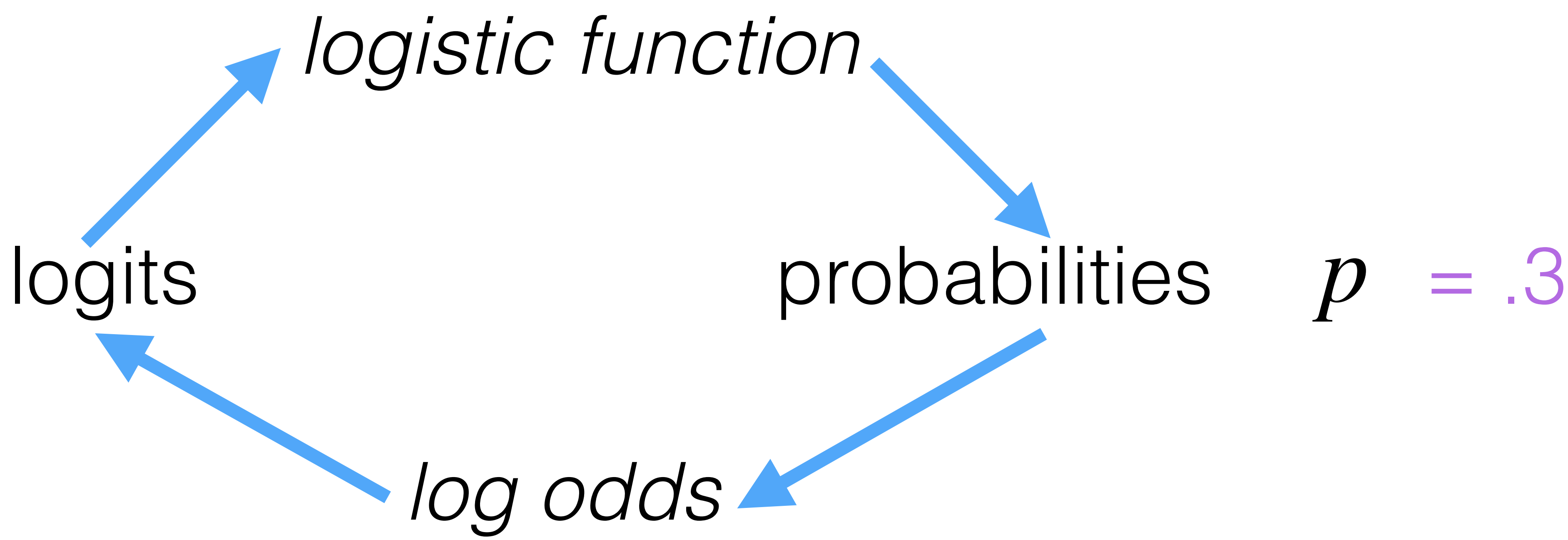

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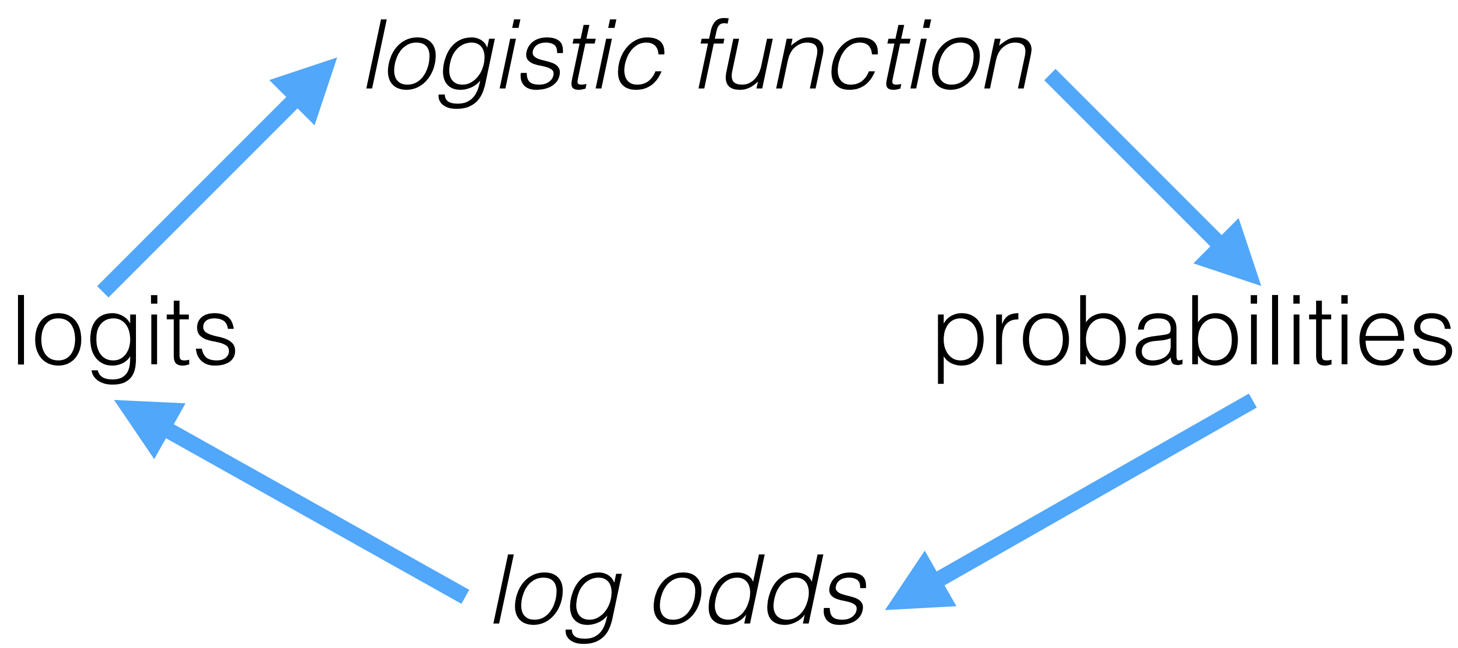

p = .3

$$\text{logit} = \ln \frac{p}{1-p}$$ = ln(.3/.7)

Logistic regression

logistic function

logit logits probabilities p = .3

= -.84

log odds

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Logistic regression

$$p = \frac{1}{1 + \exp^{-\text{logit}}}$$

logistic function

logit = -.84

logits

probabilities

p = .3

log odds

$$\text{logit} = \ln \frac{p}{1-p} = \ln(.3/.7)$$

Mixed effects logistic regression

linear model : mixed linear model ::
logit model : mixed logit model

Assumption: individual differences within a grouping factor are normally distributed in log-odds of event

$$\ln\left(\frac{p}{1-p}\right) = \overbrace{\mathbf{X}\beta}^{\text{Fixed effects}} + \overbrace{\mathbf{Z}b}^{\text{Random effects}} \quad , \quad \overbrace{b_i}^{\sim N(0,\sigma_{b_i})}$$

Dative alternation

Realization of recipient:
NP: John gave [the children] [toys]
PP: John gave [toys] [to the children]

What governs the syntactic choice?

Bresnan, J., Cueni, A., Nikitina, T., & Baayen, H. (2007). Predicting the Dative Alternation. In G. Boume, I. Kraemer, & J. Zwarts (Eds.), Cognitive Foundations of Interpretation (pp. 1–33). Amsterdam: Royal Netherlands Academy of Science.

Let’s translate it into R!