Q3) $\mathbb{Q}$ is not a discrete topology. A topology is discrete if the topolgy on X includes $\emptyset$, X, & all of it's subsets. Since $\mathbb{Q}$ is not isolated & there dense. $\mathbb{Z}$ hower isn't & is a discrete topolgy. There are gaps in a discrete topolm. In discrete topolgy you can define a neighborhood that contain/ itself & no other points in the set. Take $\varphi(\{a,b,c\})$ we can isolate $\{a\}$, $\{b\}$, $\{c\}$.

Sol: We know the following $\mathbb{R}$ is the standard topology

ii) $\mathbb{Q}$ is on the subspace topology on $\mathbb{R}$.

iii) A function f is continuous if the preimage of any open subset is open.

$\mathbb{Q}$ is dense in $\mathbb{R}$. Assume the contradiction f is not constant so for x_1, x_2 s.t $x_1 \neq x_2$ $f(x_1) = q_1 \in U_1$, $f(x_2) = q_2 \in U_2$

$f(x_1) \neq f(x_2)$ suppose U_1, U_2 are disjoint sets

$f^{-1}(U_1) \cup f^{-1}(U_2)$ are open b/c since since U_1 V_2 we have $f^{-1}(U_1), f^{-1}(U_2)$ is disjoint. But $\mathbb{R}$ is connected since it cannot be written as two disjoint subset. The assumption $f(x_1) \neq f(x_2)$ leads to a contradiction therefore

$f: \mathbb{R} \to \mathbb{Q}$ is constant.