Q.6) Proposition 4.27 If f is continuous $f: S \to T$ & S is compact then T is also compact.

Proof Let $\{V_\alpha\}_{\alpha \in A}$ be an open cover of $f(S)$ in T. $f(S) \subseteq \bigcup_{\alpha \in A} V_\alpha$. where each V_α is open in T

$S \subseteq f^{-1}(f(S)) \subseteq \bigcup_{\alpha \in A} f^{-1}(V_\alpha)$ since S has a finite subcover

$f(S) \subseteq \bigcup_{i=1}^{n} f(f^{-1}(V_{\alpha_i})) \subseteq \bigcup_{i=1}^{n} V_{\alpha_i}$

$GL(3, \mathbb{R})$ general linear

system of degree one

$\mathbb{R}$. det $\neq 0$.

$O(3)$ orthogonal matrix

det $= 1$ or -1 $A^T = A^{-1}$

$SO(3)$ $\det(A) = 1$

maintains orientation in space

Sol: We say $GL(3,\mathbb{R})$ is compact

: Consider the det function for ~~det~~ $GL(3,\mathbb{R}) \to \mathbb{R}$

The image is $\mathbb{R} \setminus \{0\}$. However the image is not compact by heinz borel. Since det is continous and the image is not compact $GL(3,\mathbb{R})$ is not compact.

O(3) det(M) = 1

maintains orientation in space

is continous and the image is not compact GL(3, ℝ) is not compact.

O(3), SO(3) is compact since each column is a unit vector. These orthogonal matrices are bounded in norm. So they are compact