Essentials of Data Science With R Software - 1

Probability and Statistical Inference

Probability Theory

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Lecture 21
Independent Events

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Independent Events:

Intuitively, two events are independent if the occurrence or non-occurrence of one event does not affect the occurrence or non-occurrence of the other event.

In other words, two events *A* and *B* are independent if the probability of occurrence of *B* has no effect on the probability of occurrence of *A*.

In such a situation, one expects that

$$P(A|B) = P(A) \text{ and } P(A|\overline{B}) = P(A)$$

which yields $P(AB) = P(A)\, P(B)$.

Independent Events:

Definition: Two random events *A* and *B* are called (stochastically) independent If $P(AB) = P(A)\,P(B)$.

i.e. if the probability of simultaneous occurrence of both events *A* and *B* is the product of the individual probabilities of occurrence of *A* and *B*.

If two random events *A* and *B* are (stochastically) independent, then

- **$\overline{A}$ and *B* are also independent.**
- ***A* and $\overline{B}$ are also independent.**
- **$\overline{A}$ and $\overline{B}$ are also independent.**

Independent Events:

Definition: The n events $A_1, A_2, \ldots, A_n$ are stochastically mutually independent, if for any subset of m events $A_{i1}, A_{i2}, \ldots, A_{im}$ $(m \leq n)$

$$P(A_{i1}\, A_{i2} \ldots A_{im}) = P(A_{i1})\, P(A_{i2}) \ldots P(A_{im})$$

A weaker form of independence is pairwise independence.

If the above condition is fulfilled only for two arbitrary events, i.e., $m = 2$, then the events are called pairwise independent.

Mutual independence implies pairwise independence.

Converse may not hold true.

Independent Events: Example

An ordinary deck has 52 playing cards.

A card is selected at random from it.

If *K* : Event that the selected card is a king and

***H* : Event that it is a heart,**

Note that $P(KH) = \frac{1}{52}$,

while $P(K) = \frac{4}{52}$ and $P(H) = \frac{13}{52}$,

then *K* and *H* are independent, as $P(KH) \neq P(K) \times P(H)$.