Essentials of Data Science With R Software - 1

Probability and Statistical Inference

Probability Theory

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Lecture 15

Basic Principle of Counting-Ordered Set, Unordered Set and Permutations

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Sample Spaces having Equally Likely Outcomes

For any event *A*,

$$P(A) = \frac{\textbf{Number of points in } A}{\boldsymbol{N}}$$

In other words, if we assume that each outcome of an experiment is equally likely to occur, then

the probability of any event *A* = the proportion of points in the sample space that are contained in *A*.

Thus, to compute probabilities it is necessary to know the number of different ways to count given events.

Basic Principle of Counting:

Suppose that two experiments are to be performed.

Suppose experiment 1 can result in any one of *m* possible outcomes and if, for each outcome of experiment 1, there are *n* possible outcomes of experiment 2,

then together there are *mn* possible outcomes of the two experiments.

$$
\begin{array}{l}
(1, 1), (1, 2), \ldots, (1, n) \\
(2, 1), (2, 2), \ldots, (2, n) \\
\ldots \\
(m, 1), (m, 2), \ldots, (m, n)
\end{array}
$$

Ordered and Unordered Sets:

Suppose three balls of different colours, black, grey, and white, are drawn.

Now there are two options:

1. **The first option is to take into account the order in which the balls are drawn.**

   In such a situation, two possible sets of balls such as (black, grey, and white) and (white, black, and grey) constitute two different sets.

Such a set is called an *ordered set*.

Ordered and Unordered Sets:

2. **In the second option, we do not take into account the order in which the balls are drawn.**

   In such a situation, the two possible sets of balls such as

   (black, grey, and white) and (white, black, and grey)

   are the same sets and constitute an *unordered set* of balls.

A group of elements is said to be *ordered* if the order in which these elements are drawn is of relevance.

Otherwise, it is called *unordered*.

Ordered and Unordered Sets: Examples

- **Ordered samples:**

– The first three places in a 100m race are determined by the order in which the athletes arrive at the finishing line.

If 8 athletes are competing with each other, the number of possible results for the first three places is of interest.

– In a lottery with two prizes, the first drawn lottery ticket gets the first prize and the second lottery ticket gets the second prize.

Ordered and Unordered Sets: Examples

- **Unordered samples:**

– The selected members for a football team. The order in which the selected names are announced is irrelevant.

– Fishing 20 fish from a lake.

– A bunch of 10 flowers made from 21 flowers of 4 different colours

Factorial function:

The factorial function $n!$ is defined as

$$n! = 1 \times 2 \times 3 \times \cdots \times n \text{ for } n > 0 \qquad \text{and} \qquad 0! = 1.$$

Thus $1! = 1$

$2! = 1 \times 2 = 2,$

$3! = 1 \times 2 \times 3 = 6\ .$

This can be calculated in R as follows:

```
factorial(n)
```

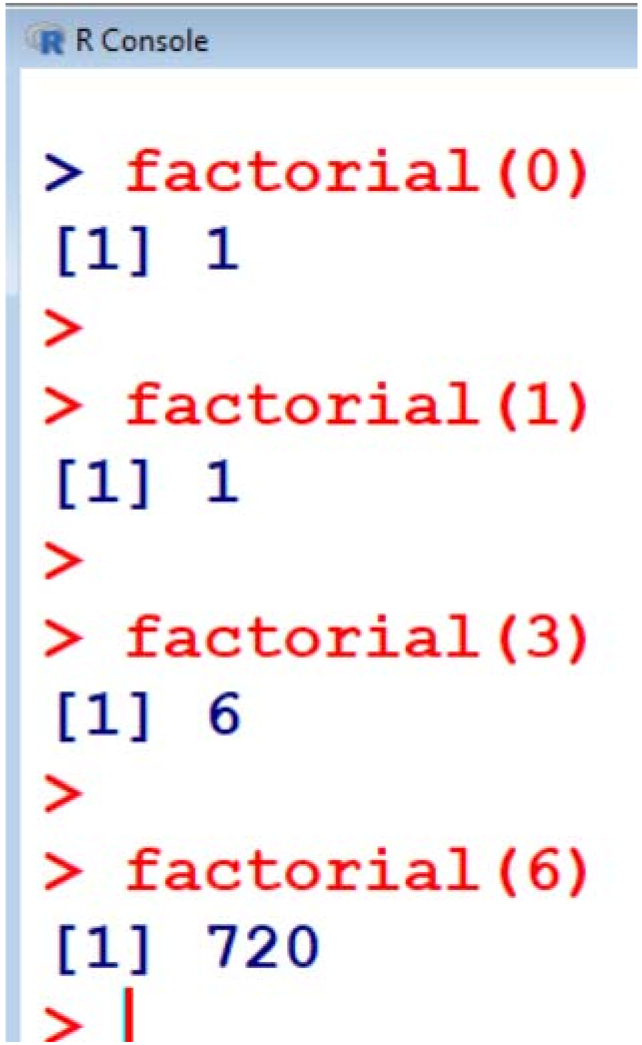
Factorial function in R:

Factorial function can be calculated in R as follows:

```
factorial(n)
```

Example:

```
> factorial(0)
[1] 1
> factorial(1)
[1] 1
> factorial(3)
[1] 6
> factorial(6)
[1] 720
```



Permutation:

Consider a set of *n* elements.

Each ordered composition of these *n* elements is called a permutation.

We distinguish between two cases:

- **If all the elements are distinguishable, then we speak of <u>permutation without replacement</u>.**
- **If some or all of the elements are not distinguishable, then we speak of <u>permutation with replacement</u>.**

Note: the meaning of "replacement" is just a convention and does not directly refer to the drawings.

Permutations Without Replacement :

Consider a set of *n* elements.

If all the *n* elements are distinguishable, then there are $n!$ different compositions of these elements.

Example: **There are 3 students who will get three ranks – First (F), Second (S) and Third (T).**

There are $3! = 6$ possible ways in which they can be ranked.

(F, S, T), (F, T, S), (S, T, F), (S, F, T), (T, F, S), (T, S, F)

Permutations Without Replacement :

Example: **A person has 10 books that he is going to put on his bookshelf. Of these,**

- **4 are mathematics books,**
- **3 are chemistry books,**
- **2 are history books, and**
- **1 is a language book.**

He wants to arrange his books so that all the books dealing with the same subject are together on the shelf.

We want to know the total number of possible different arrangements.

Permutations Without Replacement :

Solution:

There are 4! 3! 2! 1! arrangements such that the mathematics books are first in line, then the chemistry books, then the history books, and then the language book.

Similarly, for each possible ordering of the subjects, there are

4! 3! 2! 1! possible arrangements.

Hence, as there are 4! possible orderings of the subjects, the desired answer is

4! 4! 3! 2! 1! = 6,912.

Permutations With Replacement :

Consider a set of *n* elements.

Assume that not all *n* elements are distinguishable.

The elements are divided into groups, and these groups are distinguishable.

Suppose, there are *s* groups of sizes $n_1, n_2, \ldots, n_s$.

The total number of different ways to arrange the *n* elements in *s* groups is

$$\frac{n!}{n_1!n_2!\cdots n_s!}.$$

Permutations With Replacement :

Example: **There were 10 students and there are 3 types of chocolates- C1, C2 and C3. The total number of ways in which two C1, three C2 and five C3 chocolates can be given to the 10 students is obtained as follows:**

$n_1 = 2, n_2 = 3, n_3 = 5, n = 10$

The total number of different ways to arrange the *n* = 10 elements in 3 groups is

$$\frac{10!}{2!3!5!}.$$

Factorial function:

This can be calculated in R as follows:

```
factorial(10)/(factorial(2)*factorial(3)*factori
al(5))
[1] 2520
```