Essentials of Data Science With R Software - 1

Probability and Statistical Inference

Probability Theory

:::

Lecture 18

Multiplication Theorem of Probability

Shalabh

Department of Mathematics and Statistics

Indian Institute of Technology Kanpur

Multiplication Theorem of Probability:

For two arbitrary events A and B, the following holds:

$$P(A\,B) = P(A|B)P(B) = P(B|A)P(A).$$

This theorem does not require that $P(A) > 0$ and $P(B) > 0$).

A generalization of this result provides an expression for the probability of the intersection of an arbitrary number of events.

Assume that $A_1, A_2, \ldots, A_m$ are events

$$P(A_1 A_2 \ldots A_m) = P(A_1)\; P(A_2|A_1)\; P(A_3|A_1 A_2) \ldots P(A_m|A_1 A_2 \ldots A_{m-1})$$

Multiplication Theorem of Probability: Example 1

A student figures that there is a 30 percent chance that he will be selected in the cricket team. If it does, he has 60 percent certain that he will be selected as Captain of the team. What is the probability that the student will be the captain in the selected team?

Solution:

Let T : Event that the student will be selected in the team.

C : Event that the student will be made the captain,

then the desired probability is $P(TC)$.

$$P(TC) = P(T)P(C|T) = (.3)(.6) = 0.18$$

So there is an 18% chance that the student will be the captain.

Multiplication Theorem of Probability: Example 2

A student is undecided as to whether to take a French course or a chemistry course. He estimates that his probability of receiving an *A* grade would be 1/2 in a French course, and 2/3 in a chemistry course. If he decides to base his decision on the flip of a fair coin, what is the probability that he gets an *A* in chemistry?

Solution:

Let *C* : event that student takes chemistry and

***A* : Event that he receives an *A* in whatever course he takes,**

then the desired probability is *P*(*CA*). This is calculated as follows:

$$P(CA) = P(C)P(A|C) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}.$$

Multiplication Theorem of Probability: Example 3

An ordinary deck of 52 playing cards is randomly divided into 4 piles of 13 cards each. Compute the probability that each pile has exactly 1 ace.

Solution: **Define events E_i , i = 1, 2, 3, 4 as follows:**

E_1 = Event that the ace of spades is in any one of the piles.

E_2 = Event that the ace of spades and the ace of hearts are in different piles

E_3 = Event that the aces of spades, hearts, and diamonds are all in different piles

E_4 = Event that all 4 aces are in different piles

Multiplication Theorem of Probability: Example 3

The probability desired is $P(E_1E_2E_3E_4)$ and by the multiplication rule

$$P(E_1E_2E_3E_4) = P(E_1) \,.\, P(E_2|E_1) \,.\, P(E_3|E_1\,E_2) \,.\, P(E_4|E_1\,E_2\,E_3)$$

- Now $P(E_1) = 1$ since E_1 is the sample space Ω.
- $P(E_2 \mid E_1) = \frac{39}{51}$ since the pile containing the ace of spades will receive 12 of the remaining 51 cards.
- $P(E_3 \mid E_1\,E_2) = \frac{26}{50}$ since the piles containing the aces of spades and hearts will receive 24 of the remaining 50 cards; and finally,
- $P(E_4 \mid E_1\,E_2\,E_3) = \frac{13}{49}$

Multiplication Theorem of Probability: Example 3

Therefore, we obtain that the probability that each pile has exactly 1 ace is

$$P(E_1 E_2 E_3 E_4) = \frac{39}{51} \times \frac{26}{50} \times \frac{13}{49} = 0.105 \text{ (Approximately)}$$

There is approximately a 10.5 percent chance that each pile will contain an ace.