Essentials of Data Science With R Software - 1

Probability and Statistical Inference

Probability Theory

:::

Lecture 16

Basic Principle of Counting- Combinations

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Combinations:

The Binomial coefficient for any integers *m* and *n* with *n* ≥ *m* ≥ 0

is denoted and defined as

$$\binom{n}{m} = \frac{n!}{m!(n-m)!}.$$

It is read as "*n* choose *m*" and can be calculated in *R* using the

following command:

```
choose(n,m)
```

Combinations:

We now answer the question of how many different possibilities exist to draw *m* out of *n* elements, i.e. *m* out of *n* balls from an urn.

It is necessary to distinguish between the following four cases:

1. **Combinations without replacement and without consideration of the order of the elements.**
2. **Combinations without replacement and with consideration of the order of the elements.**
3. **Combinations with replacement and without consideration of the order of the elements.**
4. **Combinations with replacement and with consideration of the order of the elements.**

1. Combinations without replacement and without consideration of the order of the elements:

When there is no replacement and the order of the elements is also not relevant, then the total number of distinguishable combinations in drawing *m* out of *n* elements is

$$\binom{n}{m} = \frac{n!}{m!(n-m)!} \; .$$

This result can be obtained in *R* by using the command

```
choose(n,m)
```

1. Combinations without replacement and without consideration of the order of the elements:

Example: Suppose a company elects a new board of directors. The board consists of 6 members and 10 people are eligible to be elected. How many combinations for the board of directors exist?

Since a person cannot be elected twice, we have a situation where there is no replacement. The order is also of no importance: either one is elected or not.

$$\binom{10}{6} = \frac{10!}{6!\,(10-6)!} = 210$$

possible combinations.

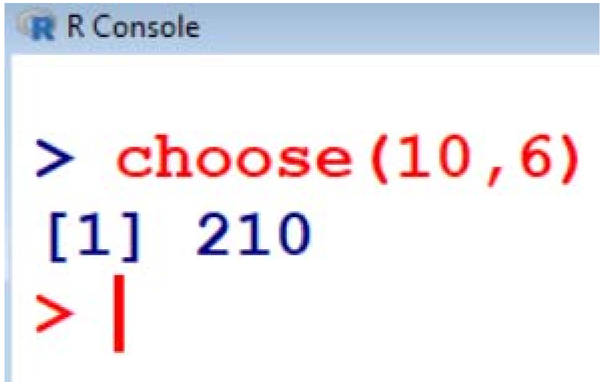
1. Combinations without replacement and without consideration of the order of the elements:

Example: **(Contd.)**

This result can be obtained in *R* by using the command

```
choose(10,6)
```

```
> choose(10,6)
[1] 210
```



2. Combinations without replacement and with consideration of the order of the elements:

The total number of different combinations for the setting without replacement and with consideration of the order is

$$\frac{n!}{(n-m)!} = m! \binom{n}{m}.$$

This can be calculated in *R* as follows:

```
factorial(n)/factorial(n-m)
```

or

```
factorial(m)*choose(n,m)
```

2. Combinations without replacement and with consideration of the order of the elements:

Example:

Consider a race with 10 students. A possible bet is to forecast the winner of the race, the second student of the race, and the third student of the race.

The total number of different combinations for the students in the first three places is $\frac{10!}{(10-3)!}$.

2. Combinations without replacement and with consideration of the order of the elements:

Example: (Contd.) This result can be explained intuitively:

- **For the first place, there is a choice of 10 different students.**
- **For the second place, there is a choice of 9 different students (10 students minus the winner).**
- **For the third place, there is a choice of 8 different students (10 students minus the first and second students).**
- **The total number of combinations is $10 \times 9 \times 8$.**

2. Combinations without replacement and with consideration of the order of the elements:

Example: (Contd.) This can be calculated in *R* as follows:

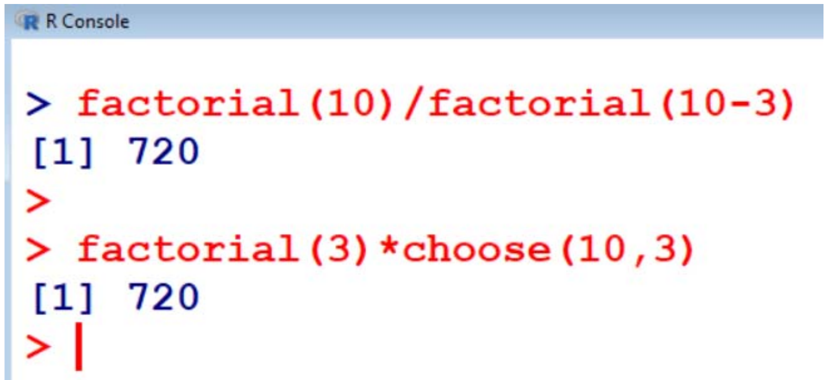
```
10 * 9 * 8
```

or

```
factorial(10)/factorial(10-3)
[1] 720
```

or

```
factorial(3)*choose(10,3)
[1] 720
```



3. Combinations with replacement and without consideration of the order of the elements:

The total number of different combinations with replacement and without consideration of the order is

$$\binom{n+m-1}{m} = \frac{n+m-1!}{m!(n-1)!} = \binom{n+m-1}{n-1}.$$

This can be calculated in *R* as follows:

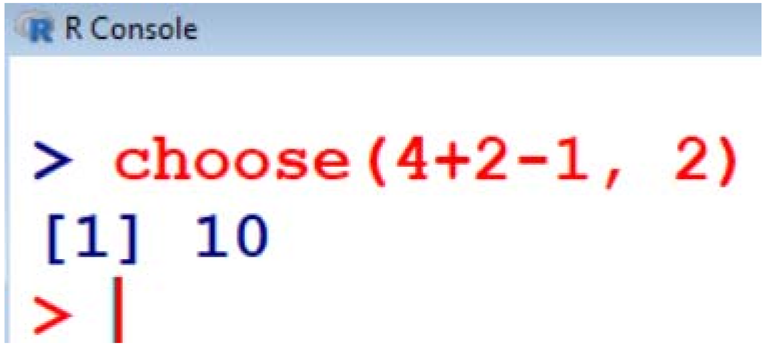
```
choose(n+m-1, m)
```

3. Combinations with replacement and without consideration of the order of the elements:

Example: **A farmer has 2 fields and aspires to cultivate one out of 4 different organic products per field. Then, the total number of choices he has is** $\binom{4+2-1}{2} = 10.$

This can be calculated in *R* as follows:

```
choose(4+2-1, 2)
[1] 10
```



3. Combinations with replacement and without consideration of the order of the elements:

Example: (Contd.)

If 4 different organic products are denoted as a, b, c, and d, then the following combinations are possible:

(a, a) (a, b) (a, c) (a, d)

(b, b) (b, c) (b, d)

(c, c) (c, d)

(d, d)

Please note that, for example, (a, b) is identical to (b, a) because the order in which the products a and b are cultivated on the first or second field is not important in this example.

4. Combinations with replacement and with consideration of the order of the elements:

The total number of different combinations for the integers *m* and *n* with replacement and when the order is of relevance is

$$n^m \,.$$

This can be calculated in *R* as follows:

```
n^m
```

or

```
n**m
```

4. Combinations with replacement and with consideration of the order of the elements:

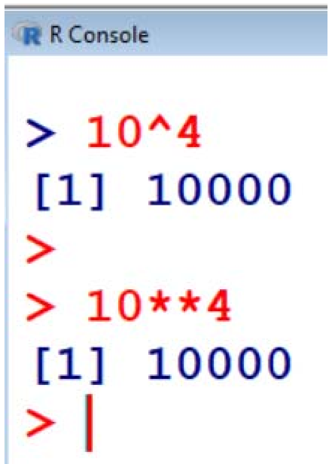
Example: **Consider a credit card with a four-digit ATM personal identification number (PIN) code. The total number of possible combinations for the PIN is**

$$n^m = 10^4 = 10000.$$

Note that every digit in the first, second, third, and fourth places (m = 4) can be chosen out of ten digits from 0 to 9 (n = 10).

This can be calculated in *R* as follows:

```
> 10^4
[1] 10000
> 10**4
[1] 10000
```



Example 1:

A committee of 5 is to be selected from a group of 6 men and 9 women. If the selection is made randomly, what is the probability that the committee consists of 3 men and 2 women?

Solution: **Let us assume that randomly selected means that each of the $\binom{15}{5}$ possible combinations is equally likely to be selected. Hence the probability that committee consists of 3 men and 2 women**

$$\frac{\binom{6}{3}\binom{9}{2}}{\binom{15}{5}} = \frac{240}{1001}$$

Example 2:

An urn contains *n* balls, of which one is special. If *k* of these balls are withdrawn one at a time, with each selection being equally likely to be any of the balls that remain at the time, what is the probability that the special ball is chosen?

Solution: **Since all of the balls are treated in an identical manner, it follows that the set of *k* balls selected is equally likely to be any of the $\binom{n}{k}$ sets of *k* balls. Therefore,**

$$P(\text{special ball is selected}) = \frac{\binom{1}{1}\binom{n-1}{k-1}}{\binom{n}{k}} = \frac{k}{n}$$

Example 3:

Following number of members of a club play tennis, squash and cricket:

	Tennis	Squash	Cricket
Number of players	36	28	18

Furthermore,

	Tennis and squash	Squash and cricket	Tennis and cricket	Tennis, squash and cricket
Number of players	22	9	12	4

How many members of this club play at least one of these sports?

Example 3:

Solution: Let

N denote the number of members of the club, and

introduce probability by assuming that a member of the club is

randomly selected.

If for any subset C of members of the club,

$P(C)$ denote the probability that the selected member is contained

in C, then

$$P(C) = \frac{\text{number of members in } C}{N}$$

Example 3:

Now, with

T **: Set of members that plays tennis,**

S **: Set that plays squash, and**

C **: Set that plays cricket, we have**

$$P(T \cup S \cup C) = P(T) + P(S) + P(C) - P(TS) - P(TC) - P(SC) + P(TSC)$$

$$= \frac{36 + 28 + 18 - 22 - 12 - 9 + 4}{N} = \frac{43}{N}$$

Hence we can conclude that 43 members play at least one of the sports.