Essentials of Data Science With R Software - 1

Probability and Statistical Inference

Probability Theory

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Lecture 9

Sample Space and Events

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Experiment:

Any activity for which the outcome is uncertain can be thought of as an "experiment."

The uncertainty concerns in the sense that the outcome of the experiment is not known until the experiment is completed.

The outcome is not predictable with certainty in advance.

For example:

- **drawing a card from a deck to observe which card is drawn ,**
- **tossing a coin to observe what turns up – Head or Tail.**

Sample Space and Events:

The outcome of the experiment is not known in advance, but we assume that all the possible outcomes of the experiment are known.

This set of all possible outcomes of an experiment is known as the sample space of the experiment and is denoted by Ω.

Any subset *E* of the sample space is known as an event.

An event is a set of possible outcomes of the experiment.

If the outcome of the experiment is contained in *E*, then we say that *E* has occurred.

Sample Space and Events:

For example:

1. If the outcome of an experiment consists in the determination of the gender of a newly born child, then

$$\Omega = \{M, F\}$$

where M and F indicates Male and Female child, respectively.

If *E* = {M}, then *E* is the event that the child is a male (boy).

Similarly, if *E* = {F}, then *E* is the event that the child is a female (girl).

Sample Space and Events:

For example:

2. If three students have to get three positions in a game – First (1), Second (2) and Third (3). Then the experiment consists of the participating in the game with positions 1, 2, and 3, then

Ω = {all orderings of (1, 2, 3)}

The outcome (2, 3, 1) means student one gets position 2, student two gets position 3 and student one gets position 1 and so on.

If *E* = {3, 2, 1} then student one gets position 3, student two gets position 2 and student one gets position 1.

Sample Space and Events:

For example:

3. An experiment is conducted to know the dosage of a medicine. The dosage is increased continuously until a patient reacts positively.

One possible sample space for this experiment is to let Ω consist of all the positive numbers, so

$$\Omega = (0, \infty)$$

where the outcome would be *x* if the patient starts getting the dosage and reacts when the value of dosage reaches *x* .

No reaction to any smaller dosage than *x*.

Sample Space and Events: Symbols and Notations

Outcome of a random experiment is called a simple event (or elementary event) and denoted by ω.

Sample space : $\Omega = \{\omega_1, \omega_2, \ldots, \omega_k\}$ is the set of all possible outcomes $\{\omega_1, \omega_2, \ldots, \omega_k\}$.

Subsets of Ω are called events and are denoted by capital letters, in general, such as *A*, *B*, *C*.

Sample Space and Events: Complementary and Sure Events

Ω_A : Set of all simple events that are contained in the event *A*

The event $\bar{A}$ refers to the non-occurring of *A* and is called a composite or complementary event.

Also Ω is an event.

Since it contains all possible outcomes, we say that Ω will always occur and is called a sure event or certain event.

Sample Space and Events: Impossible Event

On the other hand, if we consider the null set Ø = {} as an event, then this event can never occur and is called an impossible event.

The sure event therefore is the set of all elementary events, and the impossible event is the set with no elementary events.

Sample Space and Events: Example - Rolling a die:

Rolling a die: If a die is rolled once, then the possible outcomes are the number of dots on the upper surface: 1, 2, . . . , 6.

Sample space is the set of simple events

ω_1 = "1", ω_2 = "2", ω_3 = "3", ω_4 = "4", ω_5 = "5", ω_6 = "6".

$\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6\}$.

Event *A*: "An even number of dots on the upper surface of the die".

There are three possibilities that this event occurs: ω_2, ω_4, ω_6.

Complementary event of *A*: $\bar{A}$: If an odd number shows up.

There are three possibilities that this event occurs: ω_1, ω_3, ω_5.

Sample Space and Events: Example - Rolling a die:

Elementary event is an event defined to observe only one particular number, say ω_1 = "1", then it is an elementary event.

Sure event is the event that "a number which is greater than or equal to 1" because any number between 1 and 6 is greater than or equal to 1.

Impossible event is the event that "the number is 7".

Sample Space and Events: Example - Rolling two dice

Rolling two dice : Suppose we throw two dice simultaneously.

Event is defined as the "number of dots observed on the upper surface of both the dice".

Then, there are 36 simple events defined as

(number of dots on first die, number of dots on second die),

i.e. $\omega_1 = (1, 1), \omega_2 = (1, 2), \ldots, \omega_{36} = (6, 6)$.

Sample Space and Events: Example - Rolling two dice

Therefore Ω is

Ω = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}.

Sample Space and Events: Example - Rolling two dice

One can define different events and their corresponding sample spaces.

For example,

- **if an event *A* is defined as "upper faces of both the dice contain the same number of dots", then the sample space is**

$$\Omega_A = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}.$$

- **If another event *B* is defined as "the sum of numbers on the upper faces is 6", then the sample space is**

$$\Omega_B = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}.$$

Sample Space and Events: Example - Rolling two dice

A sure event is “get either an even number or an odd number”

An impossible event would be “the sum of the two dice is greater than 13”.