Essentials of Data Science With R Software - 1

Probability and Statistical Inference

Probability Theory

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Lecture 14

Some Rules of Probability

Shalabh

Department of Mathematics and Statistics

Indian Institute of Technology Kanpur

Some Rules of Probability:

- **The probability of occurrence of an impossible event ϕ is zero:**

$$P(\phi) = 1 - P(\Omega) = 0.$$

- **The probability of occurrence of a sure event is one:**

$$P(\Omega) = 1.$$

- **The probability of the complementary event of *A*, (i.e. $\bar{A}$) is**

$$P(\bar{A}) = 1 - P(A).$$

Some Rules of Probability:

- The *odds* of an event *A* is defined by

$$\frac{P(A)}{P(\bar{A})} = \frac{P(A)}{1 - P(A)}$$

Thus the odds of an event *A* tells how much more likely it is that *A* occurs than that it does not occur.

Example: if *P*(*A*) = 3/4, then $\frac{P(A)}{1-P(A)} = 3$, so the odds are 3.

Consequently, it is 3 times as likely that *A* occurs as it is that it does not.

Some Rules of Probability:

Example: Suppose a box of 30 ice creams contains ice creams of 6 different flavours with 5 ice creams of each flavour.

Suppose an event *A* is defined as *A* = {“vanilla flavour”}.

Probability of finding a vanilla flavour ice cream (without looking into the box) = *P*(“vanilla flavour”) = 5/30.

Then, the probability of the complementary event $\bar{A}$, i.e. the probability of not finding a vanilla flavour ice cream is *P*(“no vanilla flavour”) = 1 – *P*(“vanilla flavour”) = 25/30.

Some Rules of Probability: Additive Theorem of Probability

Let A_1 and A_2 be not necessarily disjoint events.

The probability of occurrence of A_1 or A_2 is

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2).$$

The meaning of "or" is in the statistical sense: either A_1 is occurring, A_2 is occurring, or both of them.

Some Rules of Probability: Additive Theorem of Probability

Example: **A total of 28% people like sweet snacks, 7% like salty snacks, and 5% like both sweet and salty snacks. The percentage of people like neither sweet nor salty snacks is obtained as follows:**

Let A_1 be the event that a randomly chosen person likes sweet snacks and A_2 be the event that a randomly chosen person likes salty snacks.

Then, the probability this person likes either sweet or salty snacks is

$$P(A_1 \cup A2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = 0.07+0.28-0.05 = 0.30$$

Thus 70% of people does not like either sweet or salty snacks.

Sample Spaces having Equally Likely Outcomes

For a large number of experiments, it is natural to assume that each point in the sample space is equally likely to occur.

For many experiments whose sample space Ω is a finite set, say $\Omega = \{1, 2, \ldots, N\}$, it is often natural to assume that

$$P(\{1\}) = P(\{2\}) = \cdots = P(\{N\}) = p \text{ (say)}$$

$$P(\Omega) = P(\{1\}) + \cdots + P(\{N\})$$

or $1 = Np$

or $P(\{i\}) = p = \frac{1}{N}$

Sample Spaces having Equally Likely Outcomes

For any event *A*,

$$P(A) = \frac{\textbf{Number of points in } A}{\boldsymbol{N}}$$

In other words, if we assume that each outcome of an experiment is equally likely to occur, then

the probability of any event *A* = the proportion of points in the sample space that are contained in *A*.

Thus, to compute probabilities it is necessary to know the number of different ways to count given events.