Essentials of Data Science With R Software - 1

Probability and Statistical Inference

Probability Theory

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Lecture 10

Set Theory and Events Using Venn Diagrams

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Union of Events:

Suppose *A* and *B* are any two events of a sample space Ω.

Define a new event $A \cup B$

$A \cup B$ is called the union of the events *A* and *B.*

$A \cup B$ consist of all outcomes that are

- **either in *A***
- **or in *B***
- **or in both *A* and *B*.**

Union of Events:

For example:

If the outcome of an experiment consists in the determination of the gender of a newly born child, then Ω = {M, F} where M and F indicates Male and Female child, respectively.

If A = {M}, then A is the event that the child is a male (boy).

Similarly, if B = {F}, then B is the event that the child is a female (girl).

Then $A \cup B$ = {M , F}, i.e., $A \cup B$ is the whole sample space Ω.

$\Omega = A \cup B$ is called as sure event

Intersection of Events:

Suppose A and B are any two events of a sample space Ω.

Define a new event $A \cap B$

$A \cap B$ is called the intersection of the events A and B.

$A \cap B$ consist of all outcomes that are in both A and B.

Event $A \cap B$ will occur if both A and B occur.

Events with Venn Diagram using Set Theory:

It is possible to view events as sets of simple events.

This helps to determine how different events relate to each other.

A popular technique to visualize this approach is to use Venn diagrams.

Events with Venn Diagram using Set Theory:

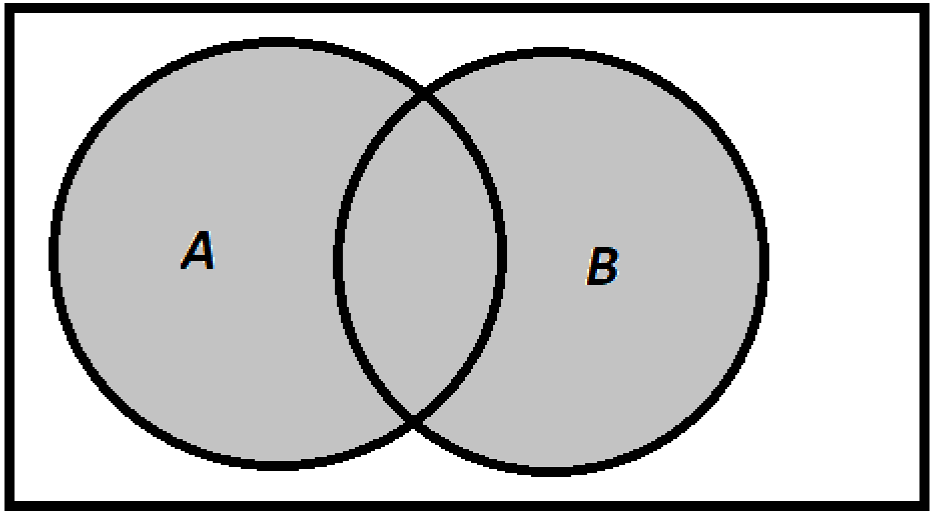
In Venn diagrams, two or more sets are visualized by circles.

Overlapping circles

Separated circles

Events with Venn Diagram using Set Theory:

We use the following notations:



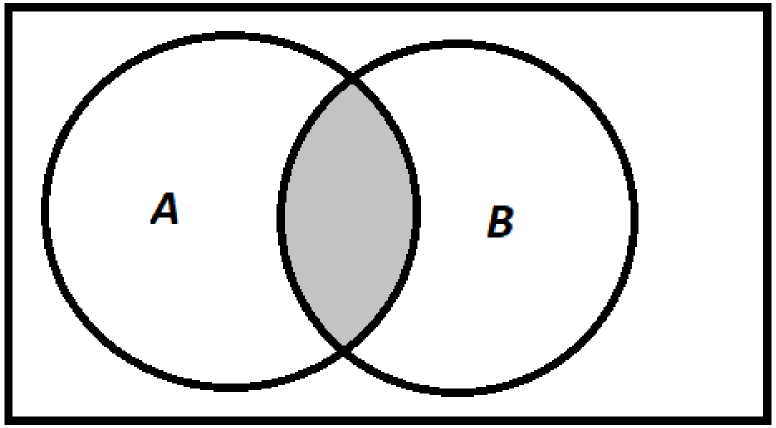
$A \cup B$: The union of events

$A \cup B$ is the set of all simple events *A* and *B* which occur when a simple Events *A* or *B* occurs (grey shaded area in figure).

Please note that we use the word "or" from a statistical perspective: "*A* or *B*" means that either a simple event from *A* occurs, or a simple event from *B* occurs, or a simple event which is part of both *A* and *B* occurs.

Events with Venn Diagram using Set Theory:

We use the following notations:



***A* ∩ *B* : The intersection of events *A* ∩ *B* is the set of all simple events of *A* and *B* which occurs when the simple events of *A* and *B* occur (grey shaded area in figure).**

Please note that we use the word "and" from a statistical perspective: "*A* and *B*" means that both simple events from *A* and from *B* occur.

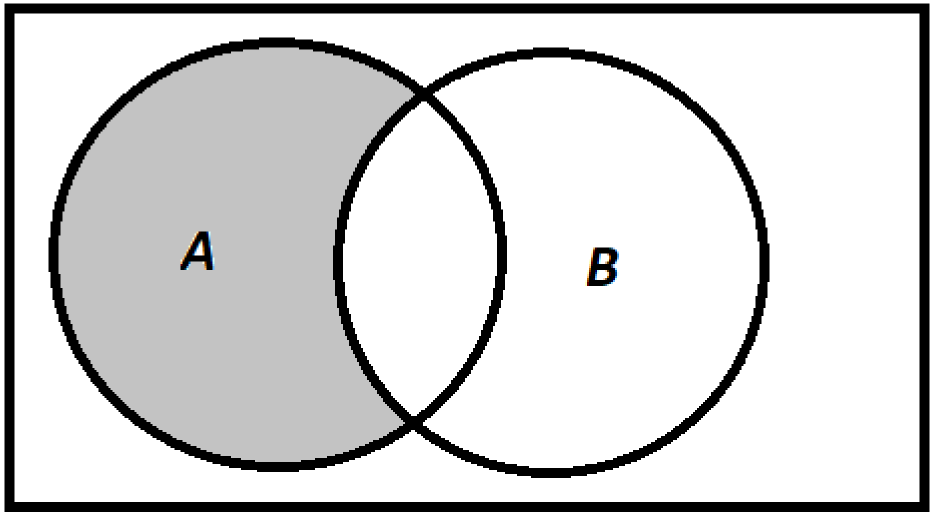
***A* ∩ *B* is also represented as *AB*.**

Events with Venn Diagram using Set Theory:

We use the following notations:

A - B : The event A - B contains all simple events of A, which are not contained in B.

(grey shaded area in figure).



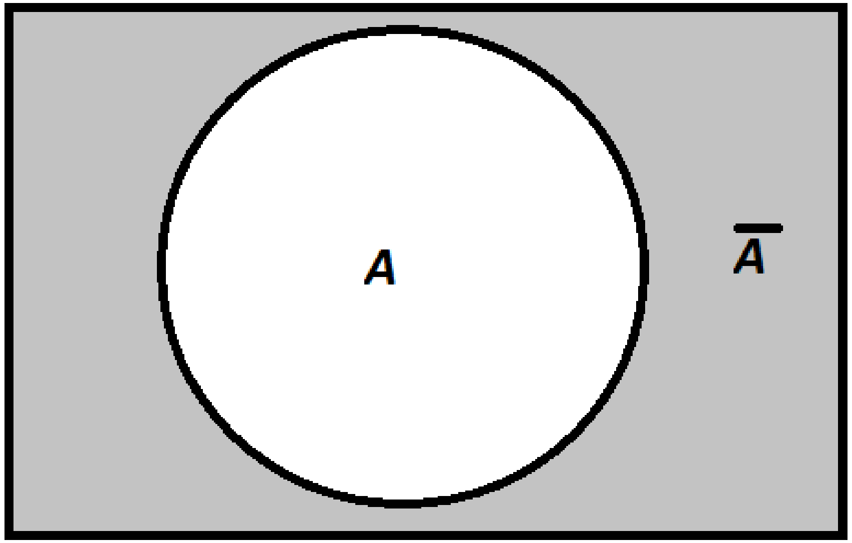
The event "A but not B" or "A minus B" occurs, if A occurs but B does not occur.

Also A - B = $A \cup \overline{B}$

Events with Venn Diagram using Set Theory:

We use the following notations:

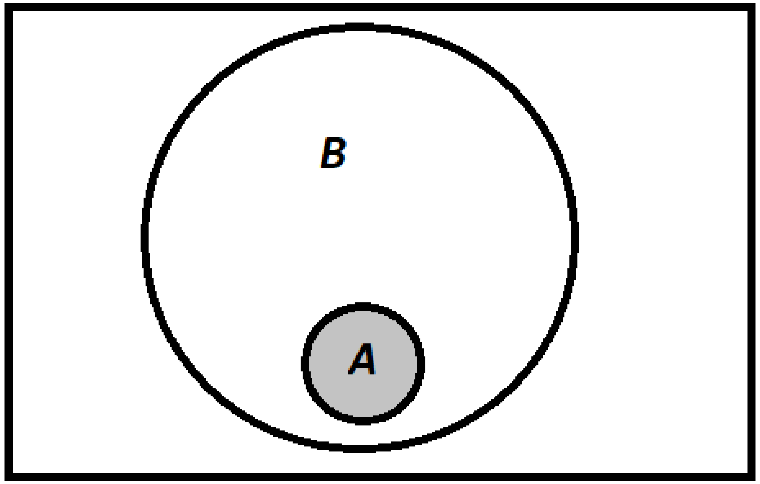
$\overline{A}$: The event $\overline{A}$ contains all simple events of Ω, which are not contained in *A*.



The complementary event of A (which is "Not-*A*" or "$\overline{A}$" occurs whenever *A* does not occur (grey shaded area in figure)

Events with Venn Diagram using Set Theory:

We use the following notations:



***A* ⊆ *B*: *A* is a subset of *B*. This means That all simple events of *A* are also part of the sample space of *B*.**

Events with Venn Diagram : Example - Rolling a die

Rolling a die: If a die is rolled once, then the possible outcomes are the number of dots on the upper surface: 1, 2, . . . , 6.

Sample space is the set of simple events

$\omega_1 = $ "1", $\omega_2 = $ "2", $\omega_3 = $ "3", $\omega_4 = $ "4", $\omega_5 = $ "5", $\omega_6 = $ "6".

$\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6\}$.

- **If $A = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$ and B is the set of all odd numbers, then $B = \{\omega_1, \omega_3, \omega_5\}$ and thus $B \subseteq A$.**

Events with Venn Diagram using Set Theory: Example - Rolling a die:

- **If $A = \{\omega_2, \omega_4, \omega_6\}$ is the set of even numbers and $B = \{\omega_3, \omega_6\}$ is the set of all numbers which are divisible by 3, then $A \cup B = \{\omega_2, \omega_3, \omega_4, \omega_6\}$ is the collection of simple events for which the number is either even or divisible by 3 or both.**

Events with Venn Diagram using Set Theory: Example - Rolling a die:

- If $A = \{\omega_1, \omega_3, \omega_5\}$ is the set of odd numbers and

  $B = \{\omega_3, \omega_6\}$ is the set of the numbers which are divisible by 3,

  then $A \cap B = \{\omega_3\}$ is the set of simple events in which the numbers are odd and divisible by 3.

- If $A = \{\omega_1, \omega_3, \omega_5\}$ is the set of odd numbers and

  $B = \{\omega_3, \omega_6\}$ is the set of the numbers which are divisible by 3,

  then $A - B = \{\omega_1, \omega_5\}$ is the set of simple events in which the numbers are odd but not divisible by 3.

Events with Venn Diagram using Set Theory: Example - Rolling a die:

- **If A = {ω_2, ω_4, ω_6} is the set of even numbers, then**

  $\bar{A}$ = {ω_1, ω_3, ω_5} is the set of odd numbers.

Disjoint Events with Set Theory

Two events *A* and *B* are disjoint if $A \cap B = \emptyset$ holds,

i.e. if both events cannot occur simultaneously.

Example:

The events *A* and $\bar{A}$ are disjoint events.

Mutually Disjoint Events with Set Theory

The events $A_1, A_2, \ldots, A_m$ are said to be mutually or pairwise disjoint, if $A_i \cap A_j = \emptyset$ whenever $i \neq j = 1, 2, \ldots, m$.

Example: Rolling a die: If a die is rolled once, then the possible outcomes are the number of dots on the upper surface: 1, 2, . . . , 6.

$$\omega_1 = \text{“1”}, \quad \omega_2 = \text{“2”}, \quad \omega_3 = \text{“3”}, \quad \omega_4 = \text{“4”}, \quad \omega_5 = \text{“5”}, \quad \omega_6 = \text{“6”}.$$

$$\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6\}.$$

If $A = \{\omega_1, \omega_3, \omega_5\}$ and $B = \{\omega_2, \omega_4, \omega_6\}$ are the sets of odd and even numbers, respectively, then the events A and B are disjoint.

Unions of More than Two Events:

We can also define unions of more than two events.

Union of the events $A_1, A_2, \ldots, A_m$, denoted by $A_1 \cup A_2 \cup \ldots \cup A_m$ is defined to be the event consisting of all outcomes that are in A_i for at least one $i = 1, 2, \ldots, m$.

In other words, the union of the A_i occurs when at least one of the events A_i occurs.

Intersections of More than Two Events:

We can also define intersections of more than two events.

Intersection of the events $A_1, A_2, \ldots, A_m$,

denoted by $A_1 \cap A_2 \cap \ldots \cap A_m$ is defined to be the event consisting of those outcomes that are in all of the events A_i, $i = 1, 2, \ldots, m$.

In other words, the intersection occurs when all of the events A_i occur.