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Kinematic Control of Free Rigid Bodies Using Dual Quaternions

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Abstract: This paper proposes a new type of control laws for free rigid bodies. The start point is the dual quaternion and its characteristics. The logarithm of a dual quaternion is defined, based on which kinematic control laws can be developed. Global exponential convergence is achieved using logarithmic feedback via a generalized proportional control law, and an appropriate Lyapunov function is constructed to prove the stability. Both the regulation and tracking problems are tackled. Omnidirectional control is discussed as a case study. As the control laws can handle the interconnection between the rotation and translation of a rigid body, they are shown to be more applicable than the conventional method.

Keywords: Kinematic control, dual quaternion, omnidirectional control, proportional control.

1 Introduction

As an efficient tool to describe rigid transformation, the dual quaternion has been applied in various fields, such as robotics, mechanical design, and inertial navigation[1−3]. By the principle of transference[4], the dual quaternion is a natural generalization of quaternion. It is a pity that dual quaternions seldom play a certain role in the control of free rigid bodies, as quaternions do in attitude control. From another viewpoint, the dual quaternion is a representation of $SE(3)$, the special Euclidean group[5]. Control problems on $SE(3)$ have been discussed by Bullo and Murray[6], where generalized PD laws using logarithmic feedback are developed. In this paper, we aim at similar results, yet using the dual quaternion language to achieve efficient computation.

To do that, the logarithm and a new type of dual quaternion norm, together with the distance between dual quaternions, will be defined. The new notations provide deeper insight into dual quaternions, and paves the way for the design of control laws.

As has been revealed by Bullo and Murray[6], the rotation of a rigid body affects its translation. In this paper we will show that the interesting fact has more than theoretic value. By compensating for the interconnection between rotations and translations, we can adjust the shape of trajectories, which is desired in practice but cannot be achieved if rotation and translation are controlled separately.

The rest of this paper is organized as follows. In Section 2, the logarithm of dual quaternion and related notations are presented, laying the foundation of the next section. Kinematic control laws based on dual quaternion are derived in Section 3, and validated by a case study on omnidirectional control in Section 4. In the same section, these laws are analyzed and compared to the conventional method. Conclusions are given in Section 5.

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2 Dual quaternion and its logarithm

To begin with, we present some necessary definitions about quaternions and dual quaternions. Some details are omitted, which can be found in [3, 7].

2.1 Quaternion and its logarithm

The usual definition of a quaternion is

$$q = (s, \boldsymbol{v}). \tag{1}$$

The quaternion multiplication $\circ$ is defined as

$$q_1 \circ q_2 = (s_1 s_2 - \boldsymbol{v}_1 \cdot \boldsymbol{v}_2, s_1 \boldsymbol{v}_2 + s_2 \boldsymbol{v}_1 + \boldsymbol{v}_1 \times \boldsymbol{v}_2).$$

The conjugation and the norm of a quaternion are defined as

$$q^* = (s, -\boldsymbol{v}) \tag{2}$$
$$\| q \|^2 = q \circ q^*. \tag{3}$$

If $\| q \| = 1$ is satisfied, we get a unit quaternion. The logarithm of a unit quaternion is defined as a 3-dimensional vector[8]:

$$\log q = \frac{\arccos s}{\sqrt{(1-s^2)}} \boldsymbol{v}.$$

A unit quaternion can also be defined as

$$q = (\cos \tfrac{\phi}{2}, \sin \tfrac{\phi}{2} \boldsymbol{n}) \tag{4}$$

where $\boldsymbol{n}$ is a 3-dimensional vector. $\boldsymbol{n}$ can be arbitrarily selected when $q = (-1, 0, 0, 0)$, but it is usually forced to be $(0, 0, 0)$ to avoid ambiguity. The logarithm of a unit quaternion is rewritten as

$$\log q = \tfrac{\phi}{2} \boldsymbol{n}, \quad 0 \leqslant \phi \leqslant 2\pi. \tag{5}$$

When $\phi = 0$ or $\boldsymbol{n} = 0$, the logarithm is a null vector. This happens when $q = (\pm 1, 0, 0, 0)$. It is noted that the logarithmic mapping is discontinuous at $q = (-1, 0, 0, 0)$, as there is an abrupt change of $\boldsymbol{n}$ around this element.

2.2 Dual quaternion

A dual quaternion is defined as

$$\hat{q} = q + \varepsilon q^o \tag{6}$$

where ε is the dual factor, q and q^o are both quaternions.

The operations of dual quaternions are similar to those of quaternions, listed as follows:

$$\begin{aligned} \lambda\hat{q} &= \lambda q + \varepsilon\lambda q^o \\ \hat{q}_1 + \hat{q}_2 &= (q_1 + q_2) + \varepsilon(q_1^o + q_2^o) \\ \hat{q}_1 \circ \hat{q}_2 &= (q_1 \circ q_2) + \varepsilon(q_2 \circ q_1^o + q_1 \circ q_2^o) \\ \hat{q}^* &= q^* + \varepsilon(q^o)^* \\ \| \hat{q} \|^2 &= \hat{q} \circ \hat{q}^* \\ \hat{q}^{-1} &= \hat{q}^* / \| \hat{q} \|^2. \end{aligned} \tag{7}$$

The kinematic equation of a rigid body is expressed as

$$2\dot{\hat{q}} = \hat{\omega}^s \circ \hat{q} \tag{8}$$

where

$$\hat{\omega}^s = \omega^s + \varepsilon(\dot{\boldsymbol{p}} + \boldsymbol{p} \times \omega^s) \tag{9}$$

represents the generalized spatial velocity.

Suppose there is a translation $\boldsymbol{p}$ succeeded by a rotation q. The whole transformation can be represented using a dual quaternion

$$\hat{q} = q + \varepsilon\tfrac{1}{2}\boldsymbol{p} \circ q. \tag{10}$$

For convenience, we identify a vector $\boldsymbol{p}$ with a quaternion $(0, \boldsymbol{p})$ here. It is a widely used trick in dual-quaternion related deductions[3].

Given $\hat{q} = q + \varepsilon q^o$, if $q \cdot q^0 = 0$, $\hat{q}$ is said to be normalized[2]. It can be verified that, a dual quaternion acquired through (10) is naturally normalized. In the rest of this paper, we assume that all dual quaternions are normalized and have unit norm unless specifically stated.

2.3 The logarithmic of a dual quaternion

The logarithm of a dual quaternion can be defined as a dual row vector. Given a normalized dual quaternion, we know[7]

$$q + \varepsilon q^0 = q e^{\varepsilon\gamma}$$

where $\gamma = q^0/q$. Following (10),

$$\begin{aligned} \gamma &= \tfrac{1}{2}\boldsymbol{p} \\ \log(q + \varepsilon q^0) &= \log q + \varepsilon\tfrac{1}{2}\boldsymbol{p}. \end{aligned}$$

Therefore

$$\log\hat{q} = \tfrac{1}{2}\phi\boldsymbol{n} + \varepsilon\tfrac{1}{2}p\boldsymbol{s} \tag{11}$$

where p and $\boldsymbol{s}$ are the norm and the direction vector of $\boldsymbol{p}$ respectively. As any normalized dual quaternion can be expressed by (10), formula (11) is always applicable.

Let $\hat{O} = (1, 0, 0, 0) + \varepsilon(0, 0, 0, 0)$. By (11) we know $\log(\pm\hat{O})$ are null dual vectors. A deeper analysis of the logarithmic mapping reveals that discontinuity occurs at $-\hat{O}$, just like what happens to the logarithmic mapping of unit quaternions.

2.4 Inner product, error, and distance

The norm of a dual quaternion's logarithm is a dual number, defined as

$$\| \log\hat{q} \| = \sqrt{\langle \log(\hat{q}), \log(\hat{q}) \rangle}$$

where the inner product $\langle , \rangle$ is calculated as

$$\langle \log(\hat{q}), \log(\hat{q}) \rangle = \log\hat{q} \cdot (\log\hat{q})^{\mathrm{T}} = \frac{|\phi|^2 + 2\varepsilon\phi p\cos\alpha}{4}.$$

A new type of norm is defined for a dual quaternion:

$$\mathcal{R}(\hat{q}) = 2 \| \log\hat{q} \|.$$

When $\hat{q} = \pm\hat{O}$.

$$\mathcal{R}(\hat{q}) = 0.$$

Otherwise,

$$\mathcal{R}(\hat{q}) = 2\sqrt{\log\hat{q} \cdot (\log\hat{q})^{\mathrm{T}}} = |\phi| + \varepsilon|p\cos\alpha| \tag{12}$$

where α is the angle between $\boldsymbol{n}$ and $\boldsymbol{s}$. In (12), $\mathcal{R}(\hat{q})$ is a dual number whose real part and dual part are both positive. It is named a positive dual scalar in this paper.

Given two dual scalars $\hat{a}_1$ and $\hat{a}_2$, if $\hat{a}_1 - \hat{a}_2$ is a positive dual scalar, we have

$$\hat{a}_1 > \hat{a}_2. \tag{13}$$

With (12) and (13), two dual quaternions can be compared by their norms.

Given two dual quaternions $\hat{q}_1$ and $\hat{q}_2$, their difference is evaluated by

$$\hat{e}_{12} = \hat{q}_1 \circ \hat{q}_2^*. \tag{14}$$

The distance between two dual quaternions can be computed as

$$\mathcal{D}(\hat{q}_1, \hat{q}_2) = \mathcal{R}(\hat{e}_{12}). \tag{15}$$

Definitions similar to (14) or (15) have been established on $SE(3)$[6] and on a sphere[9].

2.5 One useful claim

Following (8) and (11) there is an important conclusion, which will be used later.

Theorem 1. Set

$$W = \tfrac{1}{2}\mathcal{R}^2(\hat{q}).$$

The following equality holds:

$$\frac{\mathrm{d}}{\mathrm{d}t}W = \langle 2\log(\hat{q}), 2\tfrac{\mathrm{d}}{\mathrm{d}t}\log(\hat{q}) \rangle = \langle 2\log(\hat{q}), \hat{\omega}^s \rangle. \tag{16}$$

A similar conclusion has been derived on $SE(3)$[6]. Here, we present an easier proof.

Proof. Standard calculation gives

$$2\frac{\mathrm{d}}{\mathrm{d}t}\log(\hat{q}) = (\dot{\phi}\boldsymbol{n} + \varepsilon\dot{p}\boldsymbol{s}) + (\phi\dot{\boldsymbol{n}} + \varepsilon\phi\dot{\boldsymbol{n}}) = D_{\shortparallel} + D_{\perp}.$$

As $\boldsymbol{n}$ and $\boldsymbol{s}$ are both unit vectors,

$$\boldsymbol{n} \cdot \dot{\boldsymbol{n}} = 0, \qquad \boldsymbol{s} \cdot \dot{\boldsymbol{s}} = 0.$$

It follows that

$$\begin{aligned}\langle 2\log(\hat{q}), D_{\perp}\rangle &= 0\\ \langle 2\log(\hat{q}), 2\tfrac{\mathrm{d}}{\mathrm{d}t}\log(\hat{q})\rangle &= \langle 2\log(\hat{q}), D_{\shortparallel}\rangle.\end{aligned}$$

Solving (8) yields

$$\hat{\omega}^s = 2\dot{\hat{q}} \circ \hat{q}^*. \tag{17}$$

Substituting (7)(10) into (17) gives

$$\begin{aligned}\hat{\omega}^s &= 2\dot{\hat{q}} \circ \hat{q}^* =\\ &2[\dot{q} + \tfrac{\varepsilon}{2}\tfrac{\mathrm{d}}{\mathrm{d}t}(p\boldsymbol{s}) \circ q + \tfrac{\varepsilon}{2}p\boldsymbol{s} \circ \dot{q}] \circ [q^* - \varepsilon\tfrac{1}{2}q^* \circ p\boldsymbol{s}] =\\ &2[\dot{q} \circ q^* + \tfrac{\varepsilon}{2}(\dot{p}\boldsymbol{s} + p\dot{\boldsymbol{s}}) + \varepsilon p\boldsymbol{s} \circ \dot{q} \circ q^*)] =\\ &(\dot{\phi}\boldsymbol{n} + \varepsilon\dot{p}\boldsymbol{s}) +\\ &[C_{\perp} + \varepsilon p\dot{\boldsymbol{s}} + \varepsilon p\dot{\phi}\boldsymbol{s} \times \boldsymbol{n} + \varepsilon p\boldsymbol{s} \times C_{\perp}] =\\ &D_{\shortparallel} + \bar{D}_{\perp}\end{aligned}$$

where $C_{\perp} = \sin\phi\dot{\boldsymbol{n}} - 2\sin^2\frac{\phi}{2}\dot{\boldsymbol{n}} \times \boldsymbol{n}$.

It can be verified that

$$\langle 2\log(\hat{q}), \bar{D}_{\perp}\rangle = 0.$$

Therefore

$$\langle 2\log(\hat{q}), \hat{\omega}^s\rangle = \langle 2\log(\hat{q}), D_{\shortparallel}\rangle = \langle 2\log(\hat{q}), 2\tfrac{\mathrm{d}}{\mathrm{d}t}\log(\hat{q})\rangle.$$

Then (16) follows.

□

3 Control law design

3.1 The regulation problem

To stabilize (8), a logarithmic feedback law is presented:

$$\hat{\omega}^s = -2k\log\hat{q}, \quad k > 0 \tag{18}$$

It is a generalized proportional feedback law. Let

$$\log\hat{q} = \boldsymbol{V} + \varepsilon\boldsymbol{V}^0.$$

The multiplication between k and $\log\hat{q}$ follows

$$k(\boldsymbol{V} + \varepsilon\boldsymbol{V}^0) = (k\boldsymbol{V} + \varepsilon k\boldsymbol{V}^0). \tag{19}$$

To prove its stability, we consider a candidate Lyapunov function:

$$W = \tfrac{1}{2}\mathcal{R}^2(\hat{q}). \tag{20}$$

Differentiating (20) and applying Theorem 1 yields

$$\frac{\mathrm{d}}{\mathrm{d}t}W = \langle 2\log\hat{q}, -2k_p\log\hat{q}\rangle = -k_pW. \tag{21}$$

By the definition of $\mathcal{R}(\hat{q})$, $W = 0$ holds when $\hat{q} = \pm\hat{O}$; otherwise W is a positive dual scalar. As a consequence, $\frac{\mathrm{d}}{\mathrm{d}t}W$ stays non-positive, and $\frac{\mathrm{d}}{\mathrm{d}t}W = 0$ holds only when $W = 0$. Therefore, the logarithmic control law guarantees stability for all initial conditions. Moreover, through (21) we know W converges to zero exponentially.

By definition, k_p is a positive real number. Actually, k_p can be replaced by a positive dual scalar. Given a positive dual scalar

$$\hat{a} = a + \varepsilon a^0.$$

A new type of multiplication is defined between $\hat{k}$ and a dual vector:

$$\hat{a}(\boldsymbol{V} + \varepsilon\boldsymbol{V}^0) = (a\boldsymbol{V} + \varepsilon a^0\boldsymbol{V}^0). \tag{22}$$

If k is chosen as a positive dual scalar, applying the multiplication defined in (22) instead of the common multiplication defined in (19), exponential convergence can also be guaranteed, except that the real part and the dual part of W will have different converging rates. Therefore, with a little abuse of the symbol, it is unnecessary to specify whether k is a positive real number or a positive dual scalar.

When $\hat{q} = -\hat{O}$, the real part of W is zero. However, when $\hat{q}$ is near $-\hat{O}$, it can be calculated by (8) that the real part of W is close to $2\pi^2$. It reveals that there is a discontinuity at $-\hat{O}$. Excluding the special point $-\hat{O}$, the real part of W varies monotonously with $|\phi|$. As can be imaged, the control law stated by (18) will drive $\hat{q}$ to $\hat{O}$ if the initial state is not $-\hat{O}$.

3.2 The multi-equilibrium problem

As an equilibrium different from $\hat{O}$, $-\hat{O}$ cannot be reached by (18) unless it is the initial state. However, the two equilibriums are physically identical[2]. The phenomenon emerges as dual quaternions provide a double covering of $SE(3)$. When the initial state is near $-\hat{O}$, the system is supposed to be driven to $-\hat{O}$ instead of $\hat{O}$, to take a "shorter" path. This is called the multi-equilibrium problem. Readers can find an explicit explanation of the parallel problem in the study of quaternion-based attitude control[10].

To handle the multi-equilibrium problem, an extra parameter λ is introduced:

$$\lambda = \begin{cases} 1, & \text{if } \hat{q}(0) \cdot \hat{O} \geqslant 0\\ -1, & \text{otherwise}\end{cases} \tag{23}$$

where $\hat{q}(0)$ is the initial state. It is equal to saying that λ indicates the sign of the first element in $\hat{q}(0)$.

Formula (18) is then revised as

$$\hat{\omega}^s = -2k\lambda\log\lambda\hat{q}. \tag{24}$$

Given arbitrary initial conditions, the system will be driven to the nearest equilibrium by (24). The multi-equilibrium problem is properly handled.

3.3 The tracking problem

Before continuing, we propose the adjoint mapping of dual quaternions:

$$Ad_{\hat{q}_1}\hat{q}_2 = \hat{q}_1 \circ \hat{q}_2 \circ \hat{q}_1^*. \tag{25}$$

Given a reference trajectory $\hat{q}_d(t)$, the tracking error is formulated as

$$\hat{e} = \hat{q} \circ \hat{q}_d^*. \tag{26}$$

Let $\hat{\omega}_d = 2\dot{\hat{q}}_d \circ \hat{q}_d^*$. As $\parallel \hat{q}_d \parallel = 1$, it can be verified that

$$2\dot{\hat{q}}_d^* = -\hat{q}_d^* \circ \hat{\omega}_d.$$

Differentiating (26) and applying (25) yields

$$2\dot{\hat{e}} = \hat{\omega}_e \circ \hat{e} \tag{27}$$

$$\hat{\omega}_e = \hat{\omega}^s - Ad_{\hat{e}}\hat{\omega}_d. \tag{28}$$

The tracking law is then derived from (18), (27), and (28):

$$\hat{\omega}^s = -2k \log \hat{e} + Ad_{\hat{e}}\hat{\omega}_d. \quad (29)$$

4 Omnidirectional control: a case Study

As motions on $SE(3)$ are hard to illustrate, and as $SE(2)$ can embody most of the richness of $SE(3)$, a case study on $SE(2)$ will be presented here.

Among the platforms whose transformations make up $SE(2)$, the ground omnidirectional robot is a typical one. The posture of an omnidirectional robot can be described by a triple (x, y, θ), with x, y being the Cartesian coordinates, and θ being the angle between the heading direction and the x-axis. Traditionally, the three degree-of-freedoms (DOFs) of an omnidirectional robot are decoupled when performing control[11,12]. The regulation control law is

$$\begin{aligned}\dot{\theta} &= -k_0\theta \\ \dot{x} &= -k_1 x \\ \dot{y} &= -k_1 y. \quad (30)\end{aligned}$$

The tracking control law reads

$$\begin{aligned}\dot{\theta} &= -k_0\theta_e + \dot{\theta}_d \\ \dot{x} &= -k_1 x + \dot{x}_d \\ \dot{y} &= -k_1 y + \dot{y}_d. \quad (31)\end{aligned}$$

where $\theta_e = \theta - \theta_d$, and $(x_d(t), y_d(t), \theta_d(t))$ describes a reference trajectory.

4.1 A new design for omnidirectional control

Corresponding to an arbitrary element in $SE(2)$, there is a normalized dual quaternion

$$\hat{q} = (q_1, q_2, q_3, q_4) + \varepsilon(q_5, q_6, q_7, q_8) \quad (32)$$

whose relation with (x, y, θ) is

$$\begin{aligned}q_1 &= \cos(\theta/2) \quad (33) \\ q_2 &= 0 \quad (34) \\ q_3 &= 0 \quad (35) \\ q_4 &= \sin(\theta/2) \quad (36) \\ q_5 &= 0 \quad (37) \\ q_6 &= x\cos(\theta/2) + y\sin(\theta/2) \quad (38) \\ q_7 &= y\cos(\theta/2) - x\sin(\theta/2) \quad (39) \\ q_8 &= 0. \quad (40)\end{aligned}$$

Furthermore,

$$\begin{aligned}\log \hat{q} &= (0, 0, \tfrac{\theta}{2}) + \varepsilon(\tfrac{x}{2}, \tfrac{y}{2}, 0) \quad (41) \\ \hat{\omega}^s &= (0, 0, \omega_z) + \varepsilon(v_x + y\omega_z, v_y - x\omega_z, 0). \quad (42)\end{aligned}$$

If θ is limited to be within $(-\pi, \pi]$, the multi-equilibrium problem is avoided. Then, (18) and (19) is applicable to omnidirectional control. Simulations are performed for verification.

Starting from the initial posture $(-1, 1, \frac{\pi}{2})$, taking

$$k = 0.35 + 0.35\varepsilon \quad (43)$$

the simulation result of the regulation law is shown in Fig. 1. The short bar perpendicular to the curve represents the heading of the robot.

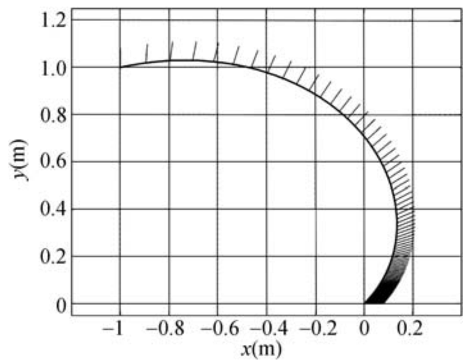

Fig. 1 Regulation result, starting from $(-1, 1, \frac{\pi}{2})$ (The short bar perpendicular to the curve represents the orientation angle.)

Given a reference trajectory

$$\begin{aligned}x_d(t) &= t \\ y_d(t) &= t \\ \theta_d(t) &= \tfrac{3}{4}\pi\end{aligned}$$

starting with the initial posture $(5, 0, 0)$, taking $k_p = 1 + \varepsilon$, applying (29), the actual trajectory is given in Fig. 2.

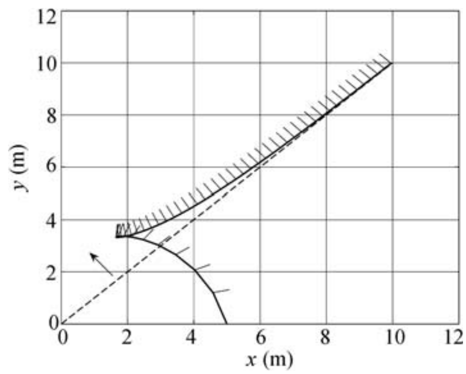

Fig. 2 Tracking a line with fixed orientation. The dashed line is the reference path, and the arrow indicates the demanded orientation.

Given another reference trajectory

$$\begin{aligned}x_d(t) &= 5\cos t \\ y_d(t) &= 5\sin t \\ \theta_d(t) &= t - \pi.\end{aligned}$$

The robot is demanded to move on a circle while pointing to the center. Starting from $(0, 0, 0)$, taking $k_p = 0.4 + 2\varepsilon$, applying (45), the actual trajectory is given in Fig. 3.

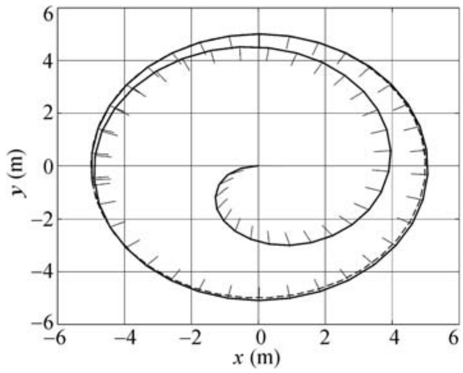

Fig. 3 Tracking a circle while pointing to its center (The dashed line represents the demanded circle.)

4.2 Comparison and analysis

Setting $k = k_0 + \varepsilon k_1$ and substituting (33)–(42), formula (18) can be rewritten as

$$\begin{aligned} \dot{\theta} &= -k_0\theta \\ \dot{x} &= -k_1(x - y\theta) \\ \dot{y} &= -k_1(y + x\theta). \end{aligned} \tag{44}$$

Given a reference trajectory $(x_d(t), y_d(t), \theta_d(t))$, the tracking law (29) can also be revised, though the calculation is somewhat tedious:

$$\begin{aligned} \dot{\theta} &= -k_0\theta_e + \dot{\theta}_d \\ \dot{x} &= -k_1 x + (\dot{x}_d \cos\theta_e - \dot{y}_d \sin\theta_e) + \\ &\quad k_1(x_d \cos\theta_e - y_d \sin\theta_e) + \\ &\quad k_1\theta_e(x_d \sin\theta_e + y_d \cos\theta_e) \\ \dot{y} &= -k_1 y + (\dot{x}_d \sin\theta_e + \dot{y}_d \cos\theta_e) + \\ &\quad k_1(x_d \sin\theta_e + y_d \cos\theta_e) - \\ &\quad k_1\theta_e(x_d \cos\theta_e - y_d \sin\theta_e). \end{aligned} \tag{45}$$

Comparing (44) to (30), we find extra terms $x\theta$ and $y\theta$. These terms represent the effect of rotation on translation. It is not easy to understand from the traditional viewpoint, where (x, y, θ) is regarded as an element in $\mathbf{R}^3$. The Lyapunov function corresponding to the conventional control laws is

$$W = \tfrac{1}{2}(x^2 + y^2 + \theta^2)$$

which is quite different from (20). When (x, y, θ) is considered as an element in $SE(3)$ and reexpressed using dual quaternion, the extra terms naturally emerge.

We use both (30) and (44) to drive an omnidirectional robot from $(5, 5, \pi/2)$ to $(0, 0, 0)$. The parameters (k_0, k_1) are chosen to be $(2, 1)$ in both control laws. Simulation results come in Figs. 4 and 5.

In practice, if synergy of orientation and position is important, formula (44) is preferred. Robot soccer game is such an example. To kick the ball, a robot should arrive at a demanded position with a demanded orientation. The orientation should be ready before the position is reached. Otherwise, a delay is needed to adjust the orientation, which is awful for real-time competition. An omnidirectional robot developed by our group, named NUBOT, has been a testbed in this case.

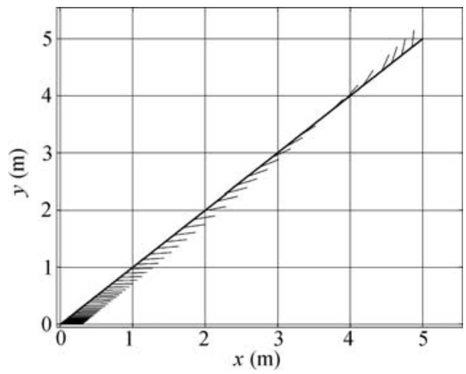

Fig. 4 The actual trajectory generated by (30)

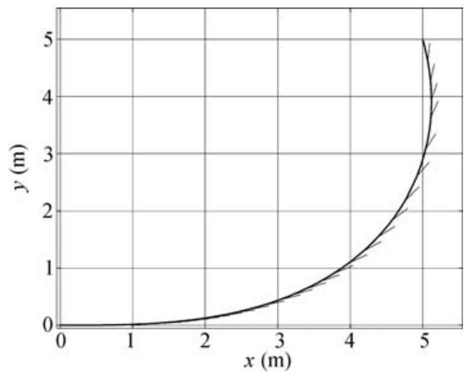

Fig. 5 The actual trajectory generated by (44)

Given a reference trajectory

$$\begin{aligned} x_d(t) &= t \\ y_d(t) &= t \\ \theta_d(t) &= \tfrac{\pi}{4} \end{aligned}$$

we apply both (31) and (45) to track it. Starting from $(-4, 5, \pi)$, taking $k_0 = 2, k_1 = 1$, the actual trajectories are shown in Figs. 6 and 7. Although the trajectory acquired by (31) converges more quickly, our choice is (45), with which the robot has a lower risk of slippage during the motion.

There is another explanation for the particularity of our method. The dual part of (9) is constituted by two parts. Traditionally, only the linear velocity $\dot{\boldsymbol{p}}$ is expected to converge. But from our viewpoint, $(\dot{\boldsymbol{p}} + \boldsymbol{p} \times \omega^s)$ is a bundled value, and our purpose is to drive the sum to zero. That is why the trajectories vary in their shapes.

Till now, we know the interconnection between rotation and translation is not negligible. This is true not only for models whose state space is $SE(2)$, but also for arbitrary free rigid bodies operating on $SE(3)$.

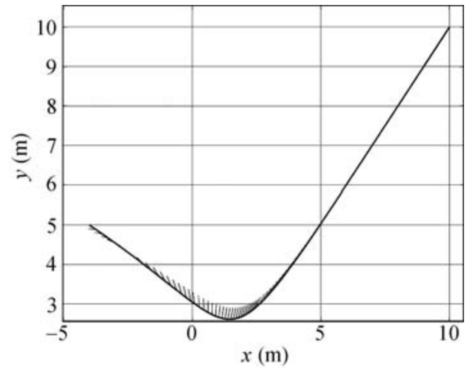

Fig. 6 The actual trajectory generated by (31)

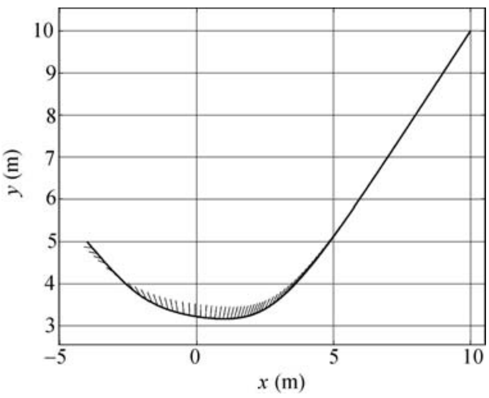

Fig. 7 The actual trajectory generated by (45)

5 Conclusions

The main result of this paper is the dual-quaternion-based kinematic control laws for free rigid bodies. In this paper, dual quaternion is analyzed, and the relation between the rotation and translation within a rigid motion are presented.

The development of this paper is under the assumption that the motion of a rigid body is described in a spatial coordinate frame. Till now, we have only finished half of the work. Description in the body coordinate frame may be more applicable in certain cases. Future work will fill the other half of the work, to show how translation is affected by rotations in a body-fixed coordinate frame.

References

[1] A. Perez, J. M. McCarthy. Dual Quaternion Synthesis of Constrained Robotic Systems. *Journal of Mechanical Design*, vol. 126, no. 3, pp. 425–435, 2004.

[2] Q. J. Ge, B. Ravani. Computer Aided Geometric Design of Motion Interpolants. *Journal of Mechanical Design*, vol. 116, no.3, pp. 756–762, 1994.

[3] Y. X. Wu, X. P. Hu, D. W. Hu, T. Li, J. X. Lian. Strapdown Inertial Navigation System Algorithms Based on Dual Quaternions. *IEEE Transactions on Aerospace and Electronic Systems*, vol. 41, no. 1, pp. 110–132, 2005.

[4] L. M. Hsia, A. T. Yang. On the Principle of Transference in Three-dimensional Kinematics. *ASME Journal of Mechnical Design*, vol. 103, no. 3, pp. 652–656, 1981.

[5] R. M. Murray, Z. X. Li, S. S. Sastry. *An Mathematical Introduction to Robotic Manipulation*, CRC Press, 1994.

[6] F. Bullo, R. M. Murray. Proportional Derivative (PD) Control on the Euclidean Group, Technical Report CIT-CDS-95-010, California Institute of Technology, USA, 1995.

[7] O. Bottema, B. Roth. *Theoretical Kinematics*, North-Holland Publishing Company, New York, USA, 1979.

[8] M. J. Kim, M. S. Kim, S. Y. Shin. A Compact Differential Formula for the First Derivative of a Unit Quaternion Curve. *Journal of Visualization and Computer Animation*, vol. 7, no. 1, pp. 43–57, 1996.

[9] F. Bullo, R. Murray, A. Sarti. Control on the Sphere and Reduced Attitude Stabilization, Technical Report CIT-CDS-95-005, California Institute of Technology, USA, 1995.

[10] P. Arambel, V. Manikonda. A New Class of Control Laws for Spacecraft Attitude Tracking Using Switching and Trajectory Rescaling. In *Proceedings of IEEE International Conference on Robotics and Automation*, IEEE Press, San Francisco, CA, USA, pp. 4033–4038, 2000.

[11] H. Asama, M. Sato, L. Bogoni, H. Kaetsu, A. Mitsumoto, et al. Development of an Omni-drectional Mobile Robot with 3 DOF Decoupling Drive Mechanism. In *Proceedings of IEEE International Conference on Robotics and Automation*, IEEE Press, Nagoya, Aichi, Japan, pp. 1925–1390, 1995.

[12] T. Kalmár-Nagy, R. D′Andrea, P. Ganguly. Near-optimal Dynamic Trajectory Generation and Control. *Robotics and Autonomous Systems*, vol. 46, no. 1, pp. 47–64, 2004.

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