

Sign Test Exercise

Sign Test Exercise

For the problem described in Pagano and Gauvreau, Chapter 11, exercise 1, suppose that we are unwilling to assume that the differences have a normal probability distribution. So instead of the t-test we decide to use a simple "non-parametric" test, the sign test. The sign test is based on the number of positive differences (the number of persons whose LDL cholesterol level on the cornflakes diet is greater than on the oat bran diet).

Pagano, Page 278

Subject	LDL (mmol/l)		Sign
	Corn Flakes	Oat Brand	
1	4.61	3.84	+
2	6.42	5.57	+
3	5.40	5.85	-
4	4.54	4.80	-
5	3.98	3.68	+
6	3.82	2.96	+
7	5.01	4.41	+
8	4.34	3.72	+
9	3.8	3.49	+
10	4.56	3.84	+
11	5.35	5.26	+
12	3.89	3.73	+
13	2.25	1.84	+
14	4.24	4.14	+

Under the hypothesis of no difference between the two diets the probability that a given individual's difference will be positive is $1/2$. Under the alternative (LDL cholesterol tends to be greater on the cornflake diet), a positive difference is more likely than a negative one.

Sign Test Exercise

- (a) Give a reasonable probability model (one with one unknown parameter) for the probability distribution of the number of positive differences in a sample of $n = 14$ subjects.
- (b) In terms of the parameter in your model, state null and alternative hypotheses analogous to those that we used in the original (Pagano) version of this problem.
- (c) The sign test rejects the null hypothesis if the number of positive differences observed is greater than or equal to some critical value, k . If we use $k = 13$ as the critical value, what is our Type I error probability?
- (d) If we use $k = 13$ as the critical value, what is the power of the sign test if in fact the probability that the difference is positive is $5/8$? $3/4$? $7/8$?
- (e) If we want a test with α no greater than 0.05, what critical value should we use?
- (f) If we apply the sign test to the data what is the p-value?
- (g) Suppose the probability distribution of the differences is actually normal with mean $\mu = 0.3$ and $\sigma = 0.4$.
- (i) What is the power of the sign test (with $k=13$)?
[Hint: What is $P(D > 0)$?]
- (ii) Calculate the power of the z-test with the same α , 0.001 (i.e. the z-test with critical value 3.090). Which test has greater power, the z-test or the sign test? Can you explain why the sign test has less power?