Notice the difference in assumptions:

P(82 consecutive "male years" | θ = 0.5) = P("male year"| θ = 0.5)^{82} = P(Y > 5000| θ = 0.5)^{82} = (0.496)^{82} = ***1.072×10^{-25}***

P(82 consecutive "male years" | θ = 0.512)=P("male year"| θ = 0.512)^{82} = P(Y > 5000| θ = 0.512)^{82} = (0.99159)^{82}= ***0.500***

So 82 consecutive "male years" is no more improbable or surprising than getting "heads" on a coin toss (if θ = 0.512) -- it is just the sort of result that you would expect to see if the process were truly random.

The question

Thus the probability that boys are more than 60% of the babies born in one day at the larger hospital is:

P(X > 27) = $\sum_{k=28}^{45} P(X=k) = \sum_{k=28}^{45} \binom{45}{k} (0.5)^k (0.5)^{45-k} = 0.068$

Stata command: Binomial(n,k,p) = P(X≥k) !!

. di Binomial(45,28,0.5)

.06757823

The question

The probability that boys are more than 60% of the babies born in one day at the smaller hospital is:

P(Y > 9) = $\sum_{k=10}^{15} P(Y=k) = \sum_{k=10}^{15} \binom{15}{k} (0.5)^k (0.5)^{15-k} = 0.151$

Stata command: Binomial(n,k,p) = P(X≥k) !!

. di Binomial(15,10,0.5)

.15087891

Stata Hint (not in notes)

Suppose X~Bin(4,0.75)

Then, P(X=k) = P(X ≥ k) - P(X ≥ k+1)

$$P(X=1) = \binom{4}{1} (0.75)^1 (0.25)^3 = 0.047$$

. di Binomial(4,1,0.75)-Binomial(4,2,0.75)
.046875

. di 4*0.75*0.25^3
.046875