Standardization Wrap-up

Populations consist of (sub)groups:

Group	Number in Group	Relative Frequency	Number Positive	Group-Specific Rate
1	N_1	$N_1/\Sigma N = w_1$	D_1	$D_1/N_1 = R_1$
2	N_2	$N_2/\Sigma N = w_2$	D_2	$D_2/N_2 = R_2$
3	N_3	$N_3/\Sigma N = w_3$	D_3	$D_3/N_3 = R_3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$\sum N_i$	$\sum W_i = 1$	$\sum D_i$	

Crude Rate : $\dfrac{\Sigma D_i}{\Sigma N_i}$

Group-Specific Rates : $R_1, R_2, R_3, \ldots$

Crude rate is <u>weighted</u> <u>average</u> of group-specific rates:

$$
\begin{aligned}
\text{Crude Rate} = \frac{\sum D}{\sum N} &= \frac{D_1}{\sum N} + \frac{D_2}{\sum N} + \frac{D_3}{\sum N} + \cdots \\
&= \frac{N_1}{\sum N}\frac{D_1}{N_1} + \frac{N_2}{\sum N}\frac{D_2}{N2} + \frac{N_2}{\sum N}\frac{D_2}{N_2} + \cdots \\
&= W_1R_1 + W_2R_2 + W_3R_3 + \cdots \\
&= \sum W_iR_i
\end{aligned}
$$

Where

$W_1 = \frac{N_1}{\sum N_i}$ is the relative frequency in group 1

$W_2 = \frac{N_2}{\sum N_i}$ is the relative frequency in group 2

And so on…

The standardized (adjusted) rate is <u>also</u> a weighted average of the group-specific rates:

Group	Standard Population	Relative Frequency
1	N^*_1	$N_1^* / \Sigma N = W_1^*$
2	N^*_2	$N_2^* / \Sigma N = W_2^*$
3	N^*_3	$N_3^* / \Sigma N = W_3^*$
⋮	⋮	⋮
	$\sum N_i^*$	$\sum W_i^* = 1$

Adjusted rate : $w_1^* R_1 + w_2^* R_2 + w_3^* R_3 + \ldots$

If standard population has the same relative frequency distribution as study population, then $W_i^* = W_i$ for all i, then

Adjusted rate = Crude rate (see formula above)

If group-specific rates within a population are all equal, then all rates, both crude and adjusted, are the same for that population. In this case there is no need for adjustment. It is because of variation in subgroup rates that we adjust -- no variation, no need to adjust.

For example, if $R_1 = R_2 = R_3 = \ldots = R$
(all the same rates in each subgroup), then for any set of weights

$$\begin{aligned} w_1R_1 + w_2R_2 + w_3R_3 + \ldots &= \\ w_1R + w_2R + w_3R + \ldots &= \\ (w_1 + w_2 + w_3 + \ldots)\, R &= \\ (\Sigma w)\, R &= \\ (1)\, R &= R \end{aligned}$$

The result is always the same.

Important Special Case: Proportional Group-Specific Rates

If group-specific rates in one population are proportional to those in another (constant ratio), then the two populations' adjusted rates will also have that same ratio, no matter what standard population is used in making the adjustment.

	Population A		Population B	
Group	Relative Frequency	Rate	Relative Frequency	Rate
1	w_{A1}	R_1	w_{B1}	cR_1
2	w_{A2}	R_2	w_{B2}	cR_2
3	w_{A3}	R_3	w_{B3}	cR_3
⋮	⋮	⋮	⋮	⋮

Where $\Sigma w_{Ai} = 1$ and $\Sigma w_{Bi} = 1$.

The group-specific rates are proportional--in each group, population B's rate is c times population A's rate.

Now we know that the crude rates can differ, even when c = 1.

Q: When can this happen?
A: When there are differences in how the two populations are distributed over the groups.

If we adjust for those differences in distribution over the subgroups, using a standard population whose relative frequencies are $w_1, w_2, w_3, \ldots,$ then we find that the ratio of adjusted rates is exactly c :

A's adjusted rate: $w_1 R_1 + w_2 R_2 + w_3 R_3 + \ldots$

B's adjusted rate: $w_1 cR_1 + w_2 cR_2 + w_3 cR_3 + \ldots$

$= c(w_1 R_1 + w_2 R_2 + w_3 R_3 + \ldots)$

= c (A's adjusted rate)

For example, if each of B's group-specific rates is 20% greater than A's rate for that same group, then c = 1.20, and

$$\frac{\text{Adjusted rate for B}}{\text{Adjusted rate for A}} = 1.20$$

B's adjusted rate is 20% greater than A's, <u>no matter what standard population is used</u>.

This is the "ideal" case for adjustment. Here the two sets of group-specific rates are really "comparable" -- each member of one set is c times the corresponding member of the other. In this case, the process of adjustment completely eliminates all effects of differences between the age distributions in the two populations. We get exactly the same ratio of adjusted rates (the ratio equals c), no matter what standard population we use.

When adjustment is most useful is when the actual rates in the two study populations are not too far from this ideal, i.e., when they are roughly proportional.

On the other hand, adjustment (standardization) is <u>not</u> so useful when the ratios of group-specific rates are not nearly equal, but are instead highly variable. If some are much greater than 1, while others are much less than 1, then the comparison of standardized rates can depend <u>critically</u> on what standard population is used.

We can always choose some standard population, and then compare populations A and B in terms of their adjusted rates. But <u>when the ratios of group-specific rates are highly variable</u>, <u>if we use a different standard population</u>, <u>we can get a very different result</u>.