BIOSTATISTICS

MATH REVIEW

A. Definitions

1. Integers: (..., -3, -2, -1, 0, 1, 2, 3...)

2. Positive Integers: (1, 2, 3, 4,...)

3. Real numbers: All numbers, integers and nonintegers

4. $|a|$ is called the absolute value of the number a:

   If $a \geq 0$, then $|a| = a$

   If $a < 0$, then $|a| = -a$

5. If b is any real number and n is a positive integer, then

$$b^n = \underbrace{b \cdot b \cdot b \cdot b \cdot \ldots \cdot b}_{\text{b multiplied by itself n times}}$$

where b is referred to as the base and n as the exponent.

We can write

$$b^0 = 1$$

$$b^{-n} = \frac{1}{b^n} \quad \text{if n is a positive integer}$$

$$\sqrt[n]{b} = b^{1/n} \quad (\text{the } n^{\text{th}} \text{ root of b}).$$

6. The logarithm to base b of a positive number x is that number y which satisfies the equation $b^y = x$ and we write $\log_b x = y$.

7. Summation Sign

$$\sum_{i=1}^{n} x_i = x_1 + x_2, \ldots + x_n$$

8. Product Sign

$$\prod_{i=1}^{n} x_i = x_1 \cdot x_2 \cdot x_3 \cdot \ldots \cdot x_n$$

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B. Examples in using summation signs

Let $x_1 = 5$ $y_1 = 9$ and $n = 3$

$x_2 = 6$ $y_2 = 12$

$x_3 = 2$ $y_3 = 4$

For the values of x and y given above, compute the following by pencil and paper, not by calculator:

$\sum_{i=1}^{3} x_i =$

$\sum_{i=1}^{3} x_i^2$

$\sum_{i=1}^{3} y_i =$

$\sum_{i=1}^{3} \frac{x_i}{y_i} =$

$\sum_{i=1}^{3} (x_i + y_i) =$

$\sum_{i=1}^{3} (y_i - 2) =$

$\sum_{i=1}^{3} x_i + \sum_{i=1}^{3} y_i =$

$\left(\sum_{i=1}^{3} y_i\right) - 2 =$

$\sum_{i=1}^{3} x_i y_i =$

$\sum_{i=1}^{3} (y_i - 2)^2 =$

$\sum_{i=1}^{3} x_i \cdot \sum_{i=1}^{3} y_i =$

$\left(\sum_{i=1}^{3} y_i^2\right) - 2 =$

$\sum_{i=1}^{3} (x_i - y_i) =$

$\sum_{i=1}^{3} (x_i - y_i)^2 =$

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C. More examples

Compute:

$$\frac{\sum_{i=1}^{3} X_i}{3} = \bar{X} =$$

$$\left(\sum_{i=1}^{3} X_i \right)^2 =$$

$$\frac{\sum_{i=1}^{3} (X_i - \bar{X})^2}{n-1} =$$

$$\frac{\left(\sum_{i=1}^{3} X_i^2 \right) - n\bar{X}^2}{n-1} =$$

$$x_1^{-1} =$$

$$\sum_{i=1}^{3} |x_i| =$$

$$x_3^{-3} =$$

$$\sum_{i=1}^{3} |x_i - y_i| =$$

$$y_3^{\frac{1}{2}} =$$

$$x_2^{0} =$$

$$\log_{10}(5 \cdot x_1 y_3) =$$

$$\log_2 y_3 =$$

$$\prod_{i=1}^{3} x_i$$

$$\frac{\prod_{i=1}^{3} x_i}{\prod_{i=1}^{3} y_i}$$

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D. More on Logarithms

a. Write the following equations in terms of logarithms:

$2^3 = 8$

$10^2 = 100$

$10^{-3} = 0.001$

b. Write the following equations in terms of exponents:

$\log_2 128 = 7$

$\log_5 125 = 3$

$\log_{1/2} (1/16) = 4$

$\log_{10} (0.01) = -2$

Common logarithm = a logarithm with base 10 such that

$\log_{10} y = x$ implies $10^x = y$.

Often $\log_{10} y$ is written as log y.

Natural logarithm = a logarithm with base e such that

$\log_e y = x$ implies $e^x = y$.

Often $\log_e y$ is written as ln y.

Note: e or Euler's constant (2.71828...) is important in describing biological relationships and is useful in many statistical applications.

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Properties of logarithms:

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b(x/y) = \log_b x - \log_b y$$

$$\log_b(x^r) = r \cdot \log_b x$$

E. Scientific Notation - Expressing a number as a product of a number N between 1 and 10 and an integral power of ten in order to simplify notation of calculations: $(N) \cdot (10)^k$

(e.g., $390.672 = 3.90672 \times 10^2$

$0.0001576 = 1.576 \times 10^{-4}$)

Rules

1) The exponent of 10 is determined by counting the number of places that the decimal point was moved when going from the original number to the number between 1 and 10.
2) The exponent is
   a) negative if the original number is less than 1
   b) positive if the original number is greater than 10
   c) 0 if the original number is between 1 and 10

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a. Express the following in scientific notation:

2.14
31.79
412.9
8,000,000
0.14
0.0379
0.00000049

F. Correct to n Decimal Places - the process of rounding a number to n decimal places.

Rules for rounding

1) Round to the nearest number

2) If the number determining the rounding is 5, round to the even number

(e.g., 3.14159 is 3.14 correct to 2 decimal places

17.45 is 17.4 correct to 1 decimal place)

a. Correct the following numbers to two decimal places:

7.865
7.847
7.853
7.875

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G. Significant Figures - any digits in a number which contribute to the specification of its magnitude apart from zeroes that determine the position of the decimal point.

Hint: It helps to first write the number in scientific notation in order to determine the number of significant figures

e.g. $92,800,000 = 9.28 \times 10^{7}$ has 3 significant figures

$0.0909 = 9.09 \times 10^{-2}$ has 3 significant figures

$0.00052 = 5.2 \times 10^{-4}$ has 2 significant figures

a. Specify the number of significant figures corresponding to each number:

0.045
4.5
4.05
4.502
20.04

H. Solve for x: 8

1. $2x = 8$
2. $2x + 3 = 7$
3. $2x + 3 = 8$
4. $2x - 1 = 4$
5. $2x - 1 = 0$
6. $2x - 1 = -1$
7. $3x + 5 = 0$
8. $x - a = 0$
9. $x - a = b$
10. $ax + b = c$
11. $2x^2 = 18$
12. $5x^2 + 90 = 215$
13. $ax^2 + bx + c = 0$
    (Remember the formula?)
14. $10^x = 100$
15. $10^x + 100 = 1100$
16. $x = \log_2 4$
17. $x = \log_4 2$
18. $x = \log_4 16$
19. $x^{1/2} = 3$
20. $x^{1/2} = 5$

I. Solve for x and evaluate when y = 3 and z = 4:

1. $x - y = 0$
2. $x + 2y = 4$
3. $2x + y = 4$
4. $2x + 6y = 4$
5. $2x + 6y + 2z = 4$
6. $ax + by + c = 3$

J. Solve using calculator:

1. $10^x - 100 = 382$
2. $e^x = 20$
3. $x = \ln(1)$
4. $x = \log(1)$
5. $x = \log(100)$
6. $x = \ln(100)$
7. $\log(x) = 3$
8. $\ln(x) = 1$
9. $\ln(x) = 3$
10. $x = e^3$
11. $\ln(x) = -3$
12. $x = e^{5.3}$

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Answer Key

Page 2

$\sum_{i=1}^{3} x_i = 5 + 6 + 2 = 13$

$\sum_{i=1}^{3} y_i = 9 + 12 + 4 = 25$

$\sum_{i=1}^{3} (x_i + y_i) = 38$

$\sum_{i=1}^{3} x_i + \sum_{i=1}^{3} y_i = 38$

$\sum_{i=1}^{3} x_i y_i = 45 + 72 + 8 = 125$

$\sum_{i=1}^{3} x_i \cdot \sum_{i=1}^{3} y_i = 13 \cdot 25 = 325$

$\sum_{i=1}^{3} (x_i - y_i) = -4 + (-6) + (-2) = -12$

$\sum_{i=1}^{3} x_i^2 = 25 + 36 + 4 = 65$

$\sum_{i=1}^{3} \frac{x_i}{y_i} = \frac{5}{9} + \frac{6}{12} + \frac{2}{4} = 1\frac{5}{9}$

$\sum_{i=1}^{3} (y_i - 2) = 7 + 10 + 2 = 19$

$\left(\sum_{i=1}^{3} y_i\right) - 2 = 25 - 2 = 23$

$\sum_{i=1}^{3} (y_i - 2)^2 = 7^2 + 10^2 + 2^2 = 153$

$\left(\sum_{i=1}^{3} y_i^2\right) - 2 = (81 + 144 + 16) - 2 = 239$

$\sum_{i=1}^{3} (x_i - y_i)^2 = (-4)^2 + (-6)^2 + (-2)^2 = 56$

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$\frac{\sum_{i=1}^{3} x_i}{3} = \bar{x} = \frac{5+6+2}{3} = \frac{13}{3}$

$\left(\sum_{i=1}^{3} x_i\right)^2 = (13)^2 = 169$

$\frac{\sum_{i=1}^{3} (x_i - \bar{x})^2}{n-1} = \frac{(5-\frac{13}{3})^2 + (6-\frac{13}{3})^2 + (2-\frac{13}{3})^2}{2} = \frac{13}{3}$

$\frac{\left(\sum_{i=1}^{3} x_i^2\right) - n\bar{x}^2}{n-1} = \frac{65 - 3\left(\frac{13}{3}\right)^2}{2} = \frac{13}{3}$

Note: $n = 3$

$x_1^{-1} = \frac{1}{5}$

$x_3^{-3} = \frac{1}{2^3} = \frac{1}{8}$

$y_3^{1/2} = \sqrt{4} = 2$

$\log_{10}(5 \cdot x_1 y_3) = \log_{10}(5 \cdot 5 \cdot 4)$

$= \log_{10}(100) = 2$

$\prod_{i=1}^{3} x_i = 5 \cdot 6 \cdot 2 = 60$

$\sum_{i=1}^{3} |x_i| = 13$

$\sum_{i=1}^{3} |x_i - y_i| = 12$

$x_2^0 = 1$

$\log_2 y_3 = log_2 4 = 2$

$\frac{\prod_{i=1}^{3} x_i}{\prod_{i=1}^{3} y_i} = \frac{5 \cdot 6 \cdot 2}{9 \cdot 12 \cdot 4} = \frac{5}{36}$

Page 4	$2^3 = 8$	$\log_2 8 = 3$
	$10^2 = 100$	$\log_{10} 100 = 2$
	$10^{-3} = 0.001$	$\log_{10} 0.001 = -3$
	$\log_2 128 = 7$	$2^7 = 128$
	$\log_5 125 = 3$	$5^3 = 125$
	$\log_{1/2}(1/16) = 4$	$\left(\frac{1}{2}\right)^4 = \frac{1}{16}$
	$\log_{10}(0.01) = -2$	$10^{-2} = 0.01$
Page 6	2.14	2.14×10^0
	31.79	3.179×10^1
	412.9	4.129×10^2
	8,000.000	8×10^6
	0.14	1.4×10^{-1}
	0.0379	3.79×10^{-2}
	0.00000049	4.9×10^{-7}
	7.865	7.86
	7.847	7.85
	7.853	7.85
	7.875	7.88
Page 7	$0.045 = 4.5\times^{-2}$	2 significant figures
	4.5	2
	4.05	3
	4.502	4
	$20.04 = 2.004 \times 10^1$	4

H. Solve for x:

1. $2x = 8$	$x = 8/2 = 4$
2. $2x + 3 = 7$	$x = (7 - 3)/2 = 2$
3. $2x + 3 = 8$	$x = (8 - 3)/2 = 2.5$
4. $2x - 1 = 4$	$x = (4 + 1)/2 = 2.5$
5. $2x - 1 = 0$	$x = 1/2$
6. $2x - 1 = -1$	$x = (-1 + 1)/2 = 0$
7. $3x + 5 = 0$	$x = -5/3 = -1\frac{2}{3}$
8. $x - a = 0$	$x = a$
9. $x - a = b$	$x = b + a$
10. $ax + b = c$	$x = (c - b)/a$
11. $2x^2 = 18$	$x = 3$
12. $5x^2 + 90 = 215$	$x = 5$
13. $ax^2 + bx + c = 0$ (Remember the formula?)	$x = [-b \pm \sqrt{b^2 - 4ac}]/2a$ (It's nice if you remember this, but it's not expected.)
14. $10^x = 100$	$x = 2$
15. $10^x + 100 = 1100$	$x = 3$
16. $x = \log_2 4$	$x = 2$
17. $x = \log_4 2$	$x = ½$
18. $x = \log_4 16$	$x = 2$
19. $x^{½} = 3$	$x = 9$
20. $x^{½} = 5$	$x = 25$

I. Solve for x and evaluate when y = 3 and z = 4:

	in general	when y = 3, z = 4
1. $x - y = 0$	$x = y$	$x = 3$
2. $x + 2y = 4$	$x = 4 - 2y$	$x = -2$
3. $2x + y = 4$	$x = (4 - y)/2$	$x = ½$
4. $2x + 6y = 4$	$x = 2 - 3y$	$x = -7$
5. $2x + 6y + 2z = 4$	$x = 2 - 3y - z$	$x = -11$
6. $ax + by + c = 3$	$x = (3 - by - c)/a$	$x = (3 - 3b - c)/a$

J. Solve using calculator:

1. $10^x - 100 = 382$	$x = \log(482) = 2.683$
2. $e^x = 20$	$x = \ln(20) = 2.996$
3. $x = \ln(1)$	$x = 0$
4. $x = \log(1)$	$x = 0$
5. $x = \log(100)$	$x = 2$
6. $x = \ln(100)$	$x = 4.605$
7. $\log(x) = 3$	$x = 1000$
8. $\ln(x) = 1$	$x = e = 2.718$
9. $\ln(x) = 3$	$x = e^3 = 20.086$
10. $x = e^3$	$x = 20.086$
11. $\ln(x) = -3$	$x = e^{-3} = 1/20.086 = 0.0498$
12. $x = e^{5.3}$	$x = 200.337$