Introduction to Biostatistics BC 203

Final Exam
Thursday, December 14, 2006

(Closed Book)

There are 25 questions, each worth 4 points.

Write out your solutions and circle your final answers. Show all your work on these pages. If you need more space write on the back of the page.

You have three hours to complete this exam.

Name: Solutions

1. A (hypothetical) study of 56 men sought to evaluate the effectiveness of the drug Rogaine for promoting hair growth. For each participant, hair growth during the month prior to taking Rogaine and for one month afterward was assessed and classified as 'minimal' or 'substantial'. The data and *partial* Stata output are provided below.

study is matched

Hair Growth Before	Hair Growth after Rogaine Minimal	Substantial	Total
Minimal	18	7	25
Substantial	1	30	31
Total	19	37	56

```
chi2(1) =       XXXX    Prob > chi2 = 0.0339
Exact significance probability        = 0.0703

Proportion with factor
      Cases       .4464286
      Controls    .3392857      [95% Conf. Interval]
                  ---------     --------------------
      difference  .1071429      -.0056467   .2199324
      odds ratio         7       1.159529   42.25854   (test based)
      odds ratio         7       .8993003   315.4834   (exact)
```

(a) State the null hypothesis in terms of the difference between the two population probabilities of interest. Be specific and define your notation.

$H_0: \theta_1 = \theta_2$ $\qquad$ $\theta_1 = P(\text{Minimal} \mid \text{Before})$ $\quad \hat{\theta}_1 = 25/56$

$\theta_2 = P(\text{Minimal} \mid \text{after})$ $\quad \hat{\theta}_2 = 19/56$

OR $H_0: \theta_1^* = \theta_2^*$ $\qquad$ $\theta_1^* = P(\text{Substantial} \mid \text{Before})$ $\quad \hat{\theta}_1^* = \frac{31}{56}$

$\theta_2^* = P(\text{Substantial} \mid \text{after})$ $\quad \theta_2^* = \frac{37}{56}$

(b) Is this test one or two-sided?

two-sided

Problem 1 continued.

(c) Identify and calculate the appropriate Chi-square test-statistic.

$$\chi^2 = \frac{(b-c)^2}{b+c} = \frac{(7-1)^2}{8} = 4.5$$

Use McNemar's Test
Study is matched.

(d) Calculate the critical value for the Chi-Square test in part (b) and state if you reject the null hypothesis that Rogaine promotes hair growth at the 5% level?

$\chi^2_{0.05} = 3.84 < \chi^2_{obs} = 4.5$ so Reject Ho at 5% level.

$\Rightarrow 0.025 < p < 0.05$

notice that this test statistic agrees with the $p = 0.0339$ on the output

(e) Interpret and summarize these data using what is given in the Stata output. What do the data say about the ability of Rogaine to promote hair growth? Does this agree with your answer in part (d)? Justify your answer.

The Exact Test is not significant ($P = 0.0703$) indicating that the normal approximation used to get the p-value in part (d) may not be valid. Hence, there is no difference here, Regaine does not promote Hair growth (95% CI for $\theta_1 - \theta_2$ is $(-0.0056$ to $0.22)$).

notice the difference between the CI that are test based (approximate) and exact.

2. A small study was designed to compare the age of recipients of Medicare services in community hospitals A and B. In hospital A, there we 6 recipients with an average age of 74.5 and a sample standard deviation of 5.2. In hospital B, there were 16 recipients with an average age of 69.3 and a sample standard deviation of 5.2. A Wilcoxon rank-sum test of these data yielded a p-value of 0.0533.

   (a) Construct a 95% confidence interval for the increase in average age of Hospital A over Hospital B when the population variance is assumed to be same for both hospitals.

$$74.5 - 69.3 \pm t_{\alpha/2}^{n+m-2}\sqrt{S_p^2\left(\tfrac{1}{n_1}+\tfrac{1}{n_2}\right)}$$

$$5.2 \pm t_{.025}^{20}\sqrt{5.2^2\left(\tfrac{1}{6}+\tfrac{1}{16}\right)}$$

$$5.2 \pm 2.086\,(2.489)$$

$$5.2 \pm 5.193 \qquad \boxed{(0.0073,\ 10.393)}$$

Notice that $S_p^2 = 5.2^2$. S_p^2 is a weighted average of S_1^2 and S_2^2, but both are equal to $5.2^2 = S_1^2 = S_2^2$, so S_p^2 must equal 5.2^2 (OR you can use the formula).

   (b) Calculate the test statistic and the critical value for a test with size 5% under the assumptions stated in part (a).

Pooled df $6+16-2$

$$T^*_{obs} = \frac{74.5-69.3-0}{\sqrt{5.2^2\left(\frac{1}{6}+\frac{1}{16}\right)}} = \frac{\cancel{5.2}}{\cancel{5.2}\sqrt{\left(\frac{1}{6}+\frac{1}{16}\right)}} = 2.0889 \quad \text{and} \quad t_{.025}^{20} = 2.086$$

   (c) Under these assumptions, would you reject the null hypothesis that the age difference is zero?

Yes, Reject $H_0: \mu_A = \mu_B$. (b/c $T^*_{obs} = 2.0889 > t_{.025}^{20} = 2.086$)

So $p < 0.05$ (Just barely)

and we Reject at the 5% level.

Problem 2 continued.

(d) Construct a 95% confidence interval for the increase in average age of Hospital A over Hospital B without assuming anything about the population variances.

$\gamma = \min(6-1, 16-1) = 5$

OR use Satterthwaite's df

$\gamma = 9.02985$ OR 9

$$74.5 - 69.3 \pm t^{\gamma}_{\alpha/2}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}$$

$$5.2 \pm \begin{Bmatrix} 2.262 \\ 2.571 \end{Bmatrix}\sqrt{5.2^2\left(\tfrac{1}{6}+\tfrac{1}{16}\right)}$$

$t^{9}_{0.025} = 2.262$

$t^{5}_{0.025} = 2.571$

$5.2 \pm 2.262(2.489)$ $(-0.428, 10.83)$

OR $5.2 \pm 2.571(2.489)$ $(-1.199, 11.599)$ ← either is ok.

(e) Calculate the test statistic and the critical value for a test with size 5% under the assumptions stated in part (d).

$$T^*_{obs} = \frac{74.5-69.3-0}{\sqrt{5.2^2\left(\frac{1}{6}+\frac{1}{16}\right)}} = 2.0889$$

$t^{9}_{0.025} = 2.262$

$t^{5}_{0.025} = 2.571$

(f) Under these assumptions, would you reject the null hypothesis that the age difference is zero?

No, Fail to Reject $H_0: \mu_A = \mu_B$. (b/c $T^*_{obs} < t^{9}_{0.025}$ OR $t^{5}_{0.025}$)

So $p > 0.05$ and we Fail to Reject at the 5% level.

Problem 2 continued.

(g) The sample variances are equal, so it is quite tempting to assume that the population variances are equal. Does the Wilcoxon rank-sum test impact the decision to assume equal variances or not? Answer 'yes' or 'no' and justify your answer. (You do not need to fill the entire page with your answer.)

Yes it should impact our decision. The Wilcoxon R/S Test does not assume equal variances (Just a shift in the distribution), so we see that the assumption of equal variances with the T-test makes a big difference here. So from the Wilcoxon R/S Test and the t-test that does not assume equal variances, we would choose to not assume equal variances and Fail to Reject the Null Hypothesis.

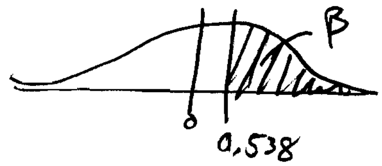
3. Over the course of a month, Drug X is lowers the concentration of LDL cholesterol in the blood stream by 60 mg/dl (sd=25 mg/dl). A Phase II study with 39 participants is planned with the hopes of showing that the drug lowers LDL cholesterol by at least 50 mg/dl.

(a) What is the power of a two-sided test, with type I error of 5%?

$$n = \frac{(Z_{\alpha/2} + Z_\beta)^2 \sigma^2}{(\mu_0 - \mu_a)^2} \quad \text{so} \quad Z_\beta = \sqrt{\frac{n(\mu_0-\mu_a)^2}{\sigma^2}} - Z_{\alpha/2}$$

$$= \sqrt{\frac{39(60-50)^2}{25^2}} - 1.96$$

$$Z_\beta = 0.538$$



$\beta = 0.2946$

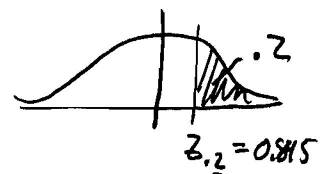
$\text{power} = 1-\beta = 1 - 0.2946$

$\boxed{= 0.7054}$

(b) What is the smallest type I error that would yield at least 80% power?

$$n = \frac{(Z_{\alpha/2} + Z_\beta)^2 \sigma^2}{(\mu_0 - \mu_a)^2} \quad \text{so} \quad Z_{\alpha/2} = \sqrt{\frac{n(\mu_0-\mu_a)^2}{\sigma^2}} - Z_\beta$$

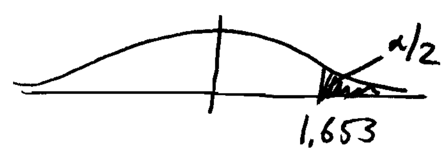
$$= 2.498 - Z_{.2}$$



$$= 2.498 - 0.845$$

$$Z_{\alpha/2} = 1.653$$

$$\Rightarrow \alpha/2 = 0.0495$$



$\boxed{\alpha = 0.0990}$

Problem 3 continued.

(c) What sample size would I need to achieve 80% power with a type I error of 5%?

$$n = \frac{(Z_{\alpha/2} + Z_{\beta})^2 \sigma^2}{(\mu_0 - \mu_a)^2} = \frac{(1.96 + 0.845)^2 \, 25^2}{10^2} = 49.175$$

$$\boxed{n = 50}$$

(d) What sample size is needed to yield a 90% confidence interval with a margin of error of 5 mg/dl?

$$n = \frac{4\sigma^2 (Z_{\alpha/2})^2}{L^2} = \frac{4 \cdot \sigma^2 (Z_{\alpha/2})^2}{2^2 MOE^2} \quad b/c \; L = 2MOE$$

$$= \frac{\sigma^2 Z_{\alpha/2}^2}{MOE^2}$$

$$= \frac{25^2 \, 1.645^2}{5^2} = 67.65$$

$$\boxed{n = 68}$$

4. The table below displays the empirical evidence for the claim that drug X lowers the concentration of LDL cholesterol in the blood stream over a one month period. 8 participants were administered the drug and their LDL levels were recorded at baseline and one month later:

	LDL cholesterol (mg/dl)	
Id	Baseline (ldl0)	One month (ldl1)
1	165	140
2	232	165
3	197	134
4	197	125
5	200	103
6	215	125
7	188	134
8	174	123

Part (e) differences

− < 45 mg/dl
+
+
+
+
+
+
+
all > 45 mg/dl

```
. ttest ldl0=ldl1, unpaired unequal

Two-sample t test with unequal variances
------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
    ldl0 |       8         196    7.540368    21.32738    178.1699    213.8301
    ldl1 |       8     131.125    6.231938    17.62658    116.3888    145.8612
---------+--------------------------------------------------------------------
combined |      16    163.5625    9.616379    38.46551    143.0657    184.0593
---------+--------------------------------------------------------------------
    diff |              64.875    9.782341                43.82395    85.92605
------------------------------------------------------------------------------
    diff = mean(ldl0) - mean(ldl1)                                t =   6.6318
Ho: diff = 0                     Satterthwaite's degrees of freedom =  13.5206

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 1.0000         Pr(|T| > |t|) = 0.0000          Pr(T > t) = 0.0000


. ttest ldl0=ldl1, unpaired

Two-sample t test with equal variances
------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
    ldl0 |       8         196    7.540368    21.32738    178.1699    213.8301
    ldl1 |       8     131.125    6.231938    17.62658    116.3888    145.8612
---------+--------------------------------------------------------------------
combined |      16    163.5625    9.616379    38.46551    143.0657    184.0593
---------+--------------------------------------------------------------------
    diff |              64.875    9.782341                43.89397    85.85603
------------------------------------------------------------------------------
    diff = mean(ldl0) - mean(ldl1)                                t =   6.6318
Ho: diff = 0                                     degrees of freedom =       14

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 1.0000         Pr(|T| > |t|) = 0.0000          Pr(T > t) = 0.0000
```

Problem 4 continued.

```
. ttest ldl0=ldl1

Paired t test
------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
    ldl0 |       8         196    7.540368    21.32738    178.1699    213.8301
    ldl1 |       8     131.125    6.231938    17.62658    116.3888    145.8612
---------+--------------------------------------------------------------------
    diff |       8      64.875     8.04327     22.7498    45.85569    83.89431
------------------------------------------------------------------------------
     mean(diff) = mean(ldl0 - ldl1)                               t =   8.0657
 Ho: mean(diff) = 0                              degrees of freedom =        7

 Ha: mean(diff) < 0           Ha: mean(diff) != 0           Ha: mean(diff)
      > 0
 Pr(T < t) = 1.0000         Pr(|T| > |t|) = 0.0001          Pr(T > t) =
      0.0000
```

(a) After testing the null hypothesis that drug X lowers the concentration of LDL cholesterol at the 5% level, we

☐ reject using a two-sample t-test with unequal variances
☐ fail to reject using a two-sample t-test with unequal variances
☐ reject using a two-sample t-test with equal variances
☐ fail to reject using a two-sample t-test with [redacted]equal variances
☒ ~~reject using a paired t-test~~
☒ fail to reject using a paired t-test

Either o.k. *
data are paired

(b) After testing the null hypothesis that drug X lowers the concentration of LDL cholesterol *by* [redacted] *45 mg/dl* at the 5% level, we

☐ reject using a two-sample t-test with unequal variances
☐ fail to reject using a two-sample t-test with unequal variances
☐ reject using a two-sample t-test with equal variances
☐ fail to reject using a two-sample t-test with [redacted]equal variances
☒ ~~reject using a paired t-test~~
☒ fail to reject using a paired t-test

Either o.k. *
data are paired.

* The wording on this question was poor and the stata output use a difference of $\bar{X}_0 - \bar{X}_1$ instead of the reverse $\bar{x}_1 - \bar{x}_0$ which is the change the question wants. It is only a matter of switching the sign, but I realize this is not easy under pressure so either answer is ok, if you realized the data are paired.

Problem 4 continued.

(c) What is the observed test-statistic from the t-test that tests whether Drug X lowers the concentration of LDL cholesterol in the blood stream *by* [redacted] *45 mg/dl?*

$H_0: \mu_0 - \mu_1 = \mu_d = 45$

$$T^*_{obs} = \frac{\bar{d} - 45}{\sqrt{S_d^2/\sqrt{n}}} = \frac{64.815 - 45}{8.04327} = \boxed{2.4710}$$

(d) What is the p-value from the t-test that tests whether drug X lowers the concentration of LDL cholesterol in the blood stream *by* [redacted] *45 mg/dl?*

From the table we have

$t^7_{0.05} = 1.895$

$t^7_{0.025} = 2.365$

$t^7_{0.02} = 2.517$

$2.4710 = t^*_{obs}$

So $\underset{t_{.025}}{2.365} < 2.4710 < \underset{t_{.02}}{2.510}$

$$\boxed{0.05 > p > 0.04}$$

$2(.025) > p > 2(.02)$

Remember the Test is 2-sided.

I also gave credit for one sided p-values b/c the Question was vague. $\boxed{.025 > p > .02}$

Problem 4 continued.

(e) What is the p-value from the sign test that tests whether drug X lowers the concentration of LDL cholesterol in the blood stream *by at least 45 mg/dl*? (Hint: this test is two-sided.)

7 of the 8 differences are greater than 45 mg/dl. So this is a Sign test with 7 +'s and 1 −'s.

Number of +'s $= X \sim Bin(8, 1/2)$ under H_0.

p-value* $= P(X \geq 7 \mid X \sim Bin(8, 1/2)) + P(X \leq 1 \mid X \sim Bin(8, 1/2))$ * 2-sided

$= \binom{8}{7} 1/2^8 + \binom{8}{8} 1/2^8 + \binom{8}{1} 1/2^8 + \binom{8}{0} 1/2^8$

$= 0.0313 + 0.0039 + 0.0313 + 0.0039 = \boxed{0.0703}$

(f) Suppose that drug X lowers the concentration of LDL cholesterol *by at least 45 mg/dl* 75% of the time, what is the power of the sign test you preformed in (e) ?

Power $= P(\text{Reject } H_0 \mid \theta = 0.75)$

$= P(X \geq 7 \mid \theta = .75) + P(X \leq 1 \mid \theta = .75)$

$= \binom{8}{7} 3/4^7 \, 1/4^1 + \binom{8}{8} 3/4^8 + \binom{8}{1} 3/4^1 \, 1/4^7 + \binom{8}{0} 1/4^8$

$= 0.267 + 0.1001 + 0.0004 + 0.0000153$

$= \boxed{0.3675}$

5. Suppose we enroll 20 participants in a study and construct a 90% confidence interval for the probability of being cured of a disease using the standard formula (i.e., $\hat{\theta}_n \pm Z_{\alpha/2}\sqrt{\hat{\theta}_n(1-\hat{\theta}_n)/n}$). Also suppose that the true probability of being cured is $\theta = 0.15$.

(a) If we repeat this study with a larger sample, say n = 50, (from the same probability distribution) how will the variance of the new sample proportion, $\hat{\theta}_{50}$, compare to the variance of the old one, $\hat{\theta}_{20}$?

$VaR(\hat{\theta}_{25}) = \frac{\theta(1-\theta)}{25}$

$VaR(\hat{\theta}_{50}) = \frac{\theta(1-\theta)}{50}$

- ☐ it will definitely be greater
- ☒ it will definitely be smaller
- ☐ it will probably be greater, although it might be smaller
- ☐ it will probably be smaller, although it might be greater
- ☐ there is no reason to expect it to be greater, and there is no reason to expect it to be smaller

(b) If we repeat this study with a larger sample, say n = 50, (from the same probability distribution) and construct a 90% confidence interval for the probability of being cured based on the new sample, how will the width of the new CI compare to the width of the old one?

width $= 2Z_{\alpha/2}\sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}}$

notice we use $\hat{\theta}$ instead of θ so the width depends on the observed estimate of θ.

- ☐ it will definitely be greater
- ☐ it will definitely be smaller
- ☐ it will probably be greater, although it might be smaller
- ☒ it will probably be smaller, although it might be greater
- ☐ there is no reason to expect it to be greater, and there is no reason to expect it to be smaller

Hence if we know θ then #2 would be correct.

(c) If we repeat this study with a larger sample, say n = 50, (from the same probability distribution) how will the coverage probability of the new confidence interval compare to the old one?

- ☐ it will be approximately the same
- ☐ there is no reason to expect it to be different
- ☒ it will be closer to 90% → why? b/c this interval is 'approximately correct' and depends on the CLT. if the interval were 'exact' the 2nd answer would be correct.
- ☐ it will be further from 90%