MODULE *ABSpec*

EXTENDS *Integers*

CONSTANT *Data* The set of all possible data values.

VARIABLES *AVar*, The last $\langle value, bit\rangle$ pair A decided to send.
BVar The last $\langle value, bit\rangle$ pair B received.

Type correctness means that *AVar* and *BVar* are tuples $\langle d, i\rangle$ where $d \in Data$ and $i \in \{0, 1\}$.

$$\begin{aligned} TypeOK \triangleq\ & \land AVar \in Data \times \{0, 1\} \\ & \land BVar \in Data \times \{0, 1\} \end{aligned}$$

It's useful to define *vars* to be the tuple of all variables, for example so we can write [*Next*]_*vars* instead of [*Next*]_$\langle \ldots \rangle$

$$vars \triangleq \langle AVar, BVar\rangle$$

Initially *AVar* can equal $\langle d, 1\rangle$ for any *Data* value *d*, and *BVar* equals *AVar*.

$$\begin{aligned} Init \triangleq\ & \land AVar \in Data \times \{1\} \\ & \land BVar = AVar \end{aligned}$$

When $AVar = BVar$, the sender can "send" an arbitrary data *d* item by setting $AVar[1]$ to *d* and complementing $AVar[2]$. It then waits until the receiver "receives" the message by setting *BVar* to *AVar* before it can send its next message. Sending is described by action A and receiving by action *B*.

$$\begin{aligned} A \triangleq\ & \land AVar = BVar \\ & \land \exists\, d \in Data : AVar' = \langle d, 1 - AVar[2]\rangle \\ & \land BVar' = BVar \end{aligned}$$

$$\begin{aligned} B \triangleq\ & \land AVar \neq BVar \\ & \land BVar' = AVar \\ & \land AVar' = AVar \end{aligned}$$

$$Next \triangleq A \lor B$$

$$Spec \triangleq Init \land \Box[Next]_{vars}$$

For understanding the spec, it's useful to define formulas that should be invariants and check that they are invariant. The following invariant *Inv* asserts that, if *AVar* and *BVar* have equal second components, then they are equal (which by the invariance of *TypeOK* implies that they have equal first components).

$$Inv \triangleq (AVar[2] = BVar[2]) \Rightarrow (AVar = BVar)$$

FairSpec is *Spec* with the addition requirement that it keeps taking steps.

$$FairSpec \triangleq Spec \land \mathrm{WF}_{vars}(Next)$$

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