Sumedh

• Ordinary differential equation

The DE is a math. eq involving an unknown function and its derivatives

eg $$\frac{d^2y}{dx^2} + \frac{3dy}{dx} + 2y = e^{-7x}$$ O = 2, D = 1

$$\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 + 2y = \sin 3x$$ O = 2, D = 1

The order of a diff eq is the order of the highest derivative of the unknown function involved in the eq.

The degree of a DE is the degree of the highest derivative of unknown function involved in the eq.

• Linear DE with const. coeffs.

$$\left(a_0 \frac{d^ny}{dx^n} + a_1 \frac{d^{n-1}y}{dx^{n-1}} + a_2 \frac{d^{n-2}y}{dx^{n-2}} + \dots a_n\right) y = F(x)$$

$a_0, a_1, \dots$ are consts. is called Linear DE of order n with const. coeff

Then the above equation can be written as

$$(a_0 D^n + a_1 D^{n-1} + \dots a_n) y = F(x)$$

$$(\phi D) y$$

The gen. or complete soln of eq (i) consists of 2 parts namely complementary func. and Particular integral

$$y = CF + PI$$

To find the CF, form the auxillary eq. by putting $D = m$ in $\phi(D) = 0$. Therefore, the auxillary eq. of eq. (i) is $\phi(m) = 0$ which will be polynomial eq. of degree n.
By solving this eq. we get n roots $m_1, m_2 \dots m_n$

Case - 1 (if all roots & unequal)

$$CF = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$

Case - 2 (if $m_1 = m_2$ & rest diff)

$$CF = (C_1 + C_2 x) e^{m x} + C_3 e^{m_3 x} + \dots \; C_n e^{m_n x}$$

Case - 3 ($m_1 = m_2 = m_3 = m$ then rest cases

$$CF = (C_1 + C_2 x + C_3 x^2)$$

Case - 4 (imaginary)

$$CF = e^{\alpha x} (C_1 \cos \beta x + C_2 \sin \beta x) \qquad m = \alpha \pm iB$$

To find PI,
let the given diffrential equation,

if $f(x) \neq 0$

$$PI = \frac{1}{\phi(D)} \cdot F(x)$$

eg $\frac{d^2y}{dx} - 7\frac{dy}{dx} + 12y = 0$

$D^2 - 7D + 12 = 0$

$m^2 - 7m + 12 = 0$

To find CF put $d = m$, the auxiliary is

Type - 1

$F(x) = e^{ax}$

If $f(x) = e^{ax}$ then $PI = \frac{1}{\phi(D)} \times f(x)$

Replace D by a

$$= \frac{1}{\phi(a)} \cdot e^{ax} \quad (\text{here } \phi(a) \neq 0)$$

if $\phi(a) = 0$, then ;

$$PI = \frac{1}{\phi(D)} \cdot e^{ax}$$

$$= x \cdot \frac{1}{\phi'(D)} e^{ax}$$

Replace D by a

$$= x \cdot \frac{1}{\phi'(a)} \cdot e^{ax} \quad (\text{provided } \phi'(a) \neq 0)$$

If $\phi'(a) = 0$ then

$$PI = x^2 . \frac{1}{\phi''(a)} e^{ax}$$

$\vdots$

goes on

eg :- $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 3e^{4x}$

Sol^n $(D^2 + 6D + 9)y = 3e^{4x}$

$$\phi(D)y = F(x)$$

To find CF

Put $D = m$ in $\phi(D) = 0$

The auxillary eq^n is $m^2 + 6m + 9 = 0$

$$(m+3)^2 = 0$$

$$m = -3, -3$$

CF : $(c_1 + c_2 x)e^{-3x}$

Now for PI

$$PI = \frac{1}{\phi(D)} . F(x)$$

$$= \frac{1}{D^2 + 6D + 9} 3e^{4x}$$

$$= 3 . \frac{1}{D^2 + 6D + 9} e^{\overset{a}{4}x}$$

$$= 3 \frac{1}{4^2 + 6(4) + 9} e^{4x}$$

$$= \frac{3}{49} \cdot e^{4x}$$

$$y = PI + CI$$

$$= (c_1 + c_2 x) e^{-3x} + \frac{3e^{4x}}{49}$$

eg $(D^2 + 2D + 1)y = e^{-x} + 3$

Soln $\phi(D) = Fx$

$\phi(D) = M$

$m^2 + 2m + 1$

$m = -1, -1$

$\therefore CF = (c_1 + c_2 x) e^{-x}$

$$PI = \frac{1}{\phi(D)} \cdot F(x)$$

$$PI = \frac{1}{(D^2 + 2D + 1)} e^{-x} + 3$$

split

$$PI = PI_1 + PI_2$$

$$= \frac{1}{D^2 + 2D + 1} \cdot e^{-x} + \frac{3}{D^2 + 2D + 1} \cdot e^{0x}$$

$$= -1 \frac{1}{(-1)^2 + 2 + 1} e^{-x} = 3$$

$$\therefore = \phi(D) = 0$$

$$\therefore \quad x \frac{1}{2D+2} e^{-x}$$

$$\phi = (D) = 0$$

$$\therefore \quad x^2 \frac{1}{2} \cdot e^{-x}$$

$$PI = \frac{x^2 \cdot e^{-x}}{2} + 3$$

$$\therefore y = (c_1 + c_2 x) e^{-x} + \left(\frac{x^2 \cdot e^{-x}}{2} \cdot 3 \right)$$

eg solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{-2x}$

solⁿ $(D^2 + 3D + 2)y = e^{-2x}$

$$\phi(D).y = e^{-2x}$$

$$m^2 + 3m + 2$$

$$m = -2, -1$$

$$CF = c_1 e^{-2x} + c_2 e^{-1x}$$

$$PI = \frac{1}{(-2)^2 + 3(-2) + 2} e^{-2x}$$

$$=$$

$$= x \frac{1}{2D+3} e^{-2x}$$

$$= -x \cdot e^{-2x}$$

$$y = c_1 e^{-2x} + c_2 e^{-x} - x \cdot e^{-2x}$$

Type -2

$$F(x) = \sin ax \;/\; \cos ax$$

$$PI = \frac{1}{\phi(D)} \cdot F(x)$$

$$= \frac{1}{\phi(D)} \cdot \sin ax \;/\; \cos ax$$

$$D^2 = -(a^2)$$

$$PI = \frac{1}{\phi(-a^2)} \cdot \sin ax \;/\cos ax$$

$$\phi(-a^2) \neq 0$$

$$PI = x \frac{1}{\phi'(D)} \cdot \sin ax$$

$$D^2 = -a^2$$

$$\vdots$$

goes on

eg $(D^2 + 3D + 2)y = \sin x$

soln $\phi(D)\, y = \sin x$

$$m^2 + 3m + 2$$

$$m = -2, -1$$

$$CF = C_1 e^{-2x} + C_2 e^{-x}$$

$$PI = \frac{1}{\phi(D)} \cdot Fx$$

$$= \frac{1}{D^2 + 3D + 2} \cdot \sin x$$

$$D^2 = -a^2$$

$$= \frac{1}{(-1)^2 + 3(D) + 2)} \sin x$$

$$= \frac{1}{3D + 1} \sin x$$

$$= \frac{1}{3D+1} \times \frac{\overset{\text{conj}}{3D-1}}{3D-1} \sin x$$

$$= \frac{3D-1}{(3D)^2 - 1^2} \sin x$$

$$= \frac{(3D-1)}{-10} \sin x$$

$$= -\left(3\overset{y'}{D} \cdot \sin x - \sin x \right) \times \frac{1}{10}$$

$$= -\frac{1}{10} \left(3\cos x - \sin x \right)$$

$$= -\frac{1}{10} \left(\qquad \right)$$

eg $(D^2 + 6D + 8)\, y = e^{-2x} + \cos^2 x$

Sol^n $m^2 + 6m + 8$

$$CF = C_1 e^{-4x} + C_2 e^{-2x}$$

$$PI = \frac{1}{D^2 + 6D + 8} \cdot e^{-2x} + \cos^2 x$$

$$PI = \frac{1}{D^2 + 6D + 8} e^{-2x} + \frac{1}{D^2 + 6D + 8} \cdot \cos^2 x$$

$$\vdots$$

$$PI = \frac{x}{2} e^{-2x} + \frac{1}{(\quad)} \frac{\cos 2x + 1}{2}$$

$$= (\qquad) + \frac{1}{2} \frac{1}{(\ )} + \frac{1}{(\ )} \frac{\cos x}{2}$$

$$= (\qquad) + \frac{1}{2} \times \frac{1}{8} + \frac{1}{2} \left(\frac{1}{6D+4} \right) \cos 2x$$

$$= (\quad) + \frac{1}{16} + \frac{1}{4} \left(\frac{1}{3D+2} \right) \cos 2x$$

$$= (\quad) + \frac{1}{16} + \frac{1}{4} \frac{3D-2}{9D^2-4} \cos 2x$$

$$(\qquad) + \frac{1}{4} \frac{3D-2}{(-40)} \cdot \cos 2x$$

$$= (\qquad) + \frac{3D-2}{-160} \cdot \cos 2x$$

$$= (\quad) + \frac{1}{-160} \left(3D \cos 2x - 2 \cos 2x \right)$$

$$= \left(\quad \right) - \frac{1}{160}\left(3\left(-2.\sin 2x - 2\cos 2x\right)\right.$$

$$y = C_1 e^{-4x} + C_2 e^{-2x} + \frac{x}{2} e^{-2x} + \frac{1}{16} + \frac{1}{16}(6\sin 2x - 2\cos 2x)$$

eg $(D^2+4)y = \sin 2x$

Soln

$$m^2+4=0$$

$$m^2 = -4$$

$$m = \pm 2i \qquad (\alpha \pm \beta i) = m$$

$$\alpha = 0 \quad \beta = 2$$

$$CF = e^{0x}\left(C_1 \cos \overset{\beta}{2}x + C_2 \sin \overset{\beta}{2}x\right)$$

$$PI = \frac{1}{D^2+4} \sin 2x$$

$$= \frac{1}{0} \; - \cdot$$

$$= x \cdot \frac{1}{2D} \sin 2x$$

$$= \frac{x \cdot D}{2D \cdot D} \sin 2x$$

$$= \frac{x}{2} \cdot \frac{D}{-4} \sin 2x$$

y'

$$= -\frac{x}{8} \quad 2\cos 2x$$

$$y = e^{0x}(c_1 \cos 2x + c_2 \sin 2x) - \frac{x}{8} 2\cos 2x$$

eg $(D^2 - 4D + 3)y = \sin 3x \cdot \cos 2x$

$(D^2 - 3D + 2)y = \cos 3x \cdot \cos x$

Type - 3

$F(x) = x^n$ where n = const. (+ve Int.) then

$$PI = \frac{1}{\phi(D)} F(x)$$

$$= \frac{1}{\phi(D)} \cdot x^n$$

$$= \frac{1}{1 \pm f(D)}$$

$$= [1 \pm f(D)]^{-1} \cdot x^n$$

Note : $(1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots.$

$(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots.$

$(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots.$

eg Solve $(D^2 + 5D + 6)y = x^2 + 4e^{3x}$

Soln $(D^2 + 5D + 6)y = f(x)$

Aux eq

$m^2 + 5m + 6$

$m = -2, -3$

$CF = C_1 e^{-2x} + C_2 e^{-3x}$

$PI = \frac{1}{\phi(D)} \cdot Fx$

$= \frac{1}{(\quad)} x^2 + 4e^{x}$

$= \frac{1}{(\quad)} x^2 + \frac{1}{(\quad)} 4e^{x}$

H.W

$= \frac{1}{D^2 + 5D + 6} \cdot x^2$

$= \frac{1}{6\left(1 + \frac{D^2 + 5D}{6}\right)} \cdot x2$

$= \frac{1}{6}\left[1 + \left(\frac{D^2 + 5D}{6}\right)\right]^{-1} \cdot x^2$

expand

$$= \frac{1}{6}\left[1-(\ )+(\ )^2+\cdots\right]x^2$$

$$= \frac{1}{6}\left[1-\frac{D^2}{6}-\frac{5D}{6}+\frac{D^4}{36}+\frac{10D^3}{36}+\frac{25D^2}{36}+\cdots\right)x^2$$

$$= \frac{1}{6}\left[1-\frac{D^2}{6}x^2 \quad - \quad \right]$$

$$= \frac{1}{6}\left[x^2 - 2/6 - 10x/6 + 0 + 0 + 50/36\right]$$

$$= \frac{1}{6}\left(x^2 - \frac{5x}{3} + \frac{19}{18}\right)$$

eg $(D^2+3D+2)y = x^2+\sin x$

sol^n m

$= -2,-1$

$$CF = C_1e^{-2x} + C_2e^{-e^{-x}}$$

$\vdots$

$$PI = \frac{1}{\left(2+\left(\frac{D^2+3D}{2}\right)\right)} \cdot x^2 + PI_2$$

$$= \frac{1}{2}$$

eg find the PI of $(D^2+2D+1)y = 1+x$

Soln

$m = -1, -1$

$$CF = (c_1 + c_2 x)e^{-x}$$

$$PI = \frac{1}{(\quad)}(1+x)$$

eg $(D^2 - 5D + 6)y = x^3 + 3x - 1$

$(D^2 + 2)y = x^2$

- **Method of Variation of parameters**

Consider the eq. $\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = F(x)$ — (i)

where a_1, a_2 are consts. $F(x)$ is a func. of x

let the complementary of the above eq. is

$\boxed{CF = C_1 f_1 + C_2 f_2}$ — (ii), where C_1, C_2 are const.
and f_1 & f_2 are func. of x

Then $PI = Pf_1 + Qf_2$ — (iii)

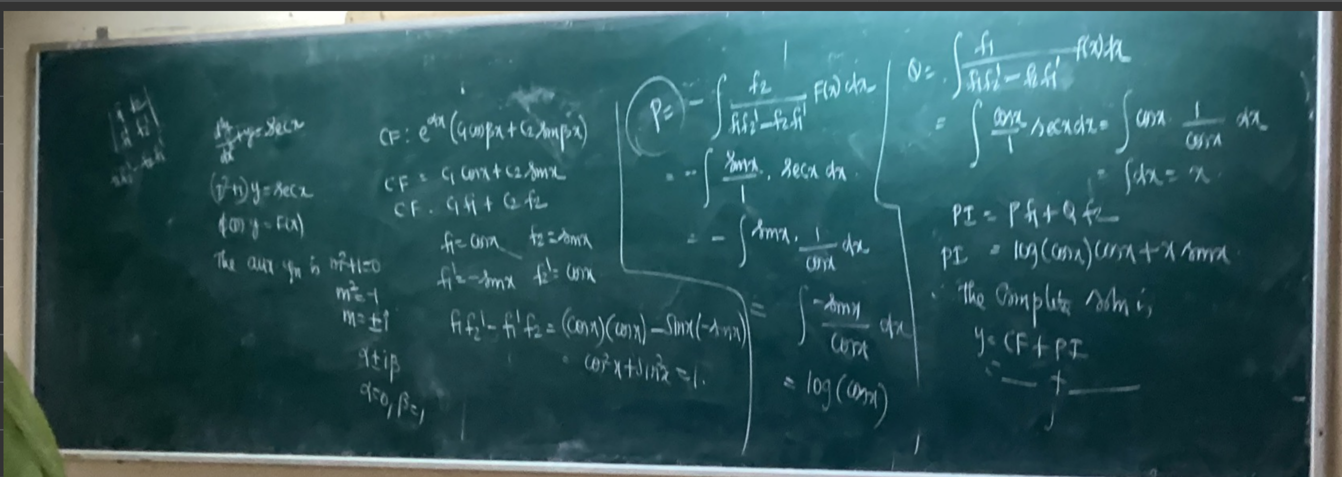
$$P = -\int \frac{f_2}{f_1f_2' - f_2f_1'} F(x)\,.dx \quad \text{— (iv)}$$

$$Q = \int \frac{f_1}{f_1f_2' - f_2f_1'} F(x)\,.dx \quad \text{— (v)}$$

Subs (iv) & (v) in (iii) we get
complete sol^n $y = CF + PI$

eg $\frac{d^2y}{dx^2} + y = \sec x$

Sol^n



- Type - 4 of PI

$F(x) = e^{ax} f(x)$

$f(x) = x^n \,/\, \sin ax \,/\, \cos ax$

$PI = \frac{1}{\phi(D)} \cdot F(x)$

$= \frac{1}{\phi(D)} \cdot e^{ax} \cdot f(x)$

Replace D with D+a

$= e^{ax} \cdot \frac{1}{\phi(D+a)} \cdot f(x)$

and then by normal way

eg $(D^2 + 9)y = (x^2+1) e^{3x}$

Soln $\phi(D) y = F(x)$

$\therefore$ aux eq

$m^2 + 9 = 0$

$m = \pm 3i$

$\therefore$ CF $= (c_1 \cos 3x + c_2 \sin 3x) e^0$

PI $= \frac{1}{\phi(D)} \cdot F(x)$

$= \frac{1}{\phi(D)} \cdot e^{3x} \cdot (x^2+1)$

$D \longrightarrow D+a$

$= e^($

Math - Thursday

eg $(D^2 - 2D + 1)\,y = x\sin x$

soln PI $\frac{1}{\phi(D)} F(x)$

$$= \frac{1}{\phi(D)} \cdot x\sin x$$

$$= \frac{1}{D^2 - 2D + 1} \cdot x\sin x$$

= Imaginary part of (IP) $\frac{1}{D^2 - 2D+1}\, x\boxed{(\cos x + i\sin x)}$

just write this step. That's it

$= (IP)\ \frac{1}{D^2 - 2D + 1}\, x \cdot e^{ix}$

$e^{i\theta} = \cos\theta + i\sin\theta$

Now Type 4

$D \rightarrow D + a$

$$= IP\ e^{ix} \frac{1}{(D+i)^2 + 2(D+i) + 1} \times x$$

$$= IP\ e^{ix} \frac{1}{D^2 + 2D(i-1) - 2i} \cdot x$$

Type - 3

$$= IP\ e^{ix} \cdot \frac{1}{-2i\left[1 + \left(\frac{D^2 + 2D(i-1)}{-2i}\right)\right]} \cdot x$$

$$IP\ \frac{e^{ix}}{-2i} \cdot \left[1 - \frac{D^2 + 2D(i-1)}{2i}\right]^{-1} x$$

$$\frac{e^{ix}}{-2i} \quad \frac{\times i}{i}$$

$$i \times i = -1$$

$$\therefore \frac{e^{ix} \times i}{-2(-1)} = \frac{i e^{ix}}{2}$$

$$= \frac{ie^{ix}}{2}\left[1 + (\qquad) + \cdots\right] x$$

$$= \frac{ie^{ix}}{2}\left[1 + \frac{D^2}{2i} + \cancel{\frac{2Di}{2i}} - \cancel{\frac{2D}{2i}}\right] x$$

$$= \frac{ie^{ix}}{2}\left[x + \cancel{\frac{D^2 x}{2i}} + Dx - \frac{Dx}{i}\right]$$

$$= \frac{ie^{ix}}{2}\left[x + 1 - 1/i\right]$$

$$\frac{1}{i} \times \frac{i}{i}$$

$$\frac{i}{-1}$$

$$= IP \; \frac{ie^{ix}}{2}\left[x + 1 - \frac{i}{(-1)}\right]$$

$$= IP \; \frac{ie^{ix}}{2}\left[x + 1 + i\right]$$

$$= IP \; \frac{1}{2} \times i \times e^{ix}\left[x + 1 + i\right]$$

$$= IP \ \frac{1}{2} x i x (\cos x + i \sin x)(x + 1 + i)$$

$$\vdots$$

$$PI = IP \ \frac{1}{2} (x \cos x + \cos x - \sin x)$$

$$y = CF + PI$$

- Type - 6

$F(x) = x f(x)$ where $f(x) = \cos ax / \sin ax$

then $PI = \frac{1}{\phi(D)} \cdot F(x)$

$$= \frac{1}{\phi(D)} \cdot x f(x)$$

$$\boxed{PI = x \cdot \frac{1}{\phi(D)} \cdot f(x) - \frac{\phi'(D)}{[\phi(D)]^2} \cdot f(x)}$$

The diff b/w Type 6 & Type 5

$x^n F(x)$

here $n = 1$

$\therefore x F(x)$

i.e special case

if $n = 2, 3, \ldots$

then type 5

eg $(D^2+4)y = x\sin x$

Soln

$$m = \pm 2i$$

$$CF = C_1 \cos 2x + C_2 \sin 2x$$

$$PI = \frac{1}{\phi(D)} \cdot F(x)$$

$$= \frac{1}{D^2+4} \cdot x\sin x$$

Applying formula

$$= x \cdot \frac{1}{D^2+4} \cdot \sin x - \frac{2D \sin x}{(D^2+4)^2}$$

Type - 2

$$= x \times \frac{1}{(-1)+4} \sin x - \frac{2 \cos x}{(D^2+4)^2}$$

Type -2

$$= \frac{x\sin x}{3} - \frac{2 \cos x}{(3)^2}$$

$$PI = \frac{x\sin x}{3} - \frac{2\cos x}{9}$$

$$y = CF + PI$$

- Linear DE with variable co-efficients

① Cauchy's homogeneous linear eq. (Euler type)

An eq. of the form :-

$$x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + a_n y = F(x)$$

Where $a_1, a_2 \dots a_n$ are const.

$F(x)$ is a func. of x is called Cauchy's euler homo. linear DE

The above eq can be transformed to linear DE with const. coeff. by the transformatⁿ :-

$$x = e^z \;/\; z = \log x$$

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}$$

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{1}{x}$$

$$x\frac{dy}{dx} = \frac{dy}{dz}$$

$$x\,Dy = D'y$$

$$D = d/dy$$

$$D' = d/dz$$

Similarly

$$x^2 D^2 y = D'(D'-1)y$$

Substituting eq (ii) & (iii) & so on, we get linear DE with const. coeff & can be solved by any one of the methods.

eg) Solve $(x^2 D^2 + 4xD + 2)y = x + 1/x$

Sol^n let $x = e^z$ / $z = \log x$

so, $xD = D'$, $x^2 D^2 = D'(D'-1)$

where $D = d/dx$, $D' = d/dz$

$\therefore \left(D'(D'-1) + 4D' + 2\right)y = e^z + 1/e^z$

$(D'^2 + 3D' + 2)y = e^z + e^{-z}$

$\cancel{\phi}(D')\, y = F(z)$

$m^2 + 3m + 2 = 0$

$CF = C_1 e^{-2z} + C_2 e^{-z}$

$PI = \dfrac{1}{D'^2 + 3D' + 2} \cdot e^z + \dfrac{1}{(\quad)} e^{-z}$

$= \dfrac{1}{6} e^z + z \dfrac{1}{2D'' + 3} e^{-z}$

$\dfrac{e^z}{6} + \dfrac{z e^{-z}}{1}$

$\dfrac{x}{6} - x \log x$

$a = b^{\log_b a}$

eg $(x^2D^2 + xD + 1)y = 4\sin(\log x)$

Sol^n $(D'(D'-1) + D' + 1)\,y = 4\sin z$

$(D'^2 + 1)y = $ "

$m = \pm i$

$CF = C_1\cos z + C_2\sin z$

$PI = \dfrac{1}{D'^2+1} \cdot 4\sin z$

$= \dfrac{4z\sin z}{D'}$

$= \dfrac{D' 4z\sin z}{D'^2}$

$= z\left(\dfrac{D' 4\sin z}{-2}\right)$

$= -2z\cos z$

$y = CF + PI$

eg $D^2 + \dfrac{D}{x} = 12\dfrac{\log x}{x^2}$

Sol^n $x^2D^2 + xD = 12\log x$

let $x = e^z \,/\, z = \log x$

so $xD = D'$, $x^2D^2 = D'(D'-1)$

$D = d/dx$, $D' = d/dz$

$(D'(D'-1) + D')\,y = 12z$

$$(D'^2)\,y = 12z$$

$$\frac{d^2y}{dz^2} = 12z$$

double int.

$$\frac{dy}{dz} = \frac{12z^2}{2} + c_1$$

$$y = \frac{12z^3}{2\times 3} + c_1 z + c_2$$

$$y = 2z^3 + c_1 z + c_2$$

$$y = 2(\log x)^3 + c_1(\log x) + c_2$$

HW $(x^2D^2 + 4xD + 2)\,y = x\log x$

$(x^2D^2 - 7xD + 12)\,y = x^2$

② lengendre's typ

An eq. of the form

$$(ax+b)^n \frac{d^n y}{dx^n} + P_1 (ax+b)^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + P_n y = F(x)$$

(i)

where $P_1, P_2, \dots$ are consts

& $F(x)$ is a func of x

is known as lengendre's linear eq

eg (i) can be reduced to linear DE with const. coeffs by putting

$$ax+b = e^z$$

$$z = \log(ax+b)$$

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}$$

$$y' = \frac{a}{ax+b} \cdot \frac{dy}{dz}$$

$$\frac{dz}{dx} = \frac{a}{ax+b}$$

$$(ax+b)\, y' = a \frac{dy}{dz}$$

$$(\quad) D = aD'$$

Sub (ii), (iii) in (i)

we get LDE with const. coeff which can be solved by any known method eg $(2x+5)^2 \frac{d^2y}{dx^2} - 6(2x+5)\frac{dy}{dz} + 8y = 0$

Sol^n $\left((2x+5)^2 D^2 - 6(2x+5) D + 8\right) y = 0$

$2x+5 = e^z \quad / \quad z = \log(2x+5)$

$(2x+5) D = 2 \cdot D'$

$(2x+5)^2 D^2 = 2^2 D'(D'-1)$

$\left(4D'^2 - 4D' - 12D' + 8\right) y = 0$

$\left(4D'^2 - 16D' + 8\right) y = 0$

$\left(D'^2 - 4D' + 2\right) y = 0$

$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{16 \pm \sqrt{128}}{8}$

$= 2 \pm \sqrt{2}$

$CF = c_1 e^{(2+\sqrt{2})z} + c_2 e^{(2-\sqrt{2})z}$

$PI = 0$

$y = CF + \cancel{PI} = CF$

eg $\left((1+x)^2 D^2 + (1+x) D + 1\right) y = 4\cos(\log(1+x))$

sol^n $\left(D'(D'-1) + D' + 1\right) y = 4\cos z$

$$(D'^2 + 1)\, y = 4\cos z$$

$$CF = c_1 \cos z + c_2 \sin z$$

$$PI = 4 \frac{1}{D'^2+1} \cdot \cos z$$

$$= 4z \frac{1}{2D'} \cdot \cos z$$

$$= 2z \frac{D'}{D'^2} \cdot \cos z$$

$$= +2z \sin z$$

$$y = CF + PI$$

$$= c_1 \cos z + c_2 \sin z + 2z \sin z$$

$$= c_1 \cos(\log(1+x)) + c_2 \sin(\log(x+1)) + 2\log(1+x)\sin(\log(1+x))$$

- Simultoneous LDE with const. coeff

eg $\frac{dx}{dt} + 2y = \sin 2t$

$\frac{dy}{dt} - 2x = \cos 2t$

Sol^n $Dx + 2y = \sin 2t$ —(i)

$Dy - 2x = \cos 2t$ —(ii)

(i) × D $\quad D^2x + 2Dy = D\sin 2t$

(ii) × 2 $\quad -4x + 2Dy = 2\cos 2t$

(−) $\quad + \quad - \quad -$

$D^2x + 4x = D(\sin 2t) - 2\cos 2t$

$D^2x + 4x = D(\sin 2t) - 2\cos 2t$

$(D^2+4)x = 2\cos 2t - 2\cos 2t$

$(D^2+4)x = 0$

$m = \pm 2i$

$CF = C_1\cos 2t + C_2 \sin 2t$

$x = CF + 0$

$x = C_1\cos 2t + C_2\sin 2t$

Now y

x in (i)

$D(C_1\cos 2t + C_2\sin 2t) + 2y = \sin 2t$

$-C_1\sin 2t \cdot 2 + C_2\cos 2t \cdot 2 + 2y = \sin 2t$

$y = \frac{\sin 2t}{2} + C_1\sin 2t - C_2\cos 2t$

eg : Solve $\frac{dx}{dt} + 2x - 3y = 5t$

$$\frac{dy}{dt} - 3x + 2y = 2e^{2t}$$

Soln $Dx + 2x - 3y = 5t$

$Dy - 3x + 2y = 2e^{2t}$

$(D+2)x - 3y = 5t$ — (i)

$(D+2)y - 3x = 2e^{2t}$ — (ii)

(i) $\times (D+2)$ $\quad (D+2)^2 x - 3(\ )y = 5t\ (D+2)$

(ii) $\times\ 3$ $\quad -9x + 3(\ \ )y = 6e^{2t}\ \ )$

$+$

$$(D+2)^2 x - 9x = (D+2)5t + 6e^{2t}$$

$$(D^2+4D-5)x = 10t + 5 + 6e^{2t}$$

Now normal eq

$F = \ 1e \quad + C_2 e^{-}$

wrong

PI $\quad \frac{10t}{D^2+4D-5} + \frac{5}{(\quad)} + \frac{6e^{2t}}{4+8-5}$

$= -2t - 1 + \frac{6}{7}e^{2t}$

$=$