Sumedh

- **Comparison test**

let $\sum u_n$ & $\sum v_n$ be two series of +ve terms such that $\lim_{n\to\infty} \frac{u_n}{v_n} = l$ [$l \neq 0$, $l \in I$] i.e limit exists

then $\sum u_n$ & $\sum v_n$ both converge or diverge together
i.e $\sum u_n$ converge if and only if $\sum v_n$ convergg , "ly
$\sum u_n$ diverge if .... $\sum v_n$ diverge.

The series $\frac{1}{1^p} + \frac{1}{2^p} + \dots + \frac{1}{n^p} + \dots = \boxed{\sum_{n=1}^{\infty} \frac{1}{n^p}}$ is

convergent if $\underline{p > 1}$ & divergent if $p \leq 1$

eg Test the convergence of the series

$$\frac{1}{1.2} + \frac{1}{2.3} + \dots \frac{1}{\infty}$$

Sol^n $U_n = \frac{1}{n(n+1)}$

1, 2, . - -
$1 + (n-1) = n$
2

$V_n = \frac{1}{n^2}$ highest Pow of n / highest Pow of n

$$\lim_{n\to\infty} \frac{U_n}{V_n} = \frac{\cancel{n^2}}{\cancel{n}(n+1)}$$

$$\lim_{n\to\infty} \frac{U_n}{V_n} = \frac{\cancel{n}}{\cancel{n}(1+1/n)}$$

$$\lim_{n\to\infty} \frac{U_n}{V_n} = 1$$

Also, $\sum v_n = \sum \frac{1}{n^2}$

comparing with $\frac{1}{n^p}$

$p = 2$, $p > 1$
$\therefore$ Converg
So, $\sum U_n$ is also converg.

eg $\frac{1\cdot 2}{3\cdot 4\cdot 5} + \frac{2\cdot 3}{4\cdot 5\cdot 6} + \dots \infty$

Sol' $\frac{n(n+1)}{(n+2)(n+3)(n+4)} = U_n$

$V_n = \frac{n^2}{n^3} = \frac{1}{n}$

$\lim_{n\to\infty} = \frac{n^2(n+1)}{(\ )(\ )(\ )}$

$= \frac{\cancel{n^3}(1 + 1/n)}{\cancel{n^3}(1+2/n)(1+3/n)(1+4/n)}$

$\lim_{n\to\infty} \frac{U_n}{V_n} = 1$

Also, $V_n = \frac{1}{n} \equiv \frac{1}{n^p}$, $\therefore$ $p = 1$

$p \leq 1$

$\therefore$ divergent

So U_n also diverg

eg $\frac{3}{1^2\cdot 2^2} + \frac{5}{2^2\cdot 3^2} + \dots \infty$

som $\frac{(2n+1)}{[n(n+1)]^2} = U_n$

$3 + (n-1)\cdot 2$

$3 + 2n - 2$

$2n + 1$

$V_n = \frac{n}{n^4} = \frac{1}{n^3}$

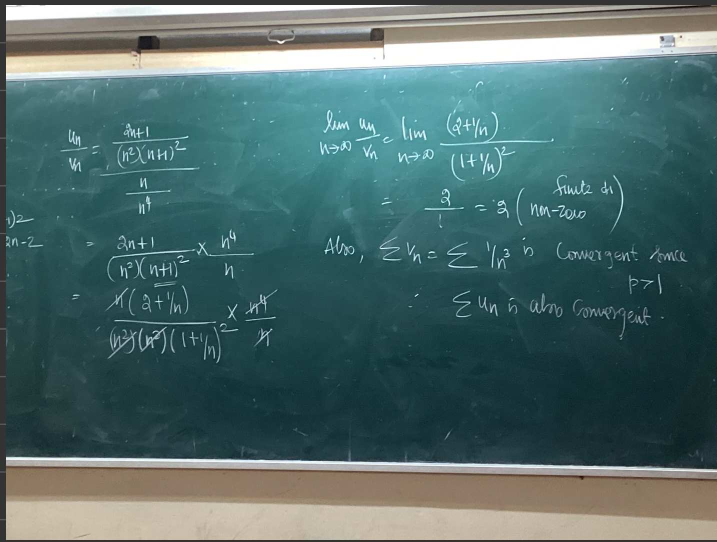
$\lim_{n\to\infty} \frac{U_n}{V_n} = \frac{2n+1}{\cancel{n^2}(n+1)^2} \times n^{\cancel{3}}$

take complete V_n (don't cut n)

$= \frac{2n^2 + n}{n(n+1)^2}$

$= \frac{n^2(2 + 1/n)}{n^3(1 + 1/n)^2}$

$$= \frac{1}{n}$$



eg $\quad \dfrac{n}{1+n(n+1)^{1/2}}$

Sol^n $\quad U_n$

$$V_n = \frac{n}{n^{3/2}}$$

$$\frac{U_n}{V_n} = \frac{\cancel{n}}{1+n(n+1)^{1/2}} \cdot \frac{n^{3/2}}{\cancel{n}}$$

$$\lim_{n\to\infty} \frac{U_n}{V_n} = \frac{\cancel{n^{3/2}}}{1+\cancel{n^{3/2}}(1+1/n)^{1/2}}$$

$$= \frac{1}{2}$$

Also $= \dfrac{n}{n^{3/2}} = \dfrac{1}{n^{1/2}} \equiv \dfrac{1}{n^p}$

$p = 1/2$
$p \leq 1$

$\therefore$ diver

Also, U_n = diver

- **Ratios Test**

$\boxed{\text{If } \lim_{n \to \infty} \frac{U_{n+1}}{U_n} = l}$, then ΣU_n is convergent if $l < 1$

& divergent if $l > 1$

Note : if $l = 1$ i.e $\lim_{n \to 0} \frac{U_{n+1}}{U_n} = 1$, the Test fails :(

i.e The series may be either conv / div

eg $\frac{3}{2 \cdot 3} + \frac{3^2}{3 \cdot 4} + \cdots \infty$

sol^n

$$\frac{3^n}{(n+1)(n+2)} = U_n$$

$$U_{n+1} = \frac{3^n . 3}{(n+2)(n+3)}$$

$$\frac{U_{n+1}}{U_n} = \frac{\cancel{3^n} . 3}{\cancel{n^2}\cancel{(1+2/n)}(1+3/n)} . \frac{\cancel{n^2}(1+1/n)\cancel{(1+2/n)}}{\cancel{3^n}}$$

$$\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = \frac{3(1+1/n)}{(1+3/n)}$$

$$= 3$$

$3 > 1$

U_n is diver

eg $\frac{1}{1^4} + \frac{1\cdot 3}{2^4} + \frac{1\cdot 3\cdot 5}{3^4} + \cdots \infty$

Sol^n $\frac{1\cdot 3\cdot 5\cdot 7\ldots(2n-1)}{n^4} = U_n$

$$U_n = \frac{(2n-1)(2n-1)}{n^4}$$

$$U_{n+1} = \frac{2n(2n)}{(n+1)^4}$$

$$\frac{U_{n+1}}{U_n}$$

eg $\frac{x}{1\cdot 2} + \frac{x^2}{3\cdot 4} + \frac{x^3}{5\cdot 6} + \cdots\infty$

Sol^n $\frac{x^n}{(2n-1)(2n)} = U_n$ $\quad 1+(n-1)2$
 $2+(n-1)2$

$$U_{n+1} = \frac{x^{n+1}}{(2n+1)(2n+2)}$$

$$\frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{(2n+1)(2n+3)} \cdot \frac{(2n-1)(2n)}{x^n}$$

$$= \frac{x}{\cancel{n^2}(2+1/n)(2+2/n)} \cdot \cancel{n^2}(2-1/n)(2)$$

$$\lim_{n\to\infty}\frac{U_{n+1}}{U_n} = \frac{\cancel{2\times 2}}{\cancel{2\times 2}}\cdot x$$

$$= x$$

$\sum U_n$ is convergent if $x < 1$ & dive $x > 1$

and if $x = 1$, the test fails

Now we'll check by comparison test

If $x = 1$

$$U_n = \frac{x^n}{(2n-1)(2n)} = \frac{1}{(2n-1)(2n)}$$

$$V_n = \frac{1}{n^2}$$

$$\frac{U_n}{V_n} = \frac{1}{(2n-1)(2n)} \times n^2$$

$$\frac{U_n}{V_n} = \frac{1}{\cancel{n^2}(2-1/n)(2)} \times \cancel{n^2}$$

$$\lim_{n\to\infty} \frac{U_n}{V_n} = \frac{1}{4}$$

limit excied

$$\sum V_n = \sum 1/n^2$$

$2 = p$

$p > 1$

$\therefore$ conv

$\therefore$ $\sum U_n$ is convergent if $x \leq 1$ & dive $x > 1$

eg $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

Soln $\quad U_n = \dfrac{n!}{n^n}$

$$U_{n+1} = \frac{(n+1)!}{(n+1)^{(n+1)}}$$

$$\frac{U_{n+1}}{U_n} = \frac{(n+1)!}{(n+1)^{(n+1)}} \times \frac{n^2}{n!}$$

$$= \frac{(n+1)\cancel{n!}}{\cancel{n}^{n+1}(1+1/n)^{n+1}} \times \frac{\cancel{n^n}}{\cancel{n!}}$$

$$\lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \frac{\cancel{n}(1+1/n)}{\cancel{n}(1+1/n)^{n+1}}$$

$$= \frac{1}{(1+1/n)^n}$$

Note : $(1+1/n)^n = e$

$$= \frac{1}{e}$$

$$= \quad < 1$$

$\therefore$ dive

- Raabe's test

let ΣU_n be a series of +ve terms if

$$\boxed{\begin{array}{l} \lim_{n\to\infty} n\left[\frac{U_n}{U_{n+1}}-1\right] = l \\ \text{Convergent if } l > 1 \\ \text{divergent if } l < 1 \end{array}}$$

eg Test the conv / div of series

$$1 + \frac{(1!)^2}{2}x + \frac{(2!)^2}{4!}x^2 + \dots \infty$$

sol

$$\frac{(n!)^2}{(2n)!}\cdot x^n = U_n$$

$$U_{n+1} = \frac{((n+1)!)^2}{(2n+2)!}\cdot x^n \cdot x$$

$$\frac{U_{n+1}}{U_n} = \frac{((n+1)!)^2}{(2n+2)!}\ \cancel{x^n}\cdot x\ \frac{(2n)!}{\cancel{x^n}\,(n!)^2}$$

$$= \frac{(n+1)^2\cdot x\ (2n)!}{(2n+2)!}$$

$$= \frac{(n+1)^2\cdot x\ \cancel{(2n)!}}{\cancel{(2n!)}\,(2n+1)(2n+2)}$$

$$= \frac{\cancel{n^2}(1+1/n)^2}{\cancel{n^2}(2+1/n)(2+1/n)}\cdot x$$

$$\lim = \frac{x}{4}$$

$\therefore$ Σu_n is convergent if $x < 4$ & dive $x > 4$

Now Rabbies

$$\lim_{n \to \infty} n \left[\frac{u_n}{u_{n+1}} - 1 \right] = l$$

$$n \left[\frac{\frac{(n!)^2}{(2n)!} \cdot x^n}{\frac{((n+1)!)^2}{(2n+2)!} \cdot x^n \cdot x} - 1 \right]$$

$$n \left[\frac{\cancel{2}(2n+1)}{{}_2\cancel{4}(n+1)} - 1 \right]$$

$$n \left[\frac{2n+1 - 2n - 2}{2n+2} \right]$$

$$\left[\frac{-n}{2n+2} \right]$$

$$\frac{-n}{n} \left[\frac{1}{2 + 2/n} \right]$$

$$= \frac{-1}{2}$$

$\therefore$ divergent

eg Test the $1 + \frac{1}{2\cdot 3} + \frac{1\cdot 3}{2\cdot 4\cdot 5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7} + \dots \infty$

Sol^n $\quad U_n = \dfrac{1\cdot 3\cdot 5 \dots (2n-1)}{(2\cdot 4\cdot 6 \dots 2n)(2n+1)}$

$$U_{n+1} = \frac{1\cdot 3\cdot 5 \dots (2n-1)(2n+1)}{(2\cdot 4\cdot 6 \quad 2n)(2n+2)(2n+3)}$$

$$\frac{U_{n+1}}{U_n} = \frac{}{}$$

lim $= 1$

$\therefore$ Test fail

Now Rab

$$n\left[\frac{U_n}{U_{n+1}} - 1\right]$$

$$n\left[\frac{(2n+2)(2n+3)}{(2n+1)(2n+1)} - 1\right]$$

$$n\left[\frac{4n^2+10n+6}{4n^2+4n+1}-1\right]$$

$$n\left[\frac{\cancel{4n^2}+10n+6-\cancel{4n^2}-4n-1}{(\quad)}\right]$$

$$n\left[\frac{6n+5}{4n^2+4n+1}\right]$$

$$=\frac{\cancel{n^2}}{\cancel{n^2}}\left(\frac{6+5/n}{4+4/n+1/n^2}\right)$$

$$=\frac{3}{2}$$

$\therefore$ Conv

- **Cauchy's Root test**

let ΣU_n be series of +ve terms then the series is

$$\boxed{\begin{array}{ll} \text{conv if} & \lim\limits_{n\to\infty} U_n^{1/n} < 1 \\ \text{div if} & \lim\limits_{n\to\infty} U_n^{1/n} > 1 \end{array}}$$

eg $\Sigma \frac{x^n}{n^n}$, $x>0$

Sol^n $U_n = \frac{x^n}{n^n}$

$$U_n^{1/n} = \left(\frac{x^n}{n^n}\right)^{1/n} = \frac{x}{n} \xrightarrow{\lim\limits_{n\to\infty}} = 0 < 1$$

$\therefore$ conv

eg $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(1+1/n)^{n^2}}$

Sol^n $U_n = \dfrac{1}{(\quad)^{n^2}}$

$$(U_n)^{1/n} = \frac{1}{(1+1/n)^n}$$

$$= \frac{1}{e}$$

$\therefore$ <1

$\therefore$ conv

eg $\dfrac{1}{2} + \dfrac{1}{3^2} + \dfrac{1}{4^3} + \cdots \dfrac{1}{(n+1)^n}$

Sol^n $U_n = \dfrac{1}{(n+1)^n}$

$$(U_n)^{1/n} = \frac{1}{n+1}$$

lim $= 0$

$\therefore$ conv

x^n

eg:- $\frac{x}{1} + \frac{1}{2}\frac{x^2}{3} + \frac{1.3}{2.4}\frac{x^3}{5} + \frac{1.3.5}{2.4.6}\frac{x^4}{7} + \dots \infty$

Solⁿ $U_n = \frac{x^{n+1}(1.3.5 \dots 2n-1)}{(2,4.6 \quad 2n)(2n+1)}$

$U_{n+1} = \frac{x^{n+2}(1.3.5 \quad 2n-1)(2n+1)}{(2,4.6 \quad 2n)(2n+2)(2n+3)}$

$$\frac{U_{n+1}}{U_n} = \frac{\cancel{x^n}.x\,(1.\cancel{3 \quad 2n-1})(2n+1)}{(2,\cancel{4 \quad 2n})(2n+2)(2n+3)} \times \frac{(2.\cancel{4. \quad 2n})(2n+1)}{x^n.\cancel{x}\,(1.\cancel{3 \quad 2n-1})}$$

$$x . \frac{(2n+1)(2n+1)}{(2n+2)(2n+3)}$$

$$x . \frac{\cancel{n^2}\,\cancel{(2\times 2)}}{\cancel{n^2}\,\cancel{2\times 2}}$$

$$= x$$

Now Rab

$$n\left[\frac{U_n}{U_{n+1}} - 1\right]$$

$$n\left[\frac{(2n+2)(2n+3)}{x(2n-1)(2n-1)} - 1\right]$$

$$n\left[\frac{(2n+2)(2n+3) - \cancel{x}(2n-1)(2n-1)}{\cancel{x}(2n-1)(2n-1)}\right]$$

n^2

- **Logarithmic Test**

If Σu_n be a series of +ve terms, if

$$\lim_{n\to\infty} \left[n \log\left(\frac{u_n}{u_{n+1}}\right)\right] = l$$

Σu_n is conv $l > 1$

Σu_n is div $l < 1$

eg Test the conv of series

$$1 + \frac{x}{1!} + \frac{2^2 x^2}{2!} + \frac{3^3 . x^3}{3!} + \dots$$

sol^n

$$u_n = \frac{n^n . x^n}{n!}$$

$$u_{n+1} = \frac{(n+1)^{(n+1)} . x^{(n+1)}}{(n+1)!}$$

$$\frac{u_{n+1}}{u_n} = \frac{(n+1)^{(n+1)} . x^{n+1}}{(n+1)!} \times \frac{n!}{n^n . x^n}$$

$$= \frac{(n+1)^{n+1} . x}{(n+1) . n^n}$$

$$\frac{u_{n+1}}{u_n} = \frac{(n+1)^n . x}{n^n}$$

$$= \left(\frac{n+1}{n}\right)^n . x$$

$$= \frac{\cancel{n^n}(1 + 1/n)^n}{\cancel{n^n}} . x$$

$$\frac{u_{n+1}}{u_n} = e.x$$

$$x = \frac{1}{e}$$

if $x > 1$ div
$x < 1$ con

$$\frac{u_n}{u_{n+1}} = \frac{1}{(1+1/n)^n \cdot x} = \frac{1}{(\quad)^n} \cdot e$$

$$n \log\left(\frac{u_n}{u_{n+1}}\right) = n \log\left(\frac{e}{(\quad)^n}\right)$$

$$= n\left[\log e - \log(\quad)^n\right]$$

$$= n\left[1 - n\log(1+1/n)\right]$$

$$= n\left[1 - n\left(\frac{1}{n} - \frac{1}{2n^2} + \frac{1}{3n^2} \cdots\right)\right]$$

$$= n\left[\cancel{1} - \cancel{1} + \frac{1}{2n} - \frac{1}{3n^2} + \cdots\right]$$

$$\lim_{n\to\infty} = \frac{1}{2} - \frac{1}{3n}$$

$$= \frac{1}{2}$$

$$\frac{1}{2} < 1$$

by log test u_n is div

u_n is con $x < 1/2$
div $x \geq 1/e$

eg Test the conv of $1 + \frac{3x}{2!} + \frac{3^2 x^2}{3!} + \ldots$

$n^{(n-1)}$

- Cauchy's Integral Test

If $x \geq 1$, $f(x)$ is non-negative monotonic dec func. of x, such that $f(n) = U_n$ for +ve integral values of n, then the series $\sum U_n$ converge or diverge, acc as the integral

$$\int_1^{\infty} f(x).dx \text{ is } \underset{\text{conv}}{\text{finite}} \;/\; \underset{\text{div}}{\text{Infinite}}$$

Note : Monotonic Sequence U_n is called inc if

$\boxed{U_n \leq U_{n+1} \text{ for all } n \geq 1, \quad U_1 \leq U_2 \leq U_3 \ldots\ldots}$

dec if $\boxed{U_n \geq U_{n+1} \text{ for all } n \geq 1, \quad U_1 \geq U_2 \geq U_3 \ldots}$

It's called monotonic either if inc. or dec.

eg $\sum \frac{1}{n^2+1}$

Sol^n

for $x \geq 1$, $f(x)$ is +ve & mono dec.

$\int f(x).dx$

$$\int_1^{\infty} \frac{1}{x^2+1}$$

$$= \tan^{-1}(x) \Big|_0^{\infty}$$

$$= \pi/2 - \pi/4 = \pi/4$$

$\int_1^{\infty} f(x).dx$ is finite

$\therefore \sum U_n$ is conv

eg $u_n = \frac{1}{\sqrt{n}}$

$f(x) = \frac{1}{\sqrt{x}}$

for $x \geq 1$ $f(x)$ is +ve & mono dec

$$\int_1^\infty f(x).dx$$

$$\int_1^\infty x^{-1/2} = \frac{x^{-1/2+1}}{1/2} = \left.\frac{\sqrt{x}}{1/2}\right|_1^\infty$$

$$= \infty$$

$\because f(x)$ is infinite $\therefore \sum u_n$ is infinite

$\therefore$ div

eg $\sum \frac{1}{n(n+1)}$

Soln $\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$

$\frac{1}{\cancel{n(n+1)}} = \frac{(n+1)A + B(n)}{\cancel{n(n+1)}}$

$1 = A(n+1) + Bn$

put $n=0$ $A=1$

$n=-1$ $B=-1$

$\therefore \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$

for $n > 1$ $f(x)$ is mono inc.

$$f(x) = \int_1^{\infty} \left(\frac{1}{x} - \frac{1}{x+1}\right) . dx$$

$$= \log x - \log(x+1) \Big|_1^{\infty}$$

$$= \log\left(\frac{x}{x+1}\right) \Big|_1^{\infty}$$

$$= \log 1 - \log 1/2$$

$$= \cancel{\log 1} - \cancel{\log 1} + \log 2$$

$$= \log 2$$

An alternating series whose terms are alternatingly +ve & -ve

eg

- **Leibtz test**

If the alternating series $u_1 - u_2 + u_3 - u_3 + \ldots$ $u_1 > 0$ satisfy the condition

i) $U_n \leq U_{n+1}$ for all n

ii) $\lim_{n \to \infty} U_n = 0$

Then Series is convergent

else divergent

eg $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$

Sol^n $U_n > 0$

$\therefore U_n = \frac{1}{n^2}$

$U_{n+1} = \frac{1}{(n+1)^2}$

If $U_{n+1} < U_n$

$\frac{1}{(n+1)^2} < \frac{1}{n^2}$

$n^2 < (n+1)^2$

$\cancel{n^2} < \cancel{n^2} + 2n + 1$

$0 < 2n+1$

True

also $\lim_{n \to \infty} U_n = \frac{1}{n^2} = 0$

$\therefore$ conv.

eg $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

Sol^n $U_n = \frac{1}{(n)^{1/2}}$

$U_{n+1} = \frac{1}{(n+1)^{1/2}}$

$U_{n+1} < U_n$

$\frac{1}{(n+1)^{1/2}} < \frac{1}{(n)^{1/2}}$

$n^{1/2} < (n+1)^{1/2}$

$\cancel{n} < \cancel{n} + 1$

$0 < 1$

$$\lim_{n \to \infty} \frac{1}{(n)^{1/2}} = 0$$

eg $1 - \frac{1}{2^p} + \frac{1}{3^p} \dots$

Sol^n $U_n = \frac{1}{n^p}$

$$U_{n+1} = \frac{1}{(n+1)^p}$$

$$U_{n+1} < U_n$$

$$\frac{1}{(n+1)^p} < \frac{1}{n^p}$$

$$n^p < (n+1)^p$$

$$n < n+1$$

$$U_n = \lim_{n \to \infty} \frac{1}{n^p} = 0$$

$\therefore$ series is con

- **Absolute conv & Conditional conv**

A series $\sum U_n$ is called absolute conv, if the series of absoute value $\sum_{n=1}^{\infty} |U_n|$ is conv

Note: $\sum U_n$ is a series of +ve terms then $|U_n| = U_n$ & so abs. conv is same as conv

eg $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$

$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$ is ab. con

Con

A series $\sum u_n$ is cond^n conv if conv but not abs. conv.

eg $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \; 1 \dots$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$$

Note : If a series $\sum u_n$ is abs con then its ofc conv.

eg $\sum \frac{(-1)^n n^3}{3^n}$

Sol^n

$$u_n = \frac{(-1)^n n^3}{3^n}$$

$$|u_n| = \frac{n^3}{3^n}$$

$$u_{n+1} = \frac{(n+1)^3}{3^{n+1}}$$

$$\frac{u_{n+1}}{|u_n|} = \frac{(n+1)^3}{3^{n+1}} \cdot \frac{3^n}{n^3}$$

$$\lim_{n\to\infty} = \frac{\cancel{n^3}(1+1/n)^3}{3} \cdot \frac{1}{\cancel{n^3}}$$

$$= \frac{1}{3}$$

∴ Conv

∵ $\sum |u_n|$ is conv

∴ $\sum u_n$ is ab conv

eg $\sum_{n=1}^{\infty} \frac{(-2)^n}{n \cdot 3^{n+1}}$

Sol^n $U_n = \frac{(-2)^n}{n \cdot 3^{n+1}}$

$|U_n| = \left| \frac{(-2)^n}{n \cdot 3^{n+1}} \right|$

$U_n = \frac{2^n}{n \cdot 3^{n+1}}$

$U_{n+1} = \frac{2^{n+1}}{(n+1)\, 3^{n+2}}$

$\frac{U_{n+1}}{U_n} = \frac{2^{n+1}}{(n+1)\, 3^{n+2}} \cdot \frac{n \cdot 3^{n+1}}{2^n}$

$= \frac{2n}{(n+1) \cdot 3}$

$= \frac{2}{3}$

ab conv

eg $\sum \frac{(-1)^n \cdot x^n}{n!}$

Sol^n $U_n = \frac{(-1)^n \cdot x^n}{n!}$

$|U_n| = \frac{x^n}{n!}$

$U_{n+1} = \frac{x^{n+1}}{(n+1)!}$

$\frac{U_{n+1}}{U_n} = \frac{x^{n+1}}{(n+1)!} \times \frac{n!}{x^n}$

$$= \frac{x}{n+1}$$

$$= 0$$

$\therefore$ abs conv.