B.Tech. / M.Tech (Integrated) DEGREE EXAMINATION, JANUARY 2023
First Semester

21MAB101T – CALCULUS AND LINEAR ALGEBRA
(For the candidates admitted from the academic year 2022-2023)

Note:
(i) **Part – A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
(ii) **Part – B** and **Part – C** should be answered in answer booklet.

Time: 3 Hours Max. Marks: 75

PART – A (20 × 1 = 20Marks)
Answer ALL Questions

| | | Marks | BL | CO | PO |

1. The number of positive terms in the canonical form is called — 1, 1, 1, 1
   (A) Signature
   (B) Index
   (C) Quadratic form
   (D) Positive definite

2. Find the eigen values of A^2 if $A = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$ — 1, 1, 1, 1
   (A) 1, 4, 25
   (B) 2, 4, 20
   (C) 4, 4, 25
   (D) 1, 2, 25

3. If A is an orthogonal matrix then $|A|$ is — 1, 1, 1, 2
   (A) 0
   (B) ±1
   (C) 1
   (D) –1

4. Find the sum and product of the eigen values of the matrix — 1, 2, 1, 2
   $A = \begin{pmatrix} 1 & 2 & -1 \\ -2 & 0 & 0 \\ 4 & 5 & 0 \end{pmatrix}$
   (A) –1, –10
   (B) 1, –10
   (C) –1, 10
   (D) 1, 10

5. If $z = 2x^2 + 3y^2 + 5xy$, then find $\dfrac{\partial z}{\partial x}$ — 1, 2, 2, 2
   (A) $4x + 5y$
   (B) $4x - 5y$
   (C) $-4x + 6y$
   (D) $4x + 6y + 5xy$

6. If f(x, y) is an implicit function, then $\dfrac{dy}{dx}$ is — 1, 1, 2, 2
   (A) $-\dfrac{(\partial f / \partial x)}{\partial f / \partial y}$
   (B) $\dfrac{(\partial f / \partial x)}{\partial f / \partial y}$
   (C) $\dfrac{(\partial f / \partial y)}{\partial f / \partial x}$
   (D) $-\dfrac{(\partial f / \partial y)}{\partial f / \partial x}$

7. If $f(x,y) = x^2 y + \sin y + e^x$, then find $f_x(1,\pi)$ 1   2   2   2
   - (A) 2π
   - (B) $2\pi - e$
   - (C) $2\pi + e$
   - (D) 0

8. If u and v are functionally dependent, then their Jacobian value is   1   1   2   1
   - (A) Zero
   - (B) Positive
   - (C) One
   - (D) Negative

9. Solution of $(D^2 + 4)y = 0$ 1   2   3   1
   - (A) $y = (Ax + B)e^{2x}$
   - (B) $y = A\cos\sqrt{2}x + B\sin\sqrt{2}x$
   - (C) $y = A\cos 2x + B\sin 2x$
   - (D) $y = Ae^{2x} + Be^{-2x}$

10. The value of $\dfrac{e^{ax}}{D-a}$ 1   1   3   1
    - (A) xe^{ax}
    - (B) e^{ax}
    - (C) $x^2 e^{ax}$
    - (D) $\dfrac{x^2}{2}e^{ax}$

11. Solve $(D^2 + 5D + 4)y = 0$ 1   2   3   2
    - (A) $y = Ae^x + Be^{-4x}$
    - (B) $y = Ae^x + Be^{4x}$
    - (C) $y = Ae^{-x} + Be^{-4x}$
    - (D) $y = Ae^{-x} + Be^{4x}$

12. Find the Particular Integral of $(D^2 - 4)y = \cos 2x$ 1   2   3   2
    - (A) $\dfrac{x}{8}\cos 2x$
    - (B) $\dfrac{1}{8}\sin 2x$
    - (C) $\dfrac{1}{8}\cos 2x$
    - (D) $-\dfrac{1}{8}\cos 2x$

13. The curvature of the straight line is   1   1   4   1
    - (A) 1
    - (B) 2
    - (C) 0
    - (D) –1

14. The envelope of the family of curves of the form $A\alpha^2 + B\alpha + C = 0$, where α is the parameter is   1   1   4   1
    - (A) $B^2 + AC = 0$
    - (B) $B^2 - 4AC = 0$
    - (C) $B^2 + 4AC = 0$
    - (D) $B^2 - AC = 0$

15. The radius of curvature in polar co-ordinates is   1   1   4   1
    - (A) $\rho = [r^2 + r'^2]^{3/2} / r^2 - rr'' + 2r'^2$
    - (B) $\rho = [r^2 - r'^2]^{3/2} / r^2 - rr'' + 2r'^2$
    - (C) $\rho = [r^2 - (r'')^2]^{3/2} / r^2 - rr'' + 2r'^2$
    - (D) $\rho = [r^2 + r'^2]^{3/2} / r^2 + rr'' + 2r'^2$

16. The equation of circle of curvature at any point (x, y) with centre of curvature $\bar{x}, \bar{y}$ and with radius of curvature ρ is

    1   1   4   1

(A) $(x-\bar{x})^2 - (y+\bar{y})^2 = \rho^2$ (B) $(x-\bar{x})^2 + (y-\bar{y})^2 = \rho^2$

(C) $(x+\bar{x})^2 + (y-\bar{y})^2 = \rho^2$ (D) $(x-\bar{x})^2 + (y+\bar{y})^2 = \rho^2$

17. The series $\sum_{n=1}^{\infty} \dfrac{1}{n^p}$ is convergent if

    1   1   5   1

(A) $p = 1$ (B) $p > 1$

(C) $p = 0$ (D) $p < 1$

18. The convergence of $\sum_{n=1}^{\infty} \dfrac{(-1)^{n-1}}{n^p}$ is tested by _____

    1   2   5   2

(A) Ratio test   (B) Root test

(C) Leibnitz test   (D) Raabe's test

19. The value of $\underset{n \to \infty}{lt}\ (n)^{1/n}$ is equal to

    1   2   5   2

(A) 0   (B) 1

(C) 2   (D) $\dfrac{1}{2}$

20. The series $1 - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} + \ldots\ldots \infty$ is

    1   2   5   2

(A) Convergent   (B) Absolutely convergent

(C) Conditionally convergent   (D) Divergent

PART – B (5 × 8 = 40 Marks)
Answer ALL Questions

Marks BL CO PO

21. a.

Find the Eigen values and Eigen vectors of the matrix $\begin{pmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 5 \end{pmatrix}$.

    8   3   1   1

(OR)

b.

Verify Cayley Hamilton theorem for the matrix $\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$.

    8   4   1   2

22. a.

Expand $e^x \cos y$ at $\left(1, \dfrac{\pi}{4}\right)$ as a Taylor series upto second degree terms.

    8   4   2

(OR)

b. If $u = u(x, y)$ and $x = e^r \cos\theta$ and $y = e^r \sin\theta$ show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = e^{-2r}\left[\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{\partial u}{\partial \theta}\right)^2\right].$$

8 3 3 1

23. a. Solve $\left(D^2 + 2D + 1\right)y = e^{3x} + \sin 2x$.

(OR)

8 4 3 2

b. Solve $\left(D^2 + 1\right)y = \tan x$ by the method of variation of parameters.

8 3 4 2

24. a. Find the radius of curvature of the curve $r = a(1 + \cos\theta)$ at the point $\theta = \dfrac{\pi}{2}$.

(OR)

8 4 4 2

b. Find the evolute of the parabola $y^2 = 4ax$.

8 4 5 2

25. a. Test the convergence of the series $\displaystyle\sum_{n=1}^{\infty}\left(\sqrt{n^4 + 1} - \sqrt{n^4 - 1}\right)$.

(OR)

8 4 5 2

b. Test the convergence of the series, $\dfrac{2}{3.4} + \dfrac{2.4}{3.5.6} + \dfrac{2.4.6}{3.5.7.8} + \dfrac{2.4.6.8}{3.5.7.9.10} + \ldots\ldots\infty$.

PART – C (1 × 15 = 15 Marks)
Answer ANY ONE Question

Marks BL CO PO

26. Reduce the quadratic form $3x^2 - 2y^2 - z^2 - 4xy + 8xz + 12yz$ to canonical form by an orthogonal transformation. Discuss the nature of the quadratic form and also find rank, index, and signature.

15 4 1 1

27. Find the dimensions of the rectangular box open at the top, of maximum capacity whose surface area is 432 sq.cm.

15 4 2 2

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