| Reg. No. | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

**B.Tech. / M.Tech (Integrated) DEGREE EXAMINATION, JULY 2023**
Second Semester

21MAB102T – ADVANCED CALCULUS AND COMPLEX ANALYSIS
*(For the candidates admitted from the academic year 2022-2023)*

**Note:**
(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
(ii) **Part – B** and **Part – C** should be answered in answer booklet.

Time: 3 Hours | Max. Marks: 75

**PART – A (20 × 1 = 20Marks)**
Answer **ALL** Questions

| | Marks | BL | CO | PO |
|---|---|---|---|---|
| 1. Evaluation of $\int_0^{\pi}\int_0^{\pi} d\theta d\phi$ is | 1 | 1 | 1 | 1 |
| (A) 1 (B) 0 (C) $\pi/2$ (D) $\pi^2$ | | | | |
| 2. The area of an ellipse is | 1 | 1 | 1 | 1 |
| (A) $\pi r^2$ (B) $\pi a^2 b$ (C) $\pi ab$ (D) $\pi ab^2$ | | | | |
| 3. If R is the region bounded by $x=0$, $y=0$, $x+y=1$ then $\iint_R dxdy$ is equal to | 1 | 2 | 1 | 1 |
| (A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$ | | | | |
| 4. Area of the double integral in polar coordinates is equal to | 1 | 2 | 1 | 2 |
| (A) $\iint_R drd\theta$ (B) $\iint_R r^2 drd\theta$ (C) $\iint_R (r+1) drd\theta$ (D) $\iint_R rdrd\theta$ | | | | |
| 5. If $r$ is the position vector of the point $(x,y,z)$ with respect to the origin then $\nabla \cdot \vec{r}$ | 1 | 2 | 2 | 2 |
| (A) 2 (B) 3 (C) 0 (D) 1 | | | | |
| 6. The unit vector normal to the surface $x^2+y^2-z^2=1$ at $(1,1,1)$ is | 1 | 2 | 2 | 2 |
| (A) $\vec{i}+\vec{j}-\vec{k}/\sqrt{3}$ (B) $2\vec{i}+2\vec{j}-2\vec{k}/\sqrt{2}$ (C) $\frac{3\vec{i}+3\vec{j}-3\vec{k}}{2\sqrt{3}}$ (D) $\frac{\vec{i}+\vec{j}-\vec{k}}{3\sqrt{2}}$ | | | | |

7. If $\phi$ is a scalar point function then curl(grad$\phi$) is 1 2 2 1

(A) Solenoidal (B) Irrotational

(C) Constant (D) 0

8. If the value of $\int_A^B \vec{F}.d\vec{r}$ does not depend on the curve C but only on terminal points A and B then $\vec{F}$ is called 1 2 2 1

(A) Solenoidal vector (B) Irrotational vector

(C) Conservative vector (D) Neither conservative nor irrotational

9. $L\left[e^{3t}\right]=$_____ 1 1 3 2

(A) $\frac{1}{s+3}$ (B) $\frac{1}{s-3}$

(C) $\frac{3}{s+3}$ (D) $\frac{s}{s-3}$

10. An example of a function for which the laplace transforms does not exits is 1 1 3 2

(A) $f(t)=t^2$ (B) $f(t)=\tan t$

(C) $f(t)=\sin t$ (D) $f(t)=e^{-at}$

11. Inverse laplace transform of $\frac{1}{s^2-a^2}$ is 1 1 3 1

(A) $\frac{\sin at}{a}$ (B) $\frac{\sinh at}{a}$

(C) $\sin at$ (D) $\sinh at$

12. Inverse laplace transform of $\frac{1}{s^2}$ is 1 1 3 1

(A) $t/2$ (B) $t^2/2$

(C) $t$ (D) $t^2$

13. The function $f(z)=u+iv$ is analytic if 1 2 4 1

(A) $u_x=v_y; u_y=-v_x$ (B) $u_x=-v_y; u_y=v_x$

(C) $u_x+v_y=0; u_y-v_x=0$ (D) $u_x=v_y; u_y=v_x$

14. If $u+iv$ is analytic, then the curves $u=c_1$ and $v=c_2$. 1 2 4 2

(A) are parallel (B) Coincides

(C) Intersect each other (D) Cut orthogonally

15. The transformation W=CZ where C is a real constant is known as 1 2 4 2

(A) Rotation (B) Reflection

(C) Magnification (D) Magnification and rotation

16. An analytic function with constant modulus is 1 2 4 1

(A) Zero (B) Analytic

(C) Constant (D) Harmonic

| | | | | | |
|---|---|---|---|---|---|
| 17. | If $f(z)$ is analytic inside and on C, the value of $\int_c \frac{f(z)}{z-a}dz$ where C is the simple closed curve and a is any point within C is<br>(A) $f(a)$ (B) $2\pi i f(a)$<br>(C) $2\pi i f'(a)$ (D) 0 | 1 | 1 | 5 | 2 |
| 18. | If $f(z)$ is not analytic at $z=z_0$ and there exists a neighborhood of $z=z_0$ containing no other singularity, then<br>(A) $z=z_0$ is isolated singularity of f(z) (B) $z=z_0$ is zero of f(z)<br>(C) $z=z_0$ is non zero of f(z) (D) $z=z_0$ is non isolated singularity of f(z) | 1 | 1 | 5 | 2 |
| 19. | If $f(z)=\frac{\sin z}{z}$, then<br>(A) z=0 is a simple pole (B) z=0 is a pole of order 2<br>(C) z=0 is a removable singularity (D) z=0 is a zero of f(z) | 1 | 1 | 5 | 2 |
| 20. | $f(z)=\frac{z+2}{(z-1)^2(z-2)}$ has<br>(A) Only poles (B) A simple pole at z=1/2<br>(C) Essential singularity (D) No poles | 1 | 1 | 5 | 1 |

**PART – B (5 × 8 = 40 Marks)**

Answer **ALL** Questions

| | | Marks | BL | CO | PO |
|---|---|---|---|---|---|
| 21.a. | Change the order of integration and evaluation $\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$. | 8 | 3 | 1 | 3 |
| | **(OR)** | | | | |
| 21.b. | Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} dzdydx$. | 8 | 3 | 1 | 3 |
| 22.a. | Find a, b, c so that the vector $\overrightarrow{F}=(x+2y+az)\vec{i}+(bx-3y-z)\vec{j}+(4x+cy+2z)\vec{k}$ is irrotational. | 8 | 3 | 2 | 3 |
| | **(OR)** | | | | |
| 22.b. | Show that $\overrightarrow{F}=\left(2xy+z^3\right)\vec{i}-x^2\vec{j}+3xz^2\vec{k}$ is a conservative vector field. Find the scalar potential also. | 8 | 3 | 2 | 2 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 23.a. | Find the laplace transform of the rectangular wave function given by $f(t)=\begin{cases}1 & ; 0<t<b\\ -1 & ; b<t<2b\end{cases}$ | 8 | 3 | 3 | 3 |
| | **(OR)** | | | | |
| 23.b. | Find $L^{-1}\left[\log\left\{\frac{s(s+1)}{s^2+1}\right\}\right]$. | 8 | 3 | 3 | 2 |
| 24.a. | Find the bilinear map which maps the points $z=1,i,-1$ on to the point $\omega=i,0,-i$. | 8 | 3 | 4 | 3 |
| | **(OR)** | | | | |
| 24.b. | Find the analytic function $f(z)=u+iv$ where $u-v=e^x(\cos y-\sin y)$. | 8 | 3 | 4 | 3 |
| 25.a. | If $f(z)=\frac{1}{(z-1)(z-2)}$ expand as Laurent's series about $z=0$ valid in<br>(i) $\lvert z\rvert<1$<br>(ii) $1<\lvert z\rvert<2$ | 8 | 4 | 5 | 3 |
| | **(OR)** | | | | |
| 25.b. | Evaluate $\oint_c \frac{\sin\pi z^2+\cos\pi z^2}{z+z^2}dz$ where c is the circle $\lvert z\rvert=2$. | 8 | 4 | 5 | 2 |

**PART – C (1 × 15 = 15 Marks)**

Answer **ANY ONE** Questions

| | | Marks | BL | CO | PO |
|---|---|---|---|---|---|
| 27. | Evaluate $\int_0^{2\pi}\frac{d\theta}{13+5\sin\theta}$ using contour integration. | 15 | 4 | 5 | 3 |
| 28. | Verify divergence theorem for $\vec{F}=\left(x^2-yz\right)\vec{i}+\left(y^2-zx\right)\vec{j}+\left(z^2-xy\right)\vec{k}$ taken over the rectangular parallelopiped $0\le x\le a;0\le y\le b;0\le z\le c$. | 15 | 4 | 2 | 3 |

* * * * *