

- Topics

- Transition
- Absorption
- Recombination
- Einstein eq
- Emmision
- DOS
- FGR
- Drude
- PV (Solar)

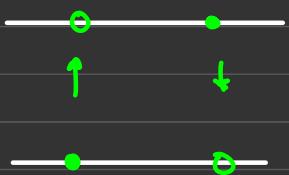
- Transition

Optical transition in bulk semi-con

e^- 's transite from one energy lvl to another

i) Band to band Transition (Inter band)

- The absorbed photon results in the transition of e^- from Valence band to Conduction band
- e^-h pair generation \rightarrow then recomb
- emission of photon
- eg GaAs



ii) Impurity lvl to band transit*

- Only in doped semicon
- e^- transits to either donor / acceptor lvl acc. to semicon
- For p-type, e^- transits from V-band to acceptor lvl creating hole in VB and ionise acceptor lvl
- eg He with Ge dope



iii) Free - carrier transition (intra)

- If an e^- is in a lower lvl CB, by absorbing photons it moves to higher energy lvl in CB
- Due to thermalisation, it relaxes down to bottom of CB releasing $h\nu$



• Optical Absorption

Absorption is the process in which photons are absorbed by the semiconductor causing transition of e^- from VB to CB.

Condition: photon energy must be equal or greater than the bandgap.

$$h\nu \geq E_g. \quad \lambda = hc/E_g.$$

Generation leads to inc. in conductivity due to inc. in mobile charge carriers. Used to detect light.

conductivity \propto photon flux.

N_{ab} :- No of atoms excited per unit V per unit T

$$N_{ab} = Q B_i z N,$$

no of atoms in ground state

• Optical Recombination

- e^- trans to CB, e-h pair generated and trans of e^- from CB to VB is recomb

i) Radiative Recomb

e^- is released from $E \geq E_g$ it recomb and releases photons

ii) Non-Radiative

if $E < E_g$, releases phonons

The excess carriers generated at non-equilibrium condn :-

Generation Rate = Recombination Rate

This process is diff for direct & indirect B-G semicon

For both Rad & Non Rad, the total lifetime is :-

$$\frac{1}{\tau} = \frac{1}{\tau_r} + \frac{1}{\tau_{nr}} ; R = \frac{\Delta n}{\tau}$$

τ = lifetime of e-h pairs

$$R = R_r + R_{nr}$$

$$n_r = \frac{1}{1 + [\tau_r / \tau_{nr}]} - \text{[efficiency]}$$

- τ_r should be higher for high rad recombination (ex: LED)
- τ should be as high as possible to get max. n_r

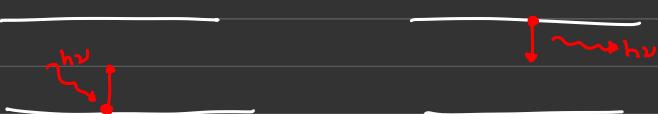
• Emission

- e^- excites with $h\nu \geq E_2 - E_1$ should deexcite with $h\nu = E_2 - E_1$ is emission

i) Spontaneous emission

- No external factor involved
- A particle after spending its lifetime in excited state comes to ground state releasing energy

$$eq := N_{sp} = A_{21} N_2 \xrightarrow{\substack{\text{no. of atoms in} \\ \text{excited state}}} \xrightarrow{\substack{\text{Probab.}}}$$



ii) Stimulated Emission

- Requires factor
- A particle in excited state is stimulated to decay before its lifetime by external factor

eq:- $N_{S1} = B_{21} N_2 Q - \text{energy density}$
 L_{probab}

$$a \sim h\nu \downarrow \approx \frac{h\nu}{h\nu}$$

• Einstein Equation

$$N_{ab} = N_{sp} + N_{sr}$$

$$N_1 B_{12} Q = N_2 A_{21} + N_2 B_{21} Q$$

$$Q = \frac{A_{21}}{\left(\frac{N_1}{N_2}\right) B_{12} - B_{21}}$$

Acc. to Max-boltz

$$\frac{N_i}{N_0} = \exp \left[-\frac{E_i}{kT} \right]$$

$$\frac{N_1}{N_2} = \exp [E_2 - E_1] / kT$$

$$\frac{N_1}{N_2} = \exp h\nu / kT$$

$$\therefore Q = \frac{A_{21}}{(e^{h\nu/kT}) B_{12} - B_{21}}$$

$$= \frac{A_{21}}{B_{12}(e^{h\nu/kT}) - B_{21}}$$

$$\therefore B_{12} = B_{21}$$

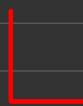
$$Q = \frac{A_{21}}{B_{21} \left(e^{\frac{hv}{kT}} - 1 \right)}$$

Acc. to planck blackbody radiation

Compare

$$Q = \frac{8\pi hc}{\lambda^5} \cdot \left(\frac{1}{e^{\frac{hv}{kT}}} - 1 \right)$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi hc}{\lambda^5}$$



This ratio shows st is finite also known as einstein relation for stimulated emission.

• Optical Joint DOS

Optical processes (emission & absorbtion) involve the energy states in VB & in CB.

It tells us the no. of states available for photons to interact simultaneously in the VB & CB

- "Explain Absorption & Emission"
- For given $h\nu$ there are several pairs of $E_2 - E_1$ states available to interact with
- Proof :-

$$E_2 = E_C + \frac{\hbar^2 k^2}{2 M_C} \quad \text{--- (i)}$$

$$E_1 = E_V - \frac{\hbar^2 k^2}{2 M_V} \quad \text{--- (ii)}$$

$$h\nu = E_2 - E_1$$

$$h\nu = (E_c - E_v) + \frac{\hbar^2 k^2}{2} \left(\frac{1}{m_c} + \frac{1}{m_v} \right)$$

$$k^2 = (h\nu - E_g) \frac{2m_v}{\hbar^2} \quad \text{--- (iii)}$$

From (i) & (iii)

$$E_2 = E_c + \frac{m_v}{m_c} (h\nu - E_g) \quad \boxed{\quad}$$

$$(E_2 - E_c) = \left(\frac{m_v}{m_c} \right) (h\nu - E_g)$$

$$(E_2 - E_c)^{1/2} = \left(\frac{m_v}{m_c} \right)^{1/2} (h\nu - E_g)^{1/2} \quad \text{--- (iv)}$$

We have

$$Z_c(E_2) dE_2 = Z(v) dv \quad \text{--- (v)}$$

$$Z(v) = Z_c(E_2) \frac{dE_2}{dv} \quad \text{--- (vi)}$$

$$\therefore Z(v) = \left[\frac{1}{2\pi^2} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} (E_2 - E_c)^{1/2} \right] \cdot \frac{dE_2}{dv}$$

$\boxed{h \cdot \frac{m_v}{m_c}}$

$$Z(v) = \frac{1}{2\pi^2} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} (E_2 - E_c)^{1/2} \cdot h \cdot \frac{m_v}{m_c}$$

Substitute eq (iv)

$$Z(v) = \frac{1}{2\pi^2} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} \cdot h \cdot \frac{m_v}{m_c} \cdot \left(\frac{m_v}{m_c} \right)^{1/2} (h\nu - E_g)^{1/2}$$

$$= \frac{1}{2\pi^2} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} \cdot h \left(\frac{m_v}{m_c} \right)^{3/2} \cdot (h\nu - E_g)^{1/2}$$

$$= \frac{h}{2\pi^2} \cdot \left(\frac{1}{\hbar^2} \right)^{3/2} \cdot (2m_v)^{3/2} \cdot (h\nu - E_g)^{1/2}$$

$$= \frac{h}{2\pi^2} \cdot \left(\frac{2\pi}{h}\right)^3 \quad . \quad "$$

$$= \left(\frac{4\pi}{h^2}\right) \times \frac{\pi}{\pi} \quad . \quad "$$

$$Z(v) = \frac{1}{\pi h^2} \cdot (2m_r)^{3/2} \cdot (hv - E_g)^{1/2}$$

Draude Model

Obtained by Draude & Lorentz in 1900.

Acc. to this theory, metal contains free e⁻ responsible for electrical conductivity & metals obey the laws of classical mechanics.

Postulates:-

i) The free e⁻s available in metal moves in random motion w/o \vec{E} similar to gas molecules in a vessel.

The collisions (e⁻-e⁻ & e⁻-ion) are elastic.

The total energy of e⁻ is purely KE.

ii) When \vec{E} is applied, e⁻s gain energy & move in one direction.

e⁻s acquire const. velocity called drift velocity & collisions are inelastic.

Collision Relaxation time is time b/w two collisions.

electrical conductivity.

$J \rightarrow$ current density
 $E \rightarrow$ electric field
 $\sigma \rightarrow$ electrical conductivity.

$$J \propto E$$

$$J = \sigma E$$

when $E = 0$, $V_d = 0$, electric current is zero

when $E \neq 0$,

free e⁻ source $(\nabla F) = eE$

$$ma = eE$$

$$-a = \frac{eE}{m}$$

$$\text{Also, } a = \frac{V_d}{\tau}$$

$$\therefore \frac{V_d}{\tau} = \frac{eE}{m}$$

$$V_d = \left(\frac{eE}{m}\right)\tau$$

$$J = \sigma E$$

$$F\alpha = eE$$

$$ma = eE$$

$$a = \frac{eE}{m}$$

$$a = \frac{V_d}{\tau}$$

$$\frac{V_d}{\tau} = \frac{eE}{m}$$

$$V_d = \frac{eE\tau}{m}$$

$$J = \frac{ne^2\tau E}{m}$$

$$\sigma = \frac{ne^2\tau}{m}$$

$$J = neV_d$$
$$= ne \left(\frac{eE}{m}\right)\tau = \frac{ne^2\tau E}{m} = \left(\frac{ne^2\tau}{m}\right)E$$

we know,
 $J \propto E \Rightarrow J = \sigma E$

$$\therefore J = \left[\frac{ne^2\tau}{m}\right]$$

$$\therefore \rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$$

electrical resistivity.

Photon DOS

DOS of photons is the total no of allowed states per unit vol in the energy range of E to $E + dE$

$$\text{DOS} : g(E) = \frac{1}{V} \times \frac{dN(E)}{dE}$$

We can calc. DOS by counting states in k-space by integrating over a vo

{ The total no of states below a given ' k ' can be determined by integrating a sphere with $r=k$ and obtain ' k ' space vol., then dividing vol. of single state to calc $N(E)$)

$$\text{Vol of sphere in k-space} = \frac{4}{3} \pi k^3$$

$$\text{Vol in single state in k-space} = \left(\frac{2\pi}{L}\right)^3 = \frac{8\pi^3}{V}$$

$$\therefore N(E) = \frac{\frac{4}{3} \pi k^3}{\frac{8\pi^3}{V}} = \frac{\frac{4}{3} \pi k^3 \times V}{8\pi^3}$$

$$N(E) = \frac{V k^3}{6\pi^2}$$

$$\frac{dN(E)}{dE} = \frac{3V}{2} \frac{k^2}{6\pi^2} \frac{dk}{dE}$$

$$\therefore g(E) = \frac{1}{V} \times \frac{1}{2} \frac{3}{\pi^2} k^2 \cdot \frac{dk}{dE}$$

for photons: $E = \hbar\omega = \hbar(c k)$ light speed

$$\therefore \frac{E}{\hbar c} = k$$

$$\frac{1}{\hbar c} = \frac{dk}{dE}$$

$$\therefore g(E) = \frac{1}{2\pi^2} \cdot \left(\frac{E}{\hbar c}\right)^2 \times \frac{1}{\hbar c}$$

$$g(E) = \frac{1}{2\pi} \frac{E^2}{(\hbar c)^3}$$

• Fermi Golden Rule

FGR is a simple expression for the transition rates b/w states of quantum system

The rates are calculated from probabilities determined by transition matrix elements in quantum mechanical.

$$\text{Transition Probability } \lambda_{if} = \frac{2\pi}{\hbar} |M_{if}|^2 \rho_f \xrightarrow{\substack{-\text{density of final state} \\ \downarrow}} \delta(E_i - E_f + i\hbar\omega)$$

$|M_{if}|^2 = |\langle b | H^{(r)}_{ab} | a \rangle|^2$
matrix element

i) Upward transition rate

$$W_{a \rightarrow b} = \frac{2\pi}{\hbar} |\langle b | H^{(r)}_{ab} | a \rangle|^2 \delta(E_a - E_b - \hbar\omega)$$

$$\text{Total rate per unit } v = 2/v \times \downarrow$$

$$R_{a \rightarrow b} = \frac{2}{v} \sum_{ka} \sum_{kb} \frac{2\pi}{\hbar} |H'_{ab}|^2 \delta(E_a - E_b - \hbar\omega) f_a (1 - f_b)$$

ii) Downward transition rate

$$W_{b \rightarrow a} = \frac{2\pi}{\hbar} |\langle a | H^{(r)}_{ba} | b \rangle|^2 \delta(E_b - E_a + \hbar\omega)$$

$$R_{b \rightarrow a} = \frac{2}{v} \sum_{ka} \sum_{kb} \frac{2\pi}{\hbar} |H'_{ba}|^2 \delta(E_b - E_a + \hbar\omega) f_b (1 - f_a)$$

Net Transition Rate

$$R_n = R_{a \rightarrow b} - R_{b \rightarrow a}$$

$$R_n = \frac{2}{v} \sum_{ka} \sum_{kb} \frac{2\pi}{\hbar} |H'_{ba}|^2 \delta(E_b - E_a - \hbar\omega) (f_a - f_b)$$

- Optical Gain & Loss

i) Gain

\downarrow Emission

When light is emitted during transition from CB to VB

Condⁿ : Net SE > Absorption

ii) Loss

Absorption of light during transition from VB to CB

Condⁿ : Net Absorp > Emi

Note:-

$$E_g < \hbar\omega < E_e - E_h \rightarrow \text{Gain}$$

$$E_g < \hbar\omega < E_e - E_h \rightarrow \text{Inversion}$$

$$E_g > \hbar\omega, E_e - E_h \rightarrow \text{Loss}$$

- Photovoltaic Cell

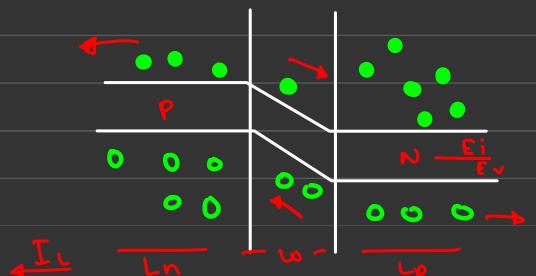
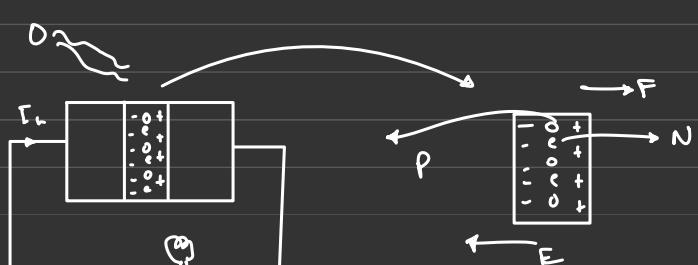
Composed of two types of semi (P N type) forming PN Junction.

When photon is absorbed then a generation of e- hole pair take place, they will be swept away due to the electric field when away due to the electric field, when applied (e- towards N-region & holes towards P-region)

Net +ve on P-side

Net -ve on N-side

This causes a pot diff to appear at PN Junction called photo voltage which drives photo current



The photocurr I_L in RB directⁿ produces Voltage drop which FB's the PN Junc. , the FB pn junc product IF

\therefore Net Current :

$$I = I_L - I_F = I_L - I_S \left[\exp\left(\frac{eV}{kT}\right) - 1 \right]$$

• The limiting Cases :-

SC condn ($R=0$, $V=0$)

$$I = I_{SC} = I_L$$

OC condn ($R \rightarrow \infty$, V_{max}/V_{oc})

$$I=0 \approx I_L - I_S \left[\exp\left(\frac{eV_{oc}}{kT}\right) - 1 \right]$$

$$V_{oc} = \frac{n k T}{e} \ln \left(1 + \frac{I_L}{I_S} \right)$$

$$V_{oc} = V_t \ln \left(1 + \frac{I_L}{I_S} \right)$$

• Power delivered

$$P = VI = I_L V - I_S \left[\exp\left(\frac{eV}{kT}\right) - 1 \right] V$$

$$\text{max } P : \frac{dP}{dV} = 0$$

$$\frac{dP}{dV} = 0 = I_L - I_S \left(\exp\left(\frac{eV}{kT}\right) - 1 \right) - I_S \frac{V}{kT} \cdot \exp\left(\frac{eV}{kT}\right)$$

$$\therefore \left(1 + \frac{V}{V_t} \right) \exp\left(\frac{eV}{kT}\right) = 1 + \frac{I_L}{I_S}$$

• Efficiency :-

It is the ratio of energy output from solar cell to input energy from sun

$$\eta = \frac{P_{max}}{Pin} \times 100$$

$$P_{max} = V_{oc} I_{sc} FF$$

$$\therefore \eta = \frac{V_{oc} I_{sc} FF \times 100}{Pin}$$

I_{sr} (SC Current): Current when V across Solar Cell = 0

$$I_{sc} = -I_L$$

I_{oc} (OC current): Max V at 0 I

$$V_{oc} = V_t \ln \left(1 + \frac{I_L}{I_e} \right)$$

FF (Fill factor) :-

Power of a Solar Cell

$$FF = \frac{P_m}{I_{sc} V_{oc}} = \frac{I_m V_m}{I_{sc} V_{oc}}$$