Sumedh

- **Basic Formulae**

1. $$\int u.v\,dx = u\int v\,dx - \int\left(u'\int v\,dx\right).dx$$

OR

$$\int u.v\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 \cdots \left[\text{till } u^n = 0\right]$$

2. Even function

$$f(x) = f(-x)$$

here, $$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

3. Odd function

$$f(-x) = -f(x)$$

here, $$\int_{-a}^{a} f(x)\,dx = 0$$

4. $$\int e^{ax}.\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin bx - b\cos bx\right]$$

$$\int e^{ax}.\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\cos bx + b\sin bx\right]$$

5. $\sin n\pi = 0$

$\sin 2n\pi = 0$

$\cos n\pi = (-1)^n$ $\left[\begin{array}{l} n: \text{even} = 1 \\ \quad\ \text{odd} = -1 \end{array}\right]$

$\cos 2n\pi = 1$

- Fourier Series

$$F(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

where, $$a_0 = \frac{1}{2\pi}\int_c^{c+2\pi} f(x)\,dx$$

[for c:
if $(0, 2\pi)$
then $c = 0$]

$$a_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x).\cos nx\,dx$$

[for $[0, a]$
here $2l = a$
$l = a/2$
l instead
of π]

$$b_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\sin nx\,dx$$

eg if given $(0, 2\pi)$ at $f(x) = x$

here $$a_0 = \frac{1}{2\pi}\int_0^{2\pi} x.\,dx$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x.\cos nx \quad ; \quad b_n = \frac{1}{\pi}\int_0^{2\pi} x.\sin nx\,dx$$

eg if given $$f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

here $$a_0 = \frac{1}{2\pi}\int_{-\pi}^{0} -\pi\,dx + \frac{1}{2\pi}\int_0^{\pi} x\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0} -\pi.\cos nx\,dx + \frac{1}{\pi}\int_0^{\pi} x.\cos nx\,dx$$

- **Half Range Series**

1. **Cosine series**

$$a_0 = \frac{1}{l} \int_0^l f(x)\, dx$$

$$a_n = \frac{2}{l} \int_0^l f(x) \cos \frac{n\pi x}{l}\, dx$$

$$b_n = 0$$

here, $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l}\, dx$

2. **sine series**

$$a_0 = 0$$

$$a_n = 0$$

$$b_n = \frac{2}{l} \int_0^l f(x) . \sin \frac{n\pi x}{l} . dx$$

here, $f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} . dx$

- **Parseval's Identity**

$$\frac{1}{2l} \int_c^{c+2l} [f(x)]^2\, dx = a_0^2 + \frac{1}{n} \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$$

$$a_0 = \frac{1}{2l} \int_c^{c+2l} f(x)\, dx$$

$$a_n = \frac{1}{l} \int_c^{c+2l} f(x) \cos \frac{n\pi x}{l}\, dx$$

$$b_n = \frac{1}{l} \int_c^{c+2l} f(x) \sin \frac{n\pi x}{l}\, dx$$

here,

half range cosine $\frac{1}{\pi}\int_0^{\pi}(f(x))^2\,dx = a_0 + \frac{1}{2}[a_1^2 + a_2^2 + \dots]$

sine $\frac{1}{\pi}\int_0^{\pi}(f(x))^2\,dx = \frac{1}{2}[b_1^2 + b_2^2 + \dots]$

- Harmonic Analysis

$$y = f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos n\theta + b_n\sin n\theta)$$

here 1st harmonic

$= \frac{a_0}{2} + (a_1\cos\theta + b_1\sin\theta)$

where Amp $= \sqrt{a_1^2 + b_1^2}$

$$a_0 = \frac{2}{N}\sum y$$

$$a_n = \frac{2}{N}\sum y\cos\theta$$

$$b_n = \frac{2}{N}\sum y\sin\theta$$

eg

x	0	1	2	3	4	5
y	9	18	24	28	26	20

here,

x	θ	y	$y\sin\theta$	$y\cos\theta$
0	$60\times0=0$	9	⋮	⋮
1	$60\times1=60$	18		
2	120	24		
3	180	28		
4	240	26		
5	300	20		

6 value

$\therefore \frac{360}{6} = 60$

$\therefore \theta = 60$