# Person re-identification based on and kernel local fisher discriminant analysis and Mahanalobis distance learning

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Abstract-In person re-identification (Re-ID) it's very important to choose robust descriptors and metric learning to improve accuracy. Mahanalobis distance based metric learning is a popular method for metric learning. However, since directly extracted descriptors usually have high dimension (thousands or more), it's intractable to learn a high dimensional semidefinite positive (SPD) matrix without dimension reduction. Many subspace learning metrics have been proposed to learn a subspace while preserving those discriminative information as much as possible. However, few work has been done to probe the metric learning on those subspace after the firsttime metric learning. In this paper the KLFDA [1] is used to reduce dimension given that kernelization method can greatly improve re-identification performance for nonlinearity. Then a Mahanalobis distance metric is learned on lower dimensional descriptors based on the limitation that the intraclass distance is at least 1 unit smaller than the minimum interclass distance. By comparing the intraclass distance only with the interclass distance the computation complexity is reduced. This method turns to have excellent performance compared with other advanced metric learning.

#### 1. Introduction

Person re-identification (Re-ID) has received increasing attention in recent years. This task is very challenging caused by many factors like low image resolution, occlusion, background noise and different camera color response, etc. In the single shot Re-ID problem, since only one image is provided in each camera for each person, it might be quite confusing when different people have similar pose or clothes. Also, in the multi-shots case, there might exist quite much difference even in different frames of the same person for different pose and illuminations [2]. Therefore, good descriptors are supposed to be robust to illumination change and occlusions.

Most previous work try to find better feature representation [3–6] and metric learning [1, 2, 7–15], or a subspace learning. Most descriptors are color and texture based descriptors. Distance metric learning has been fully studied in [16], most existing metric learning are based on Mahanalobis distance to learn a metric which outputs small intraclass distance and large interclass distance. Though much

progress has been made, there still exists some challenges caused by classical problems like small sample problem (SSS) and computation complexity on large datasets.

For descriptors with high dimension ( $d \geq 1000$ ), it's hard to directly learn a Mahanalobis distance matrix M for the small sample size n(n << d). A popular method is to use principal component analysis (PCA) to reduce dimension. PCA is a very popular dimension reducing method, but one problem is PCA doesn't consider the information between classes thus many discriminant information will be lost after dimension reduction. In this paper kernel local fisher discriminant Analysis (KLFDA) [1] is used to reduce dimension, this supervised dimension reduction combines the linear discriminant analysis and locality preserving projection. Moreover, the kernelization version of LFDA proves to improve performance and reduce computation cost. However, the metric learning on dimension reduced vectors by KLFDA hasn't been fully probed.

The contributions of this paper are as follows, (1) KLFDA and metric learning are combined together to improve Re-ID performance. Previous works mainly use KLFDA as a subspace learning method and Euclidean distance to measure the similarity after dimension reduction. The Mahanalobis distance metric learning on the projected space by KLFDA has not been fully probed. In this paper, metric learning is performed by an iterative computation. (2) Inspired by [2], this work proposes to learn a Mahanalobis distance matrix based on the limitation that intraclass distance is at least 1 unit smaller than the minimum interclass distance. Therefore, metric learning in the lower dimension space is transformed into an optimization problem. It's important to only compare the intraclass distance with the minimum interclass distance in each iteration to reduce computation complexity, because there is no need to compute every possible possible positive and negative pairs. (3) Extensive experiments are performed on VIPeR, CUHK01, prid 450s and GRID dataset. It proves the proposed work has advanced performance on those datasets.

# 2. Dimension reduction based on kernel local fisher discriminant analysis

KLFDA is the kernel version of local fisher discriminant analysis (LFDA) and it's a supervised method. Here a

brief review of LFDA is given. For a set of d-dimensional observations  $\boldsymbol{x}_i$ , where  $i \in \{1, 2, \cdots, n\}$ , the label  $l_i \in \{1, 2, \cdots, l\}$ . Two matrix are defined as the intraclass scatter matrix  $\boldsymbol{S}^{(w)}$  and between class matrix  $\boldsymbol{S}^{(b)}$ ,

$$S^{(w)} = \sum_{i=1}^{l} \sum_{j:l_j=i} (x_j - \mu_i)(x_j - \mu_i)^T$$

$$S^{(b)} = \sum_{i=1}^{l} n_i(\mu_i - \mu)(\mu_i - \mu)^T$$
(1)

where the  $\mu_i$  is the mean of samples whose label is i, and  $\mu$  is the mean of all samples,

$$\mu_i = \frac{1}{n_i} \sum x_i, \mu = \frac{1}{n} \sum x_i \tag{2}$$

The Fisher Discriminant Analysis transform matrix T can be represented as

$$T = \arg\max \frac{T^T S^{(b)} T}{T^T S^{(w)} T}$$
 (3)

Fisher discriminant analysis tries to minimize the intraclass scatter matrix while maximize the interclass scatter matrix, so T is computed by the eigenvalue decomposition. T can be represented as the set of all the corresponding eigenvectors, as  $T = (\phi_1, \phi_2, \cdots, \phi_k)$ .

FDA has a form similar with signal and noise ratio, however, the FDA dimension reduction may have poor performance for it doesn't consider the locality of data. In [17] locality preserving projection (LPP) is proposed to exploit data locality. In LPP an affinity matrix is created to record the affinity of sample  $x_i$  and  $x_j$ , typically the range of elements in  $A_{i,j}$  is [0,1]. There are many manners to define a  $n \times n$  affinity matrix A, usually the two sample points with a smaller distance measured by Euclidean or other distance has a higher affinity value than those with bigger distance value. One of them is if  $x_i$  is within k-nearest neighbours of  $x_j$  then  $A_{i,j} = 1$  otherwise  $A_{i,j} = 0$ . LFDA combines FDA and LPP and have a more strong performance. The key in LFDA is it assigns weights to elements in  $A^{(w)}$  and  $A^{(b)}$ , so that,

$$S^{(w)} = \frac{1}{2} \sum_{i=1}^{l} \sum_{j:l_j=i} A_{i,j}^w (x_j - \mu_i) (x_j - \mu_i)^T$$

$$S^{(b)} = \frac{1}{2} \sum_{i=1}^{l} A_{i,j}^b (\mu_i - \mu) (\mu_i - \mu)^T$$
(4)

where

$$\mathbf{A}_{i,j}^{(w)} = \begin{cases} \mathbf{A}_{i,j}/n_c & y_i = y_j \\ 0 & else \end{cases}$$

$$\mathbf{A}_{i,j}^{(b)} = \begin{cases} (\frac{1}{n} - \frac{1}{n_c})\mathbf{A}_{i,j} & y_i = y_j \\ \frac{1}{n} & else \end{cases}$$
(5)

where  $y_i$  is the class label of sample point  $x_i$ .

When applying the LFDA to original high dimensional descriptors, one problem is the computation cost. Suppose

the vector data has a dimension of d, LFDA has to solve the eigenvalue a matrix with dimension  $d \times d$ . In some descriptors the d could be more than 20000 and thus the cost is not trivial.

Kernelization is proved to greatly improve performance since the non-linearity is exploited. In [13] it has been demonstrated that kernelization improves the performance of many dimension reduction and metric learning. Kernelization is a projection from low dimension to high dimension, which may make classification and clustering much more accurate. The difference of kernel version LFDA is that the interclass and intraclass scatter matrix will be transformed into kernel space and the eigenvalue decomposition will be operated on kernel space. Suppose a set of sample points  $x_i, i \in \{1, 2, \cdots, n\}$ , can be mapped to a implicit higher feature space by a function  $\phi(x_i)$ . It has been proved that kernel function can be implicit and only the inner product of mapped vectors  $\phi(x_i)$  and  $\phi(x_i)$  need to be known. The kernel trick is proposed to solve this problem by defining a function  $k(x_i, x_j) = \langle \phi(x_i), \phi(x_j) \rangle$ , the  $\langle \cdot \rangle$  is the inner product. There are many kinds of kernel like linear kernel, polynomial kernel and radial basis function (RBF) kernel. In this paper the RBF kernel is adopted. A RBF kernel is defined as  $k_{RBF}(\boldsymbol{x}_i, \boldsymbol{x}_j) = \exp^{(-\gamma ||\boldsymbol{x}_i - \boldsymbol{x}_j||^2)}$ .

# 3. Metric learning on dimension reduced space by gradient descent method

Since re-identification is a problem of ranking, it is desired that the rank-1 descriptor should be the right match. In this paper, instead of comparing all the possible positive and negative pairs, a simplified version is proposed that the intraclass distance should be at 1 unit smaller than inter distance. This will decrease computation complexity quite much. Given a Mahanalobis matrix M, for dimension reduced sample points  $x_i, i=1,2,3,\cdots,n,n$  is the number of all samples, the requirement is distance between positive pair should be at least 1 unit smaller than the minimum of all negative distance. This can be denoted as

$$D(\boldsymbol{x}_i, \boldsymbol{x}_i) + \rho < \min D(\boldsymbol{x}_i, \boldsymbol{x}_k), y_i = y_i, y_i \neq y_k.$$
 (6)

 $\rho$  is a slack variable and  $\rho \in [0,1].$  This equation can be transformed into a optimization problem with respect to descriptor  $x_i$  as

$$\arg\min \sum_{y_i=y_j} \max \{ D(\boldsymbol{x}_i, \boldsymbol{x}_j) - \min_{y_i \neq y_k} D(\boldsymbol{x}_i, \boldsymbol{x}_k) + \rho, 0 \}.$$
(7)

However, the equation above only penalizes small interclass distance. Another term is needed to penalize large intraclass distance. That is, to make the sum of intraclass distance as small as possible. This term is denoted as

$$\min \sum_{y_i = y_j} D(\boldsymbol{x}_i, \boldsymbol{x}_j), \tag{8}$$

To combine equations above, a ratio factor  $\alpha$  is assigned to term 12 so that the target function can be denote as

$$f(\mathbf{M}) = (1 - \alpha) \sum_{\mathbf{x}_i, \mathbf{x}_j, \mathbf{y}_i = y_j} D(\mathbf{x}_i, \mathbf{x}_j) + \alpha \sum_{\mathbf{x}_i, \mathbf{x}_j, \mathbf{y}_i = y_j} \max \{ D(\mathbf{x}_i, \mathbf{x}_j) - \min_{\mathbf{y}_i \neq y_k} D(\mathbf{x}_i, \mathbf{x}_k) + \rho, 0 \}$$
(9)

In this way the problem is transformed to an optimization problem. Notice that equation 9 can be denoted as

$$D(\boldsymbol{x}, \boldsymbol{y}) = (\boldsymbol{x} - \boldsymbol{y})^T \boldsymbol{M} (\boldsymbol{x} - \boldsymbol{y}) = Tr(\boldsymbol{M} \boldsymbol{X}_{i,j})$$
(10)

where  $X_{i,j} = x_i * x_j^T$ , and Tr is to compute matrix trace. Therefore, equation 14 can be transformed as follow,

$$f(\mathbf{M}) = (1 - \alpha) \sum_{y_i = y_j} Tr(\mathbf{M} \mathbf{X}_{i,j})$$
$$+\alpha \sum_{y_i = y_j, y_i \neq y_k} \max \{ Tr(\mathbf{M} \mathbf{X}_{i,j}) - Tr(\mathbf{M} \mathbf{X}_{i,k}) + \rho, 0 \}$$
(11)

To minimize equation 23, the gradient descent method is used. The gradient respect to M is computed as

$$G = \frac{\partial f}{\partial M} = (1 - \alpha) \sum_{y_i = y_j} X_{i,j}$$

$$+ \alpha \sum_{y_i = y_j, y_i \neq y_k} (X_{i,j} - X_{i,k})$$
(12)

The iteration process is summarized as in Table 1;

Table 1. Optimization algorithm on dimension reduced vectors

#### Gradient optimization algorithm for target function **Input** Descriptors of training person pairs Output A SPD matrix Initialization Initialize $M_0$ with eye matrix I; Compute the initial target function value $f_0$ with $M_0$ ; Iteration count t = 0; while(not converge) Update t = t + 1; Find $x_k$ for all sample points $x_i$ , where $y_i \neq y_k$ ; Update gradient $G_{t+1}$ with equation 12; Update M with equation : $M_{t+1} = M_t - \lambda G_t$ ; Project $M_{t+1}$ to the positive semi-positive definite space; Update the target value $f|_{\mathbf{M}=\mathbf{M}_{t+1}}$ ; end while return M

#### 4. Experiment

The hierarchical gaussian descriptors [4] are used in this paper. There are two versions of gaussian of gaussian descriptor. The first one is extracted only in RGB color space, denoted as GOGrgb. While the second one is extracted from four color space {RGB, HSV, Lab, nRGB}. nRGB means normalized RGB color space.

#### 4.1. Datasets and evaluation settings

**VIPeR** dataset is the most used dataset in person Re-ID. In this dataset there are 632 different individuals and for each person there are two outdoor images from different viewpoints. All the images are scaled into  $48 \times 128$ . In this experiment the we randomly select 316 individuals from cam a and cam b as the training set, the rest images in cam a are used as probe images and those in cam b as gallery images. This process is repeated 10 times to reduce error. **CUHK1** dataset contains 971 identities from two disjoint camera views. The cameras are static in each pair of view and images are listed in the same order. For each individual, there are two images in each view. All images are scaled into  $60 \times 160$ . In this paper, we randomly select 485 image pairs as training data and the rest person pairs are used for test data.

**Prid\_450s** dataset contains 450 image pairs recorded from two different, static surveillance cameras. Additionally, the dataset also provides an automatically generated, motion based foreground/background segmentation as well as a manual segmentation of parts of a person. The images are stored in two folders that represent the two camera views. In this test, we randomly select 225 person pairs from each of two camera views as the training set, and the remaining persons are left as gallery and probe images.

**GRID** There are two camera views in this dataset. Folder probe contains 250 probe images captured in one view. Gallery folder contains 250 true match images of the probes. Besides, in gallery folder there are a total of 775 additional images that do not belong to any of the probes. In this paper, we randomly select 125 persons from those 250 persons appeared in both camera views as training pairs, and the remaining persons in probe folder is used as probe images while the remaining 125 persons and those 775 additional persons from gallery folder are used as gallery images.

### 4.2. The influence of mean removal and $\mathcal{L}_2$ normalization

In [4], mean removal and  $L_2$  normalization is found to improve performance by 5.1%. The reason for this is mean removal and normalization can reduce the impact of extrema in a single descriptor. Original GOG means no mean removal and normalization. It shows that the mean removal and  $L_2$  normalization has a slight improvement around 0.5% on the performance on all five datasets. In [4] the mean removal and normalization is adopted, to compare with results in this paper, the mean removal and  $L_2$  normalization are also adopted in this experiment.

### 4.3. Parameters setting of gradient descent iteration

In this experiment, there are a few parameters for the iteration computing including slack variable  $\rho$ , maximal iteration T, gradient step  $\lambda$ , the interclass and intraclass

limitation factor  $\alpha$  and the updating ratio  $\beta$ . Firstly the slack variable  $\rho$  is initialized as 1 to ensure the minimum interclass distance is 1 larger than intraclass distance at least. The step size of gradient updating  $\lambda$  is initialized as 0.01. When target value f increases,  $\lambda$  is scaled by a factor 0.5, and  $\lambda$  is scaled by 1.01 when target value f decreases. To judge if target value converges, the thresh  $\beta$  is defined as the ratio target value change versus previous target value, that is,  $\beta = \frac{(f_{t+1} - f_t)}{f_t}$ . According many experiment trials, when it satisfies  $\beta = 10^{-5}$ , the target value converges and the iteration is stopped. The maximal iteration times is set to 100 since the target value f will converge in around 15 iterations. The last parameter for the iteration is  $\alpha$ , to know the best value for  $\alpha$ , we tried 11 different values ranges from 0 to 1 with a step of 0.1, and find that the rank 1 and rank 5 scores reach maxima at interval [0.7, 0.8]. Then another ten trials with alpha ranging from [0.7, 0.8] with a step of 0.01. The best  $\alpha$  value should have as large top rank scores as possible and at last we find that the optimal value for  $\alpha$ is 0.76.

Table 2. Parameters setting

Paramters	α	thresh	step	Max iteration	slack variable
Values	0.76	$10^{-5}$	0.01	100	1

#### 4.4. Performance analysis

In this paper, we compare proposed metric with other state-of-the-art metrics including NFST [14], XQDA [3]. NFST is a metric which learn a null space for descriptors so that the the same class descriptors will be projected to a single point to minimize intraclass scatter matrix while different classes are projected to different points. This metric is a good solution to small sample problem in person re-identification. XQDA is quite similar with many other metrics, which learns a projection matrix W and then a Mahanalobis SPD matrix M is learned in the subspace. Those two metric are proved to have state-of-the-art performance compared with many other methods. The GOGrgb in all forms stands for the hierarchical gaussian descriptor in RGB color space while GOGfusion stands for the one in four different color spaces {RGB, Lab, HSV, nRnG}.

VIPeR A comparison form is given in Table 3. Some of recent results are also included in this form. We can find that the rank scores are better than those of NFST and XQDA in terms of both GOGrgb and GOGfusion. More specifically, the rank 1, rank 5, rank 10, rank 15 and rank 20 scores of proposed metric learning are 0.76%, 0.92%, 1.39%, 1.08%, 1.52% higher than those of GOGrgb + XQDA, and the rank 1, rank 5, rank 10, rank 15 and rank 20 GOGfusion scores of proposed metric learning are 0.35%, -0.54%, 0.98%, 0.66%, 0.79% higher than GOGfusion + XQDA respectively. Also we can see that the proposed metric learning has a better performance than NFST.

Table 3. Performance of different metrics on VIPeR

	Rank(%)				
Methods	1	5	10	15	20
GOGrgb+NFST	43.23	73.16	83.64	89.59	92.88
GOGrgb+XQDA	43.01	73.92	83.86	89.24	92.37
GOGrgb+Proposed	43.77	74.84	85.25	90.32	93.89
GOGfusion+NFST	47.15	76.39	87.31	91.74	94.49
GOGfusion+XQDA	47.97	77.44	86.80	91.27	93.70
GOGfusion+Proposed	48.32	76.90	87.78	91.93	94.49

**CUHK1** We can find that the rank 1, rank5, rank 10, rank 15, rank 20 score of GOGrgb combined with proposed metric are 5.4%, 4.18%,3.31%,2.16%,1.46% higher than XQDA, and 0.31%,1.22%,1.34%,1.17%, 1.11% than NFST. Also the rank 1, rank5, rank 10, rank 15, rank 20 score of GOGfusion combined with proposed metric are 4.57%, 2.64%, 0.70%, 1.33%, 0.83% higher than GOGfusion combined with XQDA, and 0.41%, 0.83%, 0.88%, 1.09%, 1.14% than GOGfusion combined with NFST.

Table 4. Performance of different metrics on CUHK1

	Rank(%)				
Methods	1	5	10	15	20
GOGrgb+NFST	55.60	83.02	89.07	91.98	93.56
GOGrgb+XQDA	50.51	80.06	87.10	90.99	93.21
GOGrgb+Proposed	55.91	84.24	90.41	93.15	94.67
GOGfusion+NFST	56.26	83.66	89.63	92.22	93.70
GOGfusion+XQDA	52.10	81.85	88.81	91.98	94.01
GOGfusion+Proposed	56.67	84.49	90.51	93.31	94.84

Table 5. Performance of different metrics on prid\_450s

	Rank(%)				
Methods	1	5	10	15	20
GOGrgb+NFST	61.96	84.98	90.53	94.09	96.09
GOGrgb+XQDA	65.29	85.02	91.13	94.76	96.49
GOGrgb+Proposed	60.71	84.53	91.29	94.13	96.27
GOGfusion+NFST	64.53	86.62	92.93	95.78	97.42
GOGfusion+XQDA	68.40	87.42	93.47	95.69	97.02
GOGfusion+Proposed	62.80	86.58	92.36	95.29	96.89

**Prid\_450s** In this dataset, we can find the rank 1 score of XQDA and NFST is higher than proposed metric, but they have almost the same rank 5, rank 10, rank 15, and rank 20 scores with respect to both kinds of descriptors.

Table 6. Performance of different metrics on GRID

			Rank(%)						
Methods	1	5	10	15	20				
GOGrgb+NFST	21.84	41.28	50.96	57.44	62.88				
GOGrgb+XQDA	22.64	43.92	55.12	61.12	66.56				
GOGrgb+Proposed	22.64	43.68	52.00	59.04	65.04				
GOGfusion+NFST	23.04	44.40	54.40	61.84	66.56				
GOGfusion+XQDA	23.68	47.28	58.40	65.84	69.68				
GOGfusion+Proposed	23.92	44.64	54.88	62.32	66.40				

**GRID** We can see that the rank 1 score of proposed metric are 0.24% higher than XQDA and 0.88% higher than NFST in terms of GOGfusion, but XQDA outperforms proposed metric on rank 5, rank 10, rank 15 and rank 20

scores. Besides, proposed metric outperforms NFST on rank 5, rank 10, rank 15 scores.

In summary, the Re-ID performance is improved in VIPeR, CUHK01 dataset, and has almost the same performance with NFST and XQDA on prid\_450s dataset. Specifically, proposed metric learning has the best rank 1 score in GRID dataset and its performance is only second to XQDA. The proposed metric has superior performance for following reasons: (1) dimension reduction by KLFDA exploits the nonlinearity and the loss of discriminant information between classes are minimized. (2) the simplified relative distance limitation optimization helps to confine the Mahanalobis distance matrix M to discriminate different classes.

#### 5. Conclusion

In this paper we combined KLFDA with gradient descent method based metric learning. A semi-positive definite (SPD) matrix is learned on the lower dimension space after dimension reduction by kernel local fisher discriminative analysis. By analysis we can find the proposed metric has better performance than NFST and XQDA on VIPeR and CUHK1 datasets, but XQDA and NFST outperforms the proposed metric learning on Prid\_2011 and Prid\_450s, and the proposed metric learning has better rank 1 score than NFST and its performance is only second to XQDA on GRID dataset.

#### References

- [1] M. Sugiyama, "Local Fisher Discriminant Analysis for Supervised Dimensionality Reduction," Dec. 2016.
- [2] J. You, A. Wu, X. Li, and W.-S. Zheng, "Top-push Video-based Person Re-identification," pp. 1–9, Jan. 2017.
- [3] S. Liao, Y. Hu, X. Zhu, and S. Z. Li, "Person Reidentification by Local Maximal Occurrence Representation and Metric Learning." CVPR, Apr. 2015, pp. 1–10.
- [4] T. Matsukawa, T. Okabe, E. Suzuki, and Y. Sato, "Hierarchical Gaussian Descriptor for Person Re-Identification," Dec. 2016, pp. 1–10.
- [5] O. Tuzel, F. Porikli, and P. Meer, "Region Covariance: A Fast Descriptor for Detection and Classification," Dec. 2016, pp. 1–14.
- [6] M. Farenzena, L. Bazzani, A. Perina, V. Murino, and M. Cristani, "Person Re-Identification by Symmetry-Driven Accumulation of Local Features." CVPR, Mar. 2016, pp. 1–8.
- [7] M. Kostinger, M. Hirzer, P. Wohlhart, P. M. Roth, and H. Bischof, "Large Scale Metric Learning from Equivalence Constraints," pp. 1–8, Apr. 2012.
- [8] S. Pedagadi, J. Orwell, Velastin, Sergio, and B. Boghos, "Local fisher discriminant analysis for pedestrian re-identification," in 2013 IEEE Conference on Computer Vision and Pattern Recognition (CVPR). IEEE, Jan. 2013, pp. 3318–3325.

- [9] A. Mignon and F. Jurie, "PCCA: A New Approach for Distance Learning from Sparse Pairwise Constraints," pp. 1–7, Apr. 2012.
- [10] W. Zheng, S. Gong, and T. Xiang, "Person Reidentification by Probabilistic Relative Distance Comparison," pp. 1–8, Nov. 2016.
- [11] Weinberger, K. Q, Saul, and L. K, "Distance Metric Learning for Large Margin Nearest Neighbor Classification," pp. 1–38, Feb. 2009.
- [12] M. Welling, "Kernel Canonical Correlation Analysis," pp. 1–3, Mar. 2005.
- [13] F. Xiong, M. Gou, O. Camps, and S. Mario, "Person Re-Identification using Kernel-based Metric Learning Methods," pp. 1–16, Jul. 2014.
- [14] L. Zhang, T. Xiang, and S. Gong, "Learning a Discriminative Null Space for Person Re-identification," pp. 1–10, Mar. 2016.
- [15] J. V. Davis, B. Kulis, P. Jain, S. Sra, and I. S. Dhillon, "Information-theoretic metric learning," pp. 1–8, Jun. 2007
- [16] L. Yang, "Distance Metric Learning: A Comprehensive Survey," pp. 1–51, May 2006.
- [17] X. He and P. Niyogi, "Locality Preserving Projections," pp. 1–8, Nov. 2010.