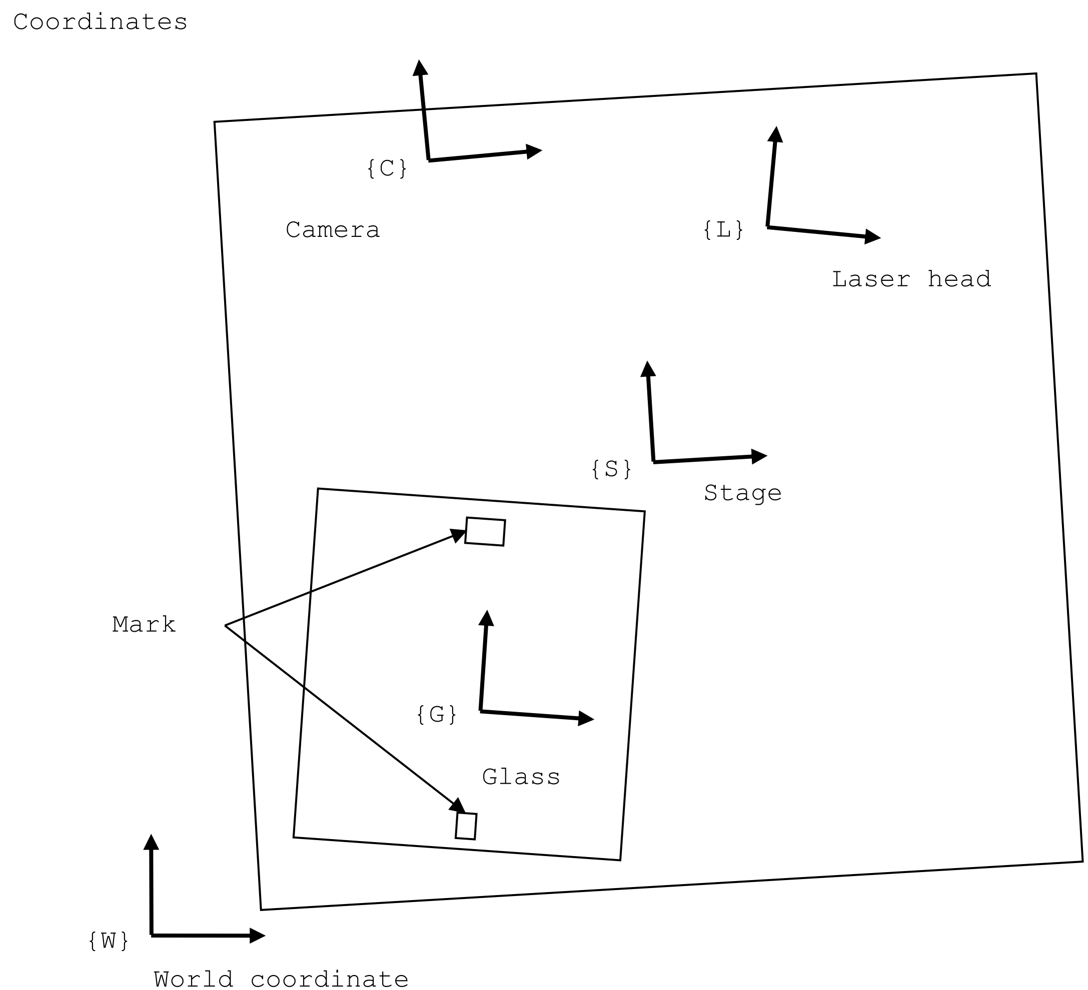
Programming Study
Align Algorithm

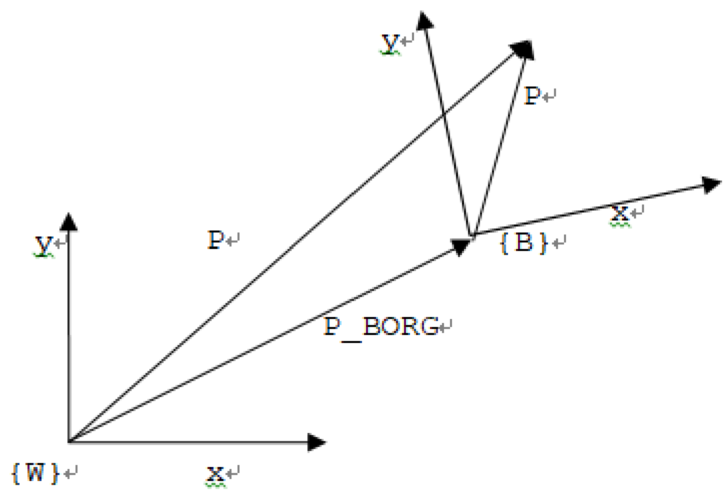
NAM SOFTWARE

2017

Geometric model - Coordinates



Coordinate Transform



$$ {}^{W}P = {}^{B}_{W}R\,{}^{B}P = \begin{bmatrix} {}^{B}_{W}T & {}^{W}P_{BORG} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & P_{BORG\_x} \\ \sin\theta & \cos\theta & P_{BORG\_y} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{B}P_{x} \\ {}^{B}P_{y} \\ 1 \end{bmatrix} $$

Geometric model – camera

Camera
Assembly to
World

Mark position
w.r.t camera pixel

$$ {}^{W}M_i = {}^{CA}_{W}R \; {}^{C}_{CA}R \; {}^{P}_{C}R \; {}^{P}M_i $$

Mark
position
w.r.t World

Camera to
Camera
Assembly

Pixel to
Camera

Geometric model – stage

Servo to World

Stage to Servo

GlassCenter to GlassLoading

$$ {}^{W}M_i = {}^{S}_{W}R \; {}^{B}_{S}R \; {}^{GL}_{B}R \; {}^{GC}_{GL}R \; {}^{G}_{GC}R \; {}^{G}M_i $$

Mark position w.r.t World

GlassLoading to Stage

Glass to GlassCenter

Mark position w.r.t Glass

Camera matrix

$$
{}^{P}P = {}^{C}_{P}R\ {}^{C}P = \begin{bmatrix} 1/r_x & 0 & W/2 \\ 0 & -1/r_y & H/2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{C}P_x \\ {}^{C}P_y \\ 1 \end{bmatrix}
$$

Resolution
mm/pixel

Width, Height

Camera matrix

$$
{}^{P}P = {}^{C}_{P}R \; {}^{C}P = \begin{bmatrix} 1/r_x & 0 & W/2 \\ 0 & -1/r_y & H/2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{C}P_x \\ {}^{C}P_y \\ 1 \end{bmatrix}
$$

Resolution
mm/pixel

Width, Height

Calibration – finding camera offset

target

Mark at the stage

$$ {}^{CA}_{W}R \; {}^{C}_{CA}R \; {}^{P}_{C}R \; {}^{P}M_i = {}^{S}_{W}R \; {}^{B}_{S}R \; {}^{B}M_i $$

$$ {}^{C}_{CA}R \; {}^{P}_{C}R \; {}^{P}M_i = {}^{CA}_{W}R^{-1} \; {}^{S}_{W}R \; {}^{B}_{S}R \; {}^{B}M_i $$

$$ {}^{C}_{CA}R\, U_i = V_i $$

Least Square Method

$$\begin{bmatrix} 1 & -\theta & x \\ \theta & 1 & y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} U_{i1} \\ U_{i2} \\ 1 \end{bmatrix} = \begin{bmatrix} V_{i1} \\ V_{i2} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -U_{i2} & U_{i1} \\ 0 & 1 & U_{i1} & U_{i2} \end{bmatrix} \begin{bmatrix} x \\ y \\ \theta \\ 1 \end{bmatrix} = \begin{bmatrix} V_{i1} \\ V_{i2} \end{bmatrix}$$

$$U \; X = V$$

$$X = (U^T U)^{-1} U^T V$$

Calibration – glass

target

$$ {}^{S}_{W}R \; {}^{B}_{S}R \; {}^{GL}_{B}R \; {}^{GC}_{GL}R \; {}^{G}_{GC}R \; {}^{G}M_i = {}^{CA}_{W}R \; {}^{C}_{CA}R \; {}^{P}_{C}R \; {}^{P}M_i $$

$$ {}^{GC}_{GL}R \; {}^{G}_{GC}R \; {}^{G}M_i = ({}^{S}_{W}R \; {}^{B}_{S}R \; {}^{GL}_{B}R)^{-1} \; {}^{CA}_{W}R \; {}^{C}_{CA}R \; {}^{P}_{C}R \; {}^{P}M_i $$

$$ {}^{GC}_{GL}R \, U_i = V_i $$

Servo control to mark

$$\vec{S} = {}^{CA}_{W}R \; {}^{C}_{CA}R \; {}^{C}M - {}^{B}_{S}R \; {}^{GL}_{B}R \; {}^{GC}_{GL}R \; {}^{G}_{GC}R \; {}^{G}M$$

$$[0,0,1]^T$$

Calibration error

$$E_i = {}^{CA}_{W}R \; {}^{C}_{CA}R \; {}^{P}_{C}R \; {}^{P}M_i - {}^{S}_{W}R \; {}^{B}_{S}R \; {}^{B}M_i$$

$$E_i = {}^{S}_{W}R^{L} \; {}^{B}_{S}R \; {}^{GL}_{B}R \; {}^{GC}_{GL}R \; {}^{G}_{GC}R \; {}^{G}M_i - {}^{CA}_{W}R \; {}^{C}_{CA}R \; {}^{P}_{C}R \; {}^{P}M_i$$

$$error = \frac{\sum_{i}^{N} |E_i|}{N}$$