Programming Study
Artificial Neural Network
v2

Sungwoo Nam
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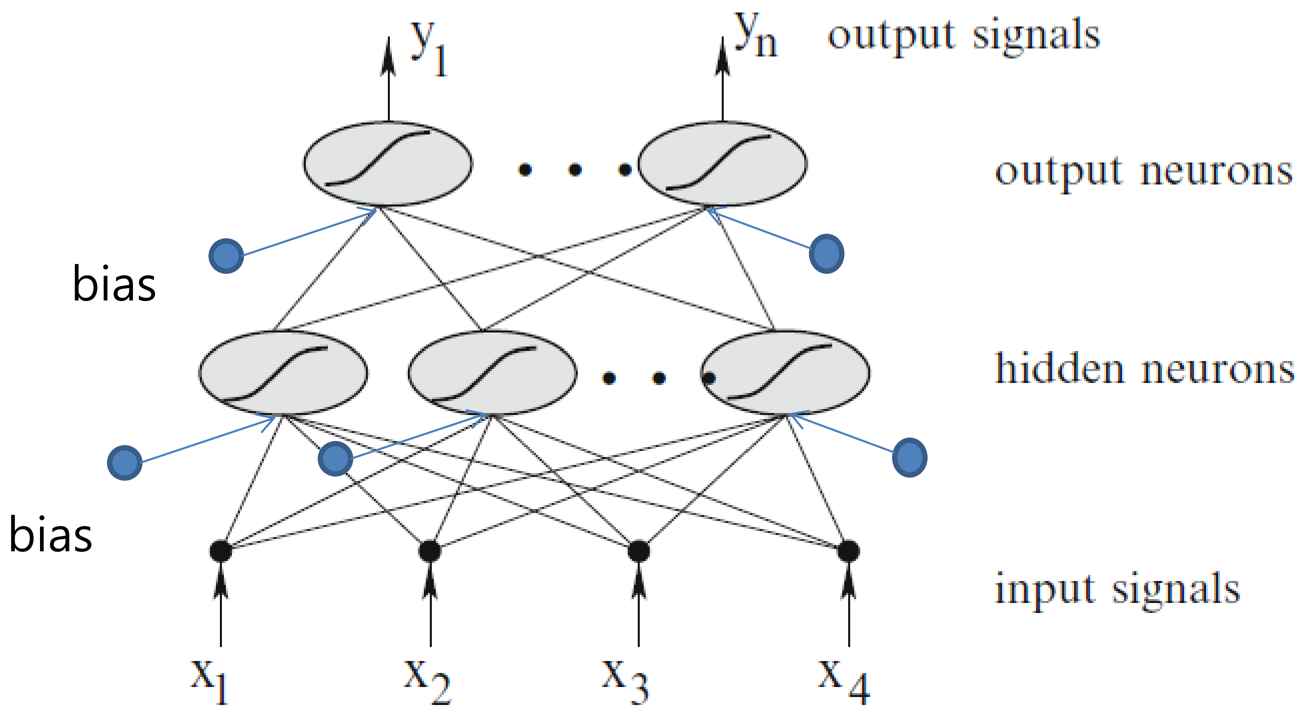
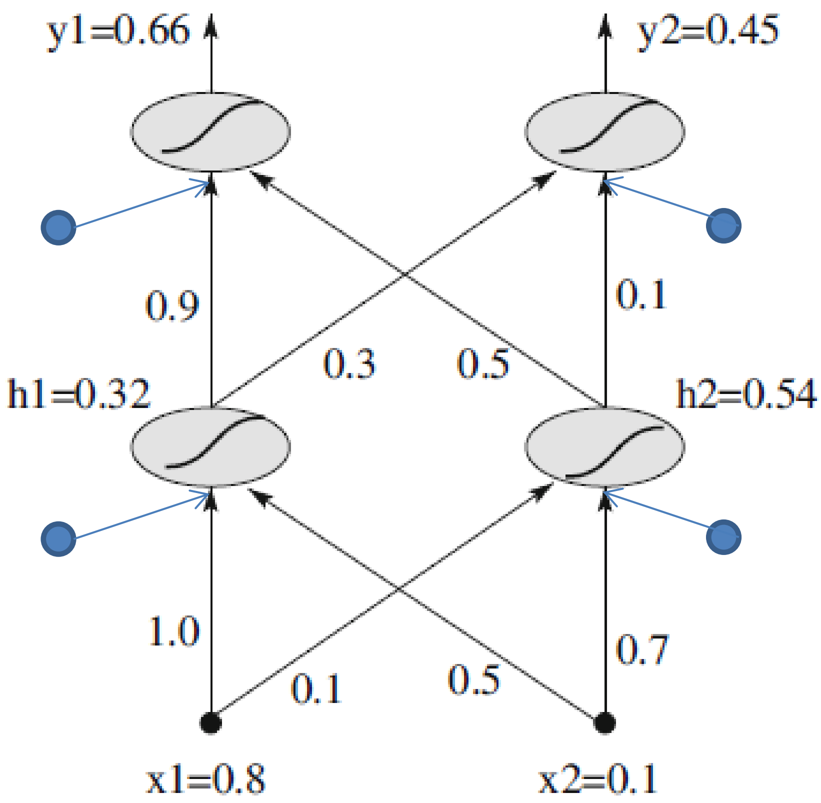
Artificial Neural Network



An Introduction to Machine Learning – Miroslav Kubat, Springer

Forward Propagation



$$y_i = f\left(\sum_i w_{ji}\, f\left(\sum_j w_{kj} x_k + b_j\right) + b_i\right)$$

$$f(\Sigma) = \frac{1}{1 + e^{-\Sigma}}$$

Forward Propagation In Code

```
Mat ForwardPropagation(const Mat& X, const Mat& HN, const Mat& ON, const Mat& HB, const Mat& OB, Mat& H)
{
    auto sigmoid = [](double &v, const int* pos) {
        v = 1.0 / (1.0 + exp(-v));
    };

    H = HN * X;
    H.forEach<double>([=](double &v, const int* pos) { v += HB.at<double>(pos[0]); });
    H.forEach<double>(sigmoid);
    Mat O = ON * H;
    O.forEach<double>([=](double &v, const int* pos) { v += OB.at<double>(pos[0]); });
    O.forEach<double>(sigmoid);

    return O;
}
```

$$f(\Sigma) = \frac{1}{1 + e^{-\Sigma}}$$

Backward Propagation

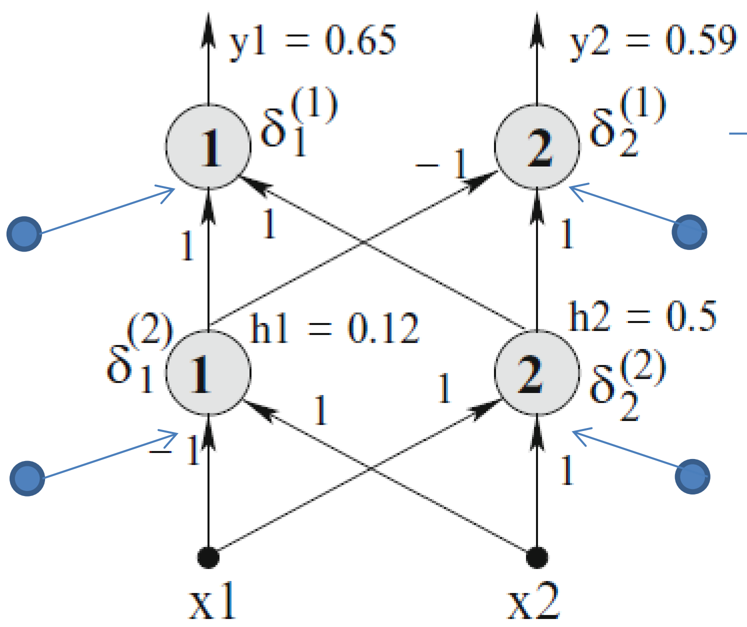
Table 5.2 *Backpropagation of error* in a neural network with one hidden layer

1. Present example $\mathbf{x}$ to the input layer and propagate it through the network.
2. Let $\mathbf{y} = (y_1, \dots y_m)$ be the output vector, and let $\mathbf{t}(\mathbf{x}) = (t_1, \dots t_m)$ be the target vector.
3. For each output neuron, calculate its responsibility, $\delta_i^{(1)}$, for the network's error:
   $\delta_i^{(1)} = y_i(1 - y_i)(t_i - y_i)$
4. For each hidden neuron, calculate its responsibility, $\delta_j^{(2)}$, for the network's error. While doing so, use the responsibilities, $\delta_i^{(1)}$, of the output neurons as obtained in the previous step.
   $\delta_j^{(2)} = h_j(1 - h_j) \sum_i \delta_i^{(1)} w_{ji}$
5. Update the weights using the following formulas, where η is the learning rate:

   output layer: $w_{ji}^{(1)} := w_{ji}^{(1)} + \eta \delta_i^{(1)} h_j$; h_j: the output of the j-th hidden neuron
   hidden layer: $w_{kj}^{(2)} := w_{kj}^{(2)} + \eta \delta_j^{(2)} x_k$; x_k: the value of the k-th attribute

6. Unless a termination criterion has been satisfied, return to step 1.

Backward Propagation



$$\delta_i^{(1)} = y_i(1 - y_i)(t_i - y_i)$$

$$\delta_j^{(2)} = h_j(1 - h_j)\sum_i \delta_i^{(1)} w_{ji}$$

$$w_{ji}^{(1)} := w_{ji}^{(1)} + \eta\delta_i^{(1)} h_j;$$
$$w_{kj}^{(2)} := w_{kj}^{(2)} + \eta\delta_j^{(2)} x_k;$$

$$b_i \coloneqq b_i + \eta\delta_i$$
$$b_j \coloneqq b_j + \eta\delta_j$$

Backward Propagation In Code

```
void BackwardPropagation(
    const Mat& X, Mat& HN, Mat& ON, Mat& HB, Mat& OB, const Mat& H,
    const Mat& Y, const Mat& T, double nu )
{
    Mat D1 = Y.clone();
    D1.forEach<double>([=](double &yi, const int* i) {
        double ti = T.at<double>(i);
        yi = yi * (1 - yi) * (ti - yi);
    });

    Mat P = ON.t() * D1;
    Mat D2 = H.clone();
    D2.forEach<double>([=](double &hi, const int*i) {
        double pi = P.at<double>(i);
        hi = hi * (1 - hi) * pi;
    });

    for (int x = 0; x < H.cols; ++x)
    {
        ON += nu * D1.col(x) * H.col(x).t();
        OB += nu * D1.col(x);
        HN += nu * D2.col(x) * X.col(x).t();
        HB += nu * D2.col(x);
    }
}
```

$$\delta_i^{(1)} = y_i(1-y_i)(t_i-y_i)$$

$$\delta_j^{(2)} = h_j(1-h_j)\sum_i \delta_i^{(1)} w_{ji}$$

$$w_{ji}^{(1)} := w_{ji}^{(1)} + \eta\delta_i^{(1)} h_j;$$
$$w_{kj}^{(2)} := w_{kj}^{(2)} + \eta\delta_j^{(2)} x_k;$$
$$b_i := b_i + \eta\delta_i$$
$$b_j := b_j + \eta\delta_j$$

Training Data

- http://archive.ics.uci.edu/ml/datasets/steel+plates+faults

notes.txt | kNN.py | SteelPlateFaults.txt

```
42	50	270900	270944	267	17	44	24220	76	108	1687	1	0	80	0.0498	0.2415	0.1818	0.0047	0.4706	1	1	2.4265	0.9031	1.6435	0.8182	-0.2913	0.5822	1	0	0	0	0	0	0
645	651	2538079	2538108	108	10	30	11397	84	123	1687	1	0	80	0.7647	0.3793	0.2069	0.0036	0.6	0.9667	1	2.0334	0.7782	1.4624	0.7931	-0.1756	0.2984	1	0	0	0	0	0	0
829	835	1553913	1553931	71	8	19	7972	99	125	1623	1	0	100	0.971	0.3426	0.3333	0.0037	0.75	0.9474	1	1.8513	0.7782	1.2553	0.6667	-0.1228	0.215	1	0	0	0	0	0	0	0
853	860	369370	369415	176	13	45	18996	99	126	1353	0	1	290	0.7287	0.4413	0.1556	0.0052	0.5385	1	1	2.2455	0.8451	1.6532	0.8444	-0.1568	0.5212	1	0	0	0	0	0	0
1289	1306	498078	498335	2409	60	260	246930	37	126	1353	0	1	185	0.0695	0.4486	0.0662	0.0126	0.2833	0.9885	1	3.3818	1.2305	2.4099	0.9338	-0.1992	1	1	0	0	0	0	0	0
430	441	100250	100337	630	20	87	62357	64	127	1387	0	1	40	0.62	0.3417	0.1264	0.0079	0.55	1	1	2.7993	1.0414	1.9395	0.8736	-0.2267	0.9874	1	0	0	0	0	0	0
413	446	138468	138883	9052	230	432	1481991	23	199	1687	0	1	150	0.4896	0.339	0.0795	0.0196	0.1435	0.9607	1	3.9567	1.5185	2.6181	0.9205	0.2791	1	1	0	0	0	0	0	0
190	200	210936	210956	132	11	20	20007	124	172	1687	0	1	150	0.2253	0.34	0.5	0.0059	0.9091	1	1	2.1206	1	1.301	0.5	0.1841	0.3359	1	0	0	0	0	0	0
330	343	429227	429253	264	15	26	29748	53	148	1687	0	1	150	0.3912	0.2189	0.5	0.0077	0.8667	1	1	2.4216	1.1139	1.415	0.5	-0.1197	0.5593	1	0	0	0	0	0	0
74	90	779144	779308	1506	46	167	180215	53	143	1687	0	1	150	0.0877	0.4261	0.0976	0.0095	0.3478	0.982	1	3.1778	1.2041	2.2148	0.9024	-0.0651	1	1	0	0	0	0	0	0
106	118	813452	813500	442	13	48	50393	76	143	1687	0	1	150	0.1257	0.2326	0.25	0.0071	0.9231	1	1	2.6454	1.0792	1.6812	0.75	-0.1093	0.8612	1	0	0	0	0	0	0
505	515	106604	106668	284	42	69	31062	97	119	1687	0	1	150	0.5987	0.5562	0.1563	0.0059	0.2381	0.9275	1	2.4533	1	1.8062	0.8438	-0.1455	0.9048	1	0	0	0	0	0	0
46	58	179258	179312	480	15	54	61966	102	158	1687	0	1	150	0.0545	0.2593	0.2222	0.0071	0.8	1	1	2.6812	1.0792	1.7324	0.7778	0.0086	0.9093	1	0	0	0	0	0	0
581	590	230644	230704	433	22	60	38917	62	111	1687	0	1	150	0.6888	0.1981	0.15	0.0053	0.4091	1	1	2.6365	0.9542	1.7781	0.85	-0.2978	0.8299	1	0	0	0	0	0	0
451	466	368143	368208	728	30	68	69258	36	133	1687	0	1	150	0.5347	0.2533	0.2308	0.0089	0.5	0.9559	1	2.8621	1.1761	1.8129	0.7692	-0.2568	0.9888	1	0	0	0	0	0	0
669	684	491552	491684	1097	59	133	119540	50	134	1687	0	1	150	0.7931	0.446	0.1136	0.0089	0.2542	0.9925	1	3.0402	1.1761	2.1206	0.8864	-0.1487	1	1	0	0	0	0	0	0
156	192	713788	714056	5044	167	282	570911	11	143	1687	0	1	150	0.1849	0.4772	0.1343	0.0213	0.2156	0.9503	1	3.7028	1.5563	2.4281	0.8657	-0.1157	1	1	0	0	0	0	0	0
90	104	751059	751132	552	38	76	59750	79	134	1687	0	1	150	0.1067	0.4599	0.1918	0.0083	0.3684	0.9605	1	2.7419	1.1461	1.8633	0.8082	-0.1543	0.9918	1	0	0	0	0	0	0
82	89	844704	844729	137	8	25	14907	92	126	1687	0	1	150	0.0972	0.2171	0.28	0.0041	0.875	1	1	2.1367	0.8451	1.3979	0.72	-0.1499	0.2998	1	0	0	0	0	0	0
1601	1613	21349	21376	209	15	27	24807	96	141	1687	0	1	200	0.0877	0.3549	0.4444	0.0071	0.8	1	1	2.3201	1.0792	1.4314	0.5556	-0.0727	0.5362	1	0	0	0	0	0	0
17	28	42683	42717	284	18	34	32604	87	141	1687	0	1	200	0.0202	0.2406	0.3235	0.0065	0.6111	1	1	2.4533	1.0414	1.5315	0.6765	-0.1031	0.6173	1	0	0	0	0	0	0
43	52	86284	86313	153	13	29	17753	101	134	1687	0	1	200	0.051	0.4138	0.3103	0.0053	0.6923	1	1	2.1847	0.9542	1.4624	0.6897	-0.0935	0.4317	1	0	0	0	0	0	0
63	72	115485	115504	106	10	19	13829	107	150	1687	0	1	200	0.0747	0.3801	0.4737	0.0053	0.9	1	1	2.0253	0.9542	1.2787	0.5263	0.0192	0.2942	1	0	0	0	0	0	0
82	92	149044	149083	264	15	39	32175	92	141	1687	0	1	200	0.0972	0.3231	0.2564	0.0059	0.6667	1	1	2.4216	1	1.5911	0.7436	-0.0479	0.6422	1	0	0	0	0	0	0
75	84	184350	184383	201	17	35	27349	123	148	1687	0	1	200	0.0889	0.3232	0.2727	0.0053	0.5294	0.9429	1	2.3032	0.9542	1.5185	0.7273	0.063	0.4913	1	0	0	0	0	0	0
1363	1372	2128884	2128907	171	9	23	16645	77	111	1373	0	1	40	0.0015	0.1739	0.3913	0.0066	1	1	1	2.233	0.9542	1.3617	0.6087	-0.2395	0.3464	1	0	0	0	0	0	0
1358	1372	2356396	2356435	395	19	39	38722	75	118	1373	0	1	40	0.0015	0.2766	0.359	0.0102	0.7368	1	1	2.5966	1.1461	1.5911	0.641	-0.2341	0.8355	1	0	0	0	0	0	0	0
1404	1410	5202306	5202324	85	9	18	8948	97	119	1627	0	1	40	0.2668	0.213	0.3333	0.0037	0.6667	1	1	1.9294	0.7782	1.2553	0.6667	-0.1776	0.215	1	0	0	0	0	0	0	0
```

Training

```
// http://archive.ics.uci.edu/ml/datasets/steel+plates+faults
void testSteelPlateDefects0()
{
    Mat D = ReadMat("SteelPlateFaults.txt", " \t");
    D = D.rowRange(0, 340);

    Mat X = D.colRange(4, 27).t();
    Normalize<double>(X);
    Mat T = D.colRange(27, 29).t();

    int M = T.rows, N = X.rows;
    Mat HN(N+1, N, CV_64FC1, Scalar(0.01));
    Mat HB(N + 1, 1, CV_64FC1, Scalar(0.0));
    Mat ON(M, N+1, CV_64FC1, Scalar(0.01));
    Mat OB(M, 1, CV_64FC1, Scalar(0.0));

    Mat H, Y;
    for (int i = 0; i < 100; ++i)
    {
        Y = ForwardPropagation(X, HN, ON, HB, OB, H );
        BackwardPropagation(X, HN, ON, HB, OB, H, Y, T, 0.1);

        // printf("iteration %d, Error %.3f\n", i, AverageDistance(T-Y));
    }

    printf( "Trained Error %.3f\n", AverageDistance(T - Y));
}
```

```
iteration 0, Error 0.708
iteration 1, Error 0.748
iteration 2, Error 0.814
iteration 3, Error 0.872
iteration 4, Error 0.722
iteration 5, Error 0.882
iteration 6, Error 0.672
iteration 7, Error 0.705
iteration 8, Error 0.635
iteration 9, Error 0.669

…

iteration 95, Error 0.086
iteration 96, Error 0.084
iteration 97, Error 0.078
iteration 98, Error 0.095
iteration 99, Error 0.090
Trained Error 0.090
```