Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

   P(X>50) = 1 - P(X < 50)
   Z = (50-45)/8 = 0.625
   Therefore, 0.267

   ## Option B is the Answer

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ =6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.
   ## Ans is False
   Solution.
   68% of the data falls within standard deviation of the mean.

   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.
   ## Ans is True
   Solution,
   Probability of people less than 30 is (30-38)/6 = 0.09 or 9%
   Therefore, 0.09 * 400 employees = 36

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between $2X_1$ and $X_1 + X_2$? Discuss both their distributions and parameters.

X = N (μ1, σ1^2)

Therefore,

2X1 = N (2 u,4 σ^2)

X1 + x2 = N (μ + μ, σ^2 + σ^2) = N (2 u, 2σ^2)

2X1-(X1+X2) = N (4μ,6 σ^2)

4. Let $X \sim N(100, 20^2)$. Find two values, a and b, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

Probability of getting value between a and b is 0.99
Probability of not getting value between a and b is 1 – 0.99 = 0.01
Therefore, probability of left side of a = -0.005
Probability of right side of b = + 0.005

Z = (X – μ) / SD
X = Z * SD + μ
a= (2.58) * 20 + 100 = 151.5
b= (-2.58) *20+100 = 48.5
       A. Therefore, answer is Option D 48.5, 151.5

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

B. Specify the 5th percentile of profit (in Rupees) for the company

C. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:-

A) 95% of the data falls within two standard deviation of the mean.
(540-450, 540+450)=> (90,990)

B) 5th percentile is 202.5 million rupees

C) $Z=(X-\mu)/\sigma$ => (0-5)/3 => -1.66=0.0485. Division2 has a higher probability of making a loss.