COMM415DA Fundamentals Of Data Science

Ref/Def Assessment (CW2)

This continuous assessment (CA) comprises 60% of the overall module assessment. This is an individual exercise and your attention is drawn to the College and University guidelines on collaboration and plagiarism, which are available from the College website. As a rule of thumb, to understand when collaboration becomes plagiarism, consider the following:

- it is OK when students communicate and support each other in better understanding the concepts presented in the lectures;
- it is not OK when students communicate how these concepts can be combined and used to solve specific Assignments questions.

Question 1

Acquire the Iris dataset using the following procedure:

```
from sklearn.datasets import load_iris
X,y = load_iris(return_X_y=True)
```

The data matrix contains 150 vectors (also called *instances*) with 4 attributes each (i.e. it is a 150 x 4 matrix) and the vector y contains the class encoded as the integers 0,1, and 2.

a) Split the data matrix in two data matrices D_{tr} and D_{ts} each containing a balanced number of instances per class (i.e. D_{tr} contains as many instances from class k as D_{ts}). Split the class vector y into y_{tr} and y_{ts} accordingly (i.e. the first instance in y_{tr} is the class of the first instance in D_{tr}, etc).

b) Define a function to compute the *distance* between two vectors (of arbitrary dimension) as the *length* of the *difference* vector.

c) Using the distance function, build the function `knn_predict` that implements the k-nearest neighbor classifier. The function takes in input two data matrices D_{ts} and D_{tr}, a target vector y_{tr}, and the number of neighbours k. For each instance in D_{ts} it returns the most frequent class of those associated to the k closest (i.e. the least distant) instances in D_{tr}.

d) Use D_{tr} and y_{tr} to fit your implementation of the nearest neighbor classifier using the `knn_predict` function. Compute the accuracies of the nearest neighbor classifier on D_{ts} for $k =1,3,5$. The accuracy is the proportion of true results (i.e. when the class predicted was the same as the true class) over the total number of predictions.

(Total 30 marks)

Question 2

Acquire the Iris dataset as indicated in Question Q1. Select only the instances relative to a single class and denote the data matrix as D.

a) Create a function `add_missing` that takes in input a data matrix D and a number k and returns a data matrix D' and a $k \times 2$ matrix P. Each row of the matrix P contains the row i and column j indices of an entry D_{ij} in D chosen uniformly at random. The matrix D' is a copy of D except for the entries specified in P: the value in each entry D'_{ij} is the column average of D_i.

b) Create a function `impute` that takes in input a data matrix M and a number r and returns a data matrix M'. The matrix M' is the reconstruction of M using its r largest singular vectors and values (i.e. it is the truncated SVD reconstruction of M).

c) Compute E the average length of the difference vectors between the corresponding instances in D and in its reconstructed version.

d) Repeat the following 30 times:

- from D generate D' using $k = 50$
- apply `impute` to D' using a specific r to compute M
- compute E between D and M Consider the average E over the 30 trials.

Report a plot of the average E value when the procedure is repeated 30 times, for $k = 50$ and $r = 1, 2, 3, 4$.

(Total marks 30)

Question 3

[40 marks]

Aim: demonstrate an understanding of LDA.

In this exercise you will need to build a dataset using the `multivariate_normal` function provided by the numpy library. You will then fit a multiclass LDA model to your data and finally you will display the decision boundaries of the trained predictive model.

Construction of the dataset:

Given a parameter `k`, the dataset is generated using `k` multi variate normal data generators. All instances are 2 dimensional.

The multi variate normal data generators are defined by 3 parameters: a mean, a covariance matrix and the number of samples to generate.

a) The means of the data generator will lie on the vertices of a regular polygon (if `k=3` the polygon is a triangle, if `k=6` it is an hexagon, etc). Write your own code to determine the position of the vertices of a regular polygon given a radius value in input. *Hint: you can use your knowledge on linear transformations (e.g. rotations).*

b) The covariance matrices will be constructed by specifying 2 parameters: a `ratio` between the two main directions of variability (if the ratio is, say, 2:1, then the parameter is 2); and a `rotation` in degrees (i.e. 90 for a right angle) to determine the main *direction* of variability.

Example: If the ratio is 2 and the rotation parameter is 45 your covariance matrix will be equivalent to:

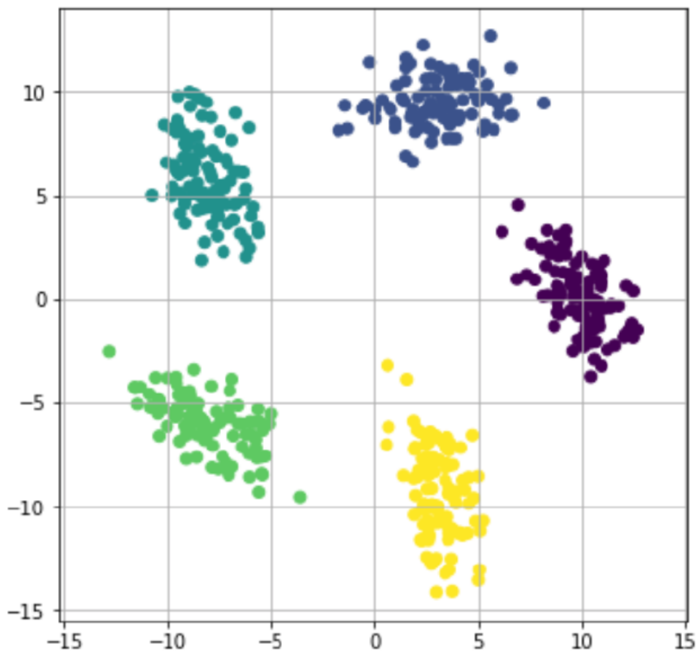
```
matrix([[2.5, 1.5],
        [1.5, 2.5]])
```

c) Using the results from the previous points, write your own function `data_matrix, targets = make_data(k, num_instances, radius, ratio)` to generate a data matrix with `num_instances` rows and 2 columns and a `targets` vector of length `num_instances` containing a class indicator for each instance (i.e. an integer between 0 and k-1). The function takes in input the number of classes `k`, the total number of instances `num_instances`, the parameter `radius` to express the distance from the origin for the means of the multi variate normal data generators, and finally the `ratio` between the two main directions of variability to determine the covariance matrices for the multi variate normal data generators.

Important: For each one of the `k` multi variate normal data generator, sample the `rotation` parameter uniformly at random between 0 and 360.

d) Display a scatter plot in 2D of the generated dataset distinguishing instances belonging to different classes by color.

You should obtain something like this:



LDA model:

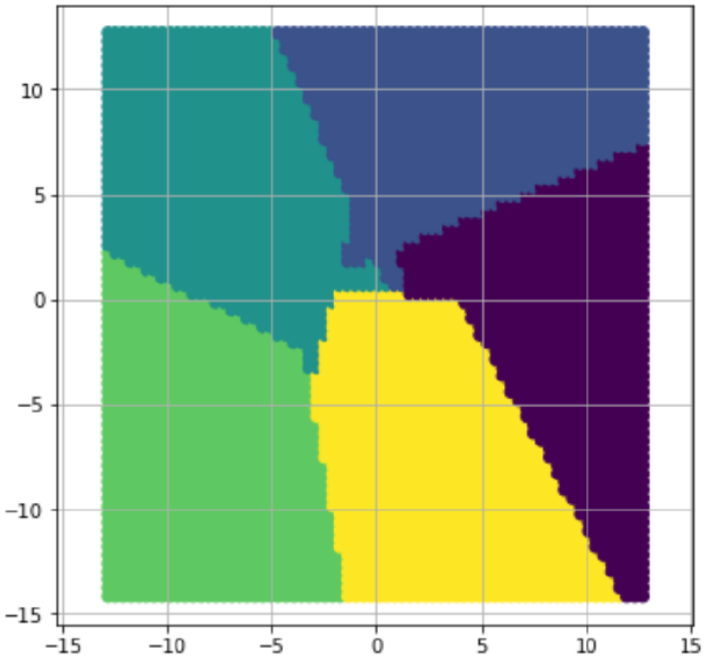
e) Write the function `params = fit_LDA(data_matrix,targets)`. The function outputs the parameters for a LDA classifier fit for the classification of the required number of classes. The number of classes will be automatically deduced from the `targets` vector.

f) Write the function `predictions = test_LDA(data_matrix, params)` to output the class predicted by the LDA model encoded in `param`.

g) Write a function `make_grid` to generate instances regularly spaced in 2D. The function should allow the user to specify the grid *density*, i.e. how many points to generate.

h) Generate 500 instances for 5 classes with radius 10 and ratio 2. Fit an LDA model. Generate a grid dataset that covers the original data set. Display a scatter plot in 2D of the grid dataset distinguishing instances belonging to different predicted classes by color.

You should obtain something like this:



Submitting your work

Please write your student ID in the first cell of the notebook.

You should submit the *Jupyter notebook* containing the code with its output for all the questions.
Make a separate cell for each point a), b), c), etc of each question.

Submit both a `PDF` copy of your notebook as a proof of execution and also the original source file with extension `.ipynb`. Make a single archive (.zip or .tar) containing the PDF **and** the notebook.

Markers will not be able to give feedback if you do not submit the PDF version of your code and marks will be deducted if you fail to do so.

Marking criteria

Work will be marked against the following criteria. Although it varies a bit from question to question they all have approximately equal weight.

- **Does your algorithm correctly solve the problem?** In most of the questions the required code has been described, but not always in complete detail and some decisions are left to you.
- **Is the code syntactically correct?** Is your program a legal Python program regardless of whether it implements the algorithm?
- **Is the code beautiful or ugly?** Is the implementation clear and efficient or is it unclear and extremely inefficient (e.g. it takes more than a few minutes to execute, or it has a quadratic complexity when a linear complexity would suffice)? Is the code well structured? Have you made good use of functions? Are you using Numpy functions on entire arrays when possible?
- **Is the code well laid out and commented?** Is there a comment describing what the code does? Have you used space to make the code clear to human readers?

There are 10% penalties for:

- Not submitting the PDF version of your programs.
- Not creating functions as instructed in the questions.