

Level 7 Research Scientist (MSc Data Science Professional)

Introduction

The Level 7 Research Scientist (MSc Data Science Professional) requires some relatively advanced Mathematics. To ensure the applicants for this programme are aware of the level of Maths we expect apprentices to have when they begin the programme, we have included these worksheets as a self-assessment tool. The worksheets will not be marked and applicants do not have to return them to us, but we do expect candidates to be able to complete the types of exercises identified in the worksheets. If candidates cannot complete such worksheets they have time between now and September to develop their understanding of Maths to ensure that they are prepared for when the programme begins. See below for some helpful materials for self-learning.

Self-learning Materials

The following self-learning materials may be helpful to either refresh the knowledge you already have, or to teach you the necessary basics if the concepts are new. If you are comfortable in your Mathematical knowledge, you can start the worksheets without reading this section.

Algebra:

- Algebra 1 = <https://www.khanacademy.org/math/algebra>
- Algebra 2 = <https://www.khanacademy.org/math/algebra2>

And ideally all A-level Maths: <https://www.examsolutions.net/a-level-maths/aqa/>
examsolutions.net has a very similar format of teaching to Khan Academy but it is specifically designed for the UK system, so it specifically covers A-Level content.

Ideally you would be comfortable with the introductory lessons here:

- Basics of statistics: <https://www.khanacademy.org/math/statistics-probability>. The first few modules (analyzing categorical data, displaying and comparing quantitative data, summarizing quantitative data and probability)
- Linear Algebra: <https://www.khanacademy.org/math/linear-algebra>
- And then to go further (not expected), they could do more of (or all) of the modules in Basics of Statistics and Linear Algebra

Worksheet 1

Q1

What is the unit vector in the direction of $\mathbf{u} = (-8, -1)$

Q2

Solve the equation below:

$$\ln(3x + 1) + \ln(3) = \ln(18)$$

Q3

Find the derivative of the following function with respect to x :

$$f(x) = e^{-x^2/2c^2}$$

Q4

If $\ln(\frac{\theta}{1-\theta}) = \alpha + \beta x$, which one of the following is correct?

1. $\theta = \frac{e^{\alpha+\beta x}}{e^{\alpha+\beta x}+1}$
2. $\theta = \frac{1}{1+e^{\alpha+\beta x}}$
3. $\theta = \frac{e^{\alpha+\beta x}}{e^{\alpha+\beta x}-1}$

Q5

What is the probability of getting a sum of 9 when two dice are thrown?

Q6

Evaluate the following:

1. $\int_{-2}^1 (6x^2 - 2x + 3)dx$
2. $\int_{-10}^{-1} (\frac{5}{e^{-x}} - \frac{1}{5x})dx$

Q7

Which of the following is an appropriate term to propose a range of plausible values for an unknown parameter?

- p-value
- coefficient of variation
- confidence interval

Q8

What is the positive square root of variance?

- root mean square error
- standard deviation
- mean square error

Q9

Solve the following equation (what is the maxima). (Hint: You need to take the derivative with respect to x)

$$x = \operatorname{argmax}_x \ln(x) \sum_{i=1}^n y_i - \sum_{i=1}^n \ln(y_i) - nx$$

Q10

A die is rolled and a coin is tossed. Find the probability that the die shows an odd number and the coin shows a head.

Worksheet 2

Q1

Let

$$A = \begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix}$$

Notice that A contains every integer from 1 to 9 and that the sums of each row, column, and diagonal of A are equal. Such a grid is sometimes called a magic square. Compute the determinant of A .

Q2

Given any constants a, b, c where $a \neq 0$, find all values of x such that the matrix A is invertible if

$$A = \begin{bmatrix} 1 & 0 & c \\ 0 & a & -b \\ -1/a & x & x^2 \end{bmatrix}$$

Q3

Find the eigenvalues and eigenvectors of the matrix:

$$A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$$

Q4

Differentiate x^7 with respect to x .

Q5

Differentiate $\tan x$ with respect to x .

Q6

Differentiate $\sin(2x + 5)$ with respect to x .

Q7

Find all of the first order partial derivatives for the multivariate function

$$f(x, y) = x^4 + 6\sqrt{y} - 10.$$

Q8

Let \mathbf{v} and \mathbf{w} be two $n \times 1$ column vectors. Prove that $(\mathbf{v}\mathbf{w}^T) = \mathbf{v}^T\mathbf{w}$.

Q9

Let A and B be $n \times n$ matrices. Is it always true that $(AB) = (A)(B)$. If it is true, prove it. If not, give a counterexample.

Q10

Prove that the distance between two parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is given by $\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$.

Worksheet 3

Q1

If $A = \{x, y, z, p, q, r\}$ and $B = \{x, y, k, w, v\}$ are sets, what is

$$A \cup B$$

Q2

If $y = x^b \exp(-k)$ what is $\ln y$?

Q3

If I flip a coin 3 times, what is the probability I get 3 heads?

Q4

Which function grows faster: $f(n) = n!$ or $g(n) = n^10$?

Q5

If $\vec{x} = (1, 2, -1)$, $\vec{y} = (1, 0, 1)$, what is $\vec{x} \cdot \vec{y}$?

Q6

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$
$$x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Compute $x^T A^T$.

Q7

For what values of x is the expression

$$x^2 - 3x + 2 = 0$$

Q8

What is the value of y as $x \rightarrow \infty$

$$y = \frac{x}{\exp(x)}$$

Q9

For some high dimensional vectors \vec{x} and \vec{y} define

$$r = \sum_{i=1}^N x_i y_i$$

If the elements of \vec{x} and \vec{y} are randomly assigned ± 1 on the basis of a coin flip, what is the expected value of r ?