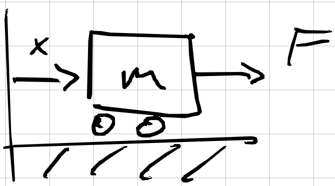
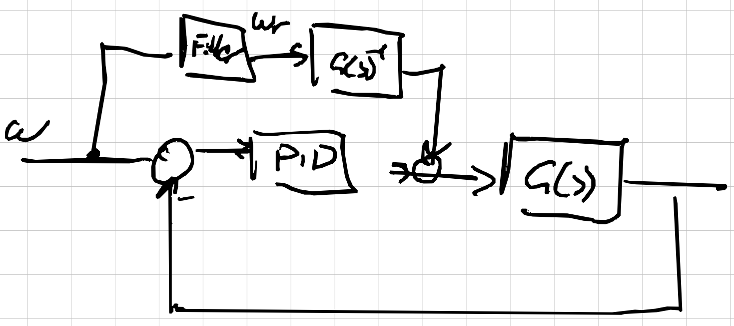
Exercise 3)



$$m\ddot{x} = F \quad (m=1) \qquad G(s) = \frac{1}{s^2}$$



$$G_F(s) = \frac{\lambda^2}{(s+\lambda)^2} = \frac{\lambda^2}{s^2+2\lambda s+\lambda^2} = \frac{\omega_r}{\omega}$$

$$\hookrightarrow \lambda^2 \cdot \omega = \omega_r \cdot s^2 + \omega_r \cdot 2\lambda s + \omega_r \cdot \lambda^2$$

$$\lambda^2 \cdot \omega = \ddot{\omega}_r + \dot{\omega}_r \cdot 2\lambda + \omega_r \cdot \lambda^2$$

$$\ddot{\omega}_r = \lambda^2 \cdot \omega - \dot{\omega}_r 2\lambda - \omega_r \lambda^2$$

↑ input u

$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 \\ -\lambda^2 & -2\lambda \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ \lambda^2 \end{bmatrix} u$$

$$\begin{bmatrix} \omega_r \\ F_F \end{bmatrix} = \vec{y} = \begin{bmatrix} 1 & 0 \\ -\lambda^2 & -2\lambda \end{bmatrix} \vec{x} + \begin{bmatrix} 0 \\ \lambda^2 \end{bmatrix} u$$

$$m\ddot{x} = k_L \cdot \left(\frac{k_v}{A_{sp}} \omega - \frac{A_B}{A_{sp}} \dot{x} \right)^2 - mg \quad \Big/ :m$$

$$u = \frac{1}{k_F} \cdot (F_F \cdot \dot{\omega} + \omega)$$

$$\ddot{x} = \underbrace{a \cdot (b\omega - c\dot{x})^2}_{v} - g$$

$$\hookrightarrow \sqrt{\frac{v}{a}} = b\omega - c\dot{x}$$

$$\ddot{x} = v - g \qquad b\omega = \sqrt{\frac{v}{a}} + c\dot{x}$$

$$\omega = \frac{1}{b} \cdot \sqrt{\frac{\ddot{x}+g}{a}} + \frac{c}{b}\dot{x}$$

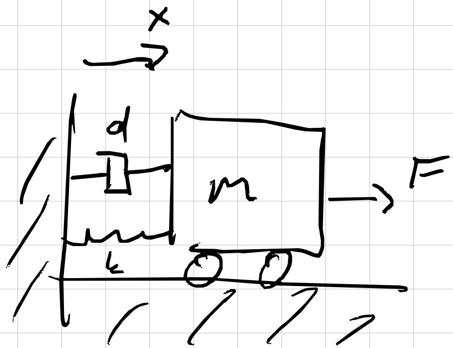
Assume: x is flat output

$$v = \ddot{x} + g$$

$$\frac{d}{dt} \quad \dot{\omega} = \frac{1}{2} \cdot \frac{1}{b} \cdot \frac{1}{\sqrt{\frac{\ddot{x}+g}{a}}} \cdot \frac{1}{a} \cdot x^{(3)} + \frac{c}{b}\ddot{x}$$

$n = 3$

↪ planed trajectory is of 3rd order



$$\ddot{x} = -\frac{d}{m}\dot{x} - \frac{k}{m}x + \frac{1}{m}F$$

$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{d}{m} \end{bmatrix} \cdot x + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \vec{x}$$

char. Poly.: $\quad s^2 + \underset{a_1}{10}s + \underset{a_0}{1}$

Wish: $(s+10)^2 = s^2 + \underset{\hat{a}_1}{20}s + \underset{\hat{a}_0}{100}$

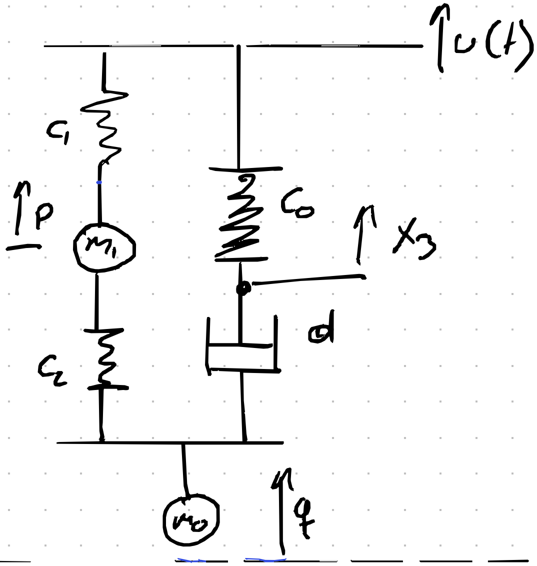
$$k_1 = \hat{a}_0 - a_0 = 99$$

$$k_2 = \hat{a}_1 - a_1 = 10$$

$$u = -k^T \cdot \vec{x}$$

Zero Dynamics

Beispiel Lunze I S. 190
Bsp. 5.10



1) $\ddot{p} \cdot m_1 = c_1 \cdot (p + u) + c_2 \cdot (q + p)$

2) $\ddot{q} \cdot m_0 = c_2 \cdot (p - q) + c_0 \cdot (u - x_3)$

3) $0 = c_0 \cdot (u - x_3) + d \cdot (\dot{q} - x_3)$

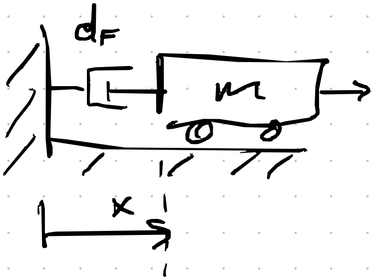
$c_0 = 1\ N/m$ $m_{0,1} = 1\ kg$

$c_1 = 0.75\ N/m$ no gravity

$c_2 = 0.4\ N/m$

$d = 3\ Ns/m$

Observer (in class)



$m\ddot{x} = F$ ⇐ model we use for design

$m\ddot{x} + d_F\dot{x} = F$ ⇐ model we don't know and use for simulation

$$\dot{\vec{x}} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\vec{x} + \begin{bmatrix}0\\ \frac{1}{m}\end{bmatrix} u$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\vec{x}$$

State feedback:

$u = -\vec{k}^T \vec{x}$

feedback gain

>>acker(A,b,s)   s... desired Eigenvalues

$\boxed{\text{Observer is needed because not all states are measured}}$

Model copy
$$\dot{\hat{\vec{x}}} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\hat{\vec{x}} + \begin{bmatrix}0\\ \frac{1}{m}\end{bmatrix} u + L^T \cdot \tilde{y} \qquad \tilde{y} = y - \hat{y}$$

observer gain

$L^T = \text{acker}(A^T, c^T, s)$

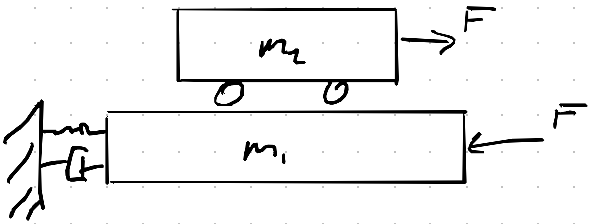
$$\hat{y} = \begin{bmatrix}1 & 0\end{bmatrix}\hat{\vec{x}}$$

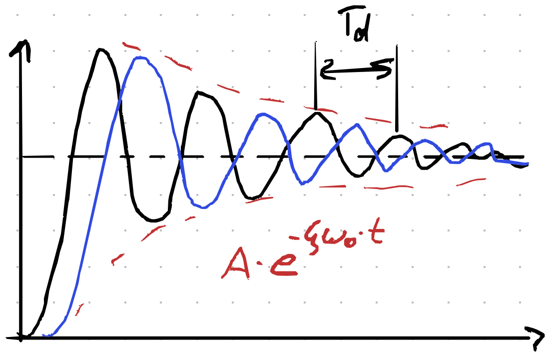
two inputs

$$\vec{b}_{new} = \begin{bmatrix}\vec{b}, -\vec{L}\end{bmatrix} \cdot \begin{bmatrix}u\\ \tilde{y}\end{bmatrix}$$ ⇐ two inputs

minus !

Filter





$$\ddot{x} + 2\xi\omega_0 \cdot \dot{x} + \omega_0 \cdot x = F$$

$$T_d = \frac{1}{f_d} = \frac{2\pi}{\omega_d} \qquad \omega_d = \omega_0 \cdot \sqrt{1-\xi^2}$$

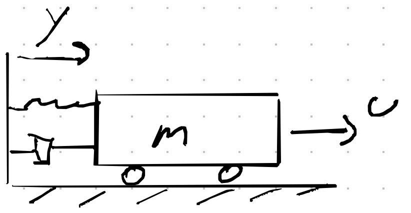
$$G(s) = \frac{1}{s^2 + 2\xi\omega_0 \cdot s + \omega_0^2}$$

Poles: $s_{1,2} = -\xi\omega_0 \pm \sqrt{\xi^2\omega_0^2 - \omega_0^2}$

$$s_{1,2} = -\xi\omega_0 \pm \omega_0\sqrt{\xi^2 - 1}$$

$$s_{1,2} = -\xi\omega_0 \pm j\,\omega_0\sqrt{1-\xi^2}$$

State Feedback Controll



$d = 10\ \mathrm{N/m/s} \qquad k = 5\ \mathrm{N/m}$

$m = 1\ \mathrm{kg}$

$$m \cdot \ddot{y} + d \cdot \dot{y} + k \cdot y = u \quad \rightarrow \quad G(s) = \frac{\overbrace{1/m}^{b_0}}{s^2 + \underbrace{\frac{d}{m}}_{a_1} \cdot s + \underbrace{\frac{k}{m}}_{a_0}}$$

$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{d}{m} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

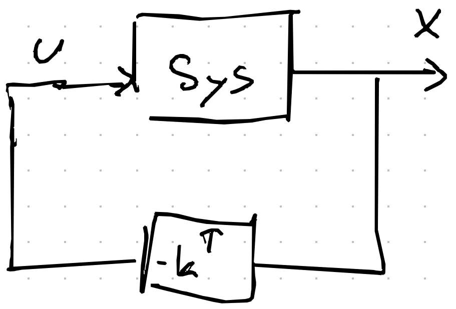
$$y = \begin{bmatrix} \frac{1}{m} & 0 \end{bmatrix} \vec{x}$$

Char. poly.: $0 = s^2 + \frac{d}{m} \cdot s + \frac{k}{m}$

$$0 = s^2 + 10s + 5 \quad \rightarrow \quad s_{1,2} = \frac{-10 \pm \sqrt{100 - 4 \cdot 5}}{2}$$

$$s_{1,2} = \frac{-10 \pm \sqrt{80}}{2} = \begin{cases} -0.53 \\ -9.47 \end{cases}$$

currently: $(s + 0.53) \cdot (s + 9.47)$



$$u = -k^T \cdot x$$

$$\dot{\vec{x}} = A \cdot \vec{x} - b \cdot k^T \cdot x$$

$$\dot{\vec{x}} = \underbrace{(A - b \cdot k^T)}_{\hat{A}} \cdot \vec{x}$$

$$\dot{\vec{x}} = \begin{bmatrix} 0 & 1 \\ \underbrace{-a_0 - k_1}_{-\hat{a}_0} & \underbrace{-a_1 - k_2}_{-\hat{a}_1} \end{bmatrix} \cdot \vec{x}$$

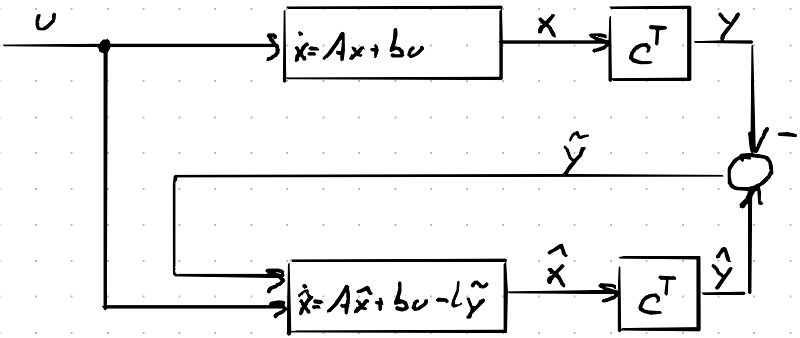
wish: $(s + 10) \cdot (s + 10) = s^2 + \underbrace{20}_{\hat{a}_1} s + \underbrace{100}_{\hat{a}_0}$

$$\left.\begin{aligned} \hat{a}_0 = a_0 + k_1 \rightarrow \quad & k_1 = \hat{a}_0 - a_0 = 100 - 5 = 95 \\ & k_2 = \hat{a}_1 - a_1 = 20 - 10 = 10 \end{aligned}\right\} k^T = \begin{bmatrix} 95 \\ 10 \end{bmatrix}$$

now up to Simulink

Observer

Block diagram



$$\dot{\hat{x}} = A \cdot \hat{x} + bu - L\tilde{y}$$