Advanced Control Engineering I: Ball in Tube

1. Laboratory

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Aim of the laboratory exercise

A position control for the ping-pong ball is investigated. This is done by introducing an upward air flow through a fan in an upright Plexiglas tube. With sufficient force, the air flow transports the ball upwards. The position of the ball is determined by means of a ToF-sensor (time of flight) attached to the tube. The aim of the experiment is to set a desired position for the float and to maintain this position even in the presence of disturbances. Furthermore, a trajectory should be planned to change the height of the ball smooth enough. The controller to be designed for this purpose is to be implemented in Matlab/Simulink and auto-code generated to a Beckhoff PLC. The implementation can be used for both, the test preparation (simulation) and directly on the test rig.

System description

A schematic representation of the test rig can be found in Figure 1. The system can be separated into two parts: On the one hand, the ball subsystem, which describes the dynamic behavior of the ball position $z(t)$ as a function of the air flow velocity $v(t)$ influenced by the fan speed $\omega(t)$ and, on the other hand, the fan subsystem, which characterizes the dynamic behavior of the flow velocity $v(t)$ as a function of the fan motor voltage $u(t)$.

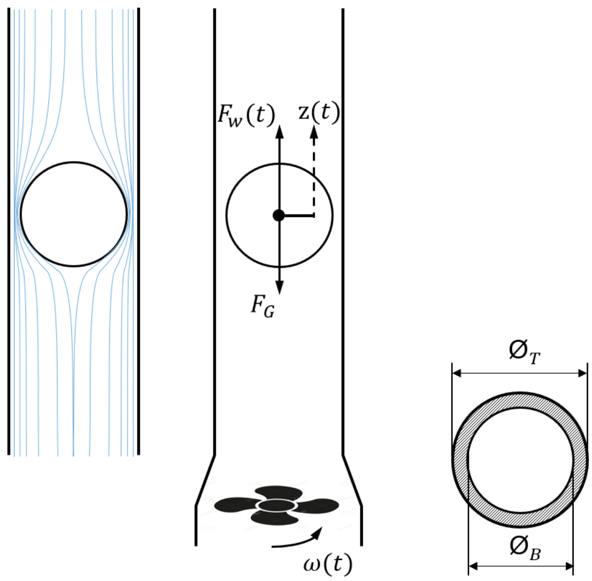

Fig. 1 – Schematic representation of the ball-in-tube test rig

All parameters for the simulation at home are listed in the following table:

Name	Symbol	Value	Unit
Area ball	A_B	13	cm^2
Mass of ball	m	2.7	g
Area tube	A_T	15	mm^2
Drag coefficient	k_L	2×10^{-5}	m^3
Air flow coefficient	k_V	1.27×10^{-4}	$\mathrm{kg\,m^{-1}}$
Fan time constant	τ_ω	1.6	s
Fan gain	k_ω	28	$\mathrm{s^{-1}\,V^{-1}}$
Density of air	ρ_L	1.225	$\mathrm{kg\,m^{-3}}$
Gravity constant	g	9.81	$\mathrm{m\,s^{-2}}$
Length of tube	h_{max}	0.4	m
max fan Voltage	u_{max}	5	V

Model of the ball

Essentially, the gravitational force F_g, the drag force $F_w(t)$ and the ball's own inertia act on the ball. Balance of forces in vertical direction yields

$$F_w(t) - F_g - m\,\ddot{z}(t) = 0\,.$$

The gravitational force F_g is determined by the mass m and the gravitational acceleration g to

$$F_g = m\,g\,.$$

The drag force $F_w(t)$ is made up of two components: The first results from the friction of the air flowing past the ball. The second is the result of the pressure difference between the top and bottom of the ball and is referred to as the pressure drag force. In this case of a so-called “voluminous body”, the frictional force can be neglected compared to the pressure drag force. The flow resistance force thus corresponds to the pressure resistance force and can be expressed as

$$F_w(t) = \frac{1}{2}\,c_W\,\rho_L\,A_B\,\tilde{v}^2(t)$$

Here, c_W corresponds to the drag coefficient, ρ_L to the density of the air, A_B to the cross-sectional area of the ball and $\tilde{v}(t)$ to the relative velocity between the ball and the medium flowing around it. The relative velocity $\tilde{v}(t) = v_{GAP}(t) - v_B(t)$ consists of the velocities $v_{GAP}(t)$ and $v_B(t)$ occurring in the air gap. Here, $v_{GAP}(t)$ describes the velocity of the air flow in the gap, which is generated by the fan. The velocity of the air flow generated by the movement of the ball in the air gap is indicated by $v_B(t)$. If we make the idealizing assumption that the density of the air in the duct and in the air gap is identical, the resulting constancy of the velocity $v_{GAP}(t)$ means that it is calculated as a function of the flow velocity $v(t)$ in the duct as follows

$$v_{GAP}(t) = \frac{A_T}{A_{GAP}}\,v(t)$$

with the inner pipe cross-sectional area A_T and the air gap cross-sectional area

$$A_{GAP} = A_T - A_B\,.$$

The speed of the air flow generated by the movement of the ball can be calculated analogously using the speed $\dot{z}(t)$ of the ball

$$v_B(t) = \frac{A_B}{A_{GAP}}\,\dot{z}(t)\,.$$

Substituting the speeds into the original equation of the drag force results in

$$F_w(t) = \frac{1}{2}\, c_W\, \rho_L\, A_B \left(\frac{A_T\, v(t) - A_B\, \dot{z}(t)}{A_{GAP}}\right)^2 .$$

As the volume flow $A_T\, v(t)$ of the fan is a variable that is difficult to measure, the absolute speed $\omega(t)$ of the fan is used instead. The relationship between the two is assumed to be proportional with the proportionality factor k_V, so that the following notation can also be used

$$F_w(t) = \frac{1}{2}\, c_W\, \rho_L\, A_B \left(\frac{k_V\, \omega(t) - A_B\, \dot{z}(t)}{A_{GAP}}\right)^2 .$$

Lets assume, that for the range of volume flow generated by the fan the drag coefficient is constant and thus, it is possible to introduce the constant variable $k_L = \frac{1}{2}\, c_W\, \rho_L\, A_B$ to simplify the overall model to

$$m\, \ddot{z}(t) = k_L \left(\frac{k_V\, \omega(t) - A_B\, \dot{z}(t)}{A_{GAP}}\right)^2 - m\, g\, .$$

Model of the fan

The equivalent circuit diagram shown in Figure 2 is used to model the fan motor.

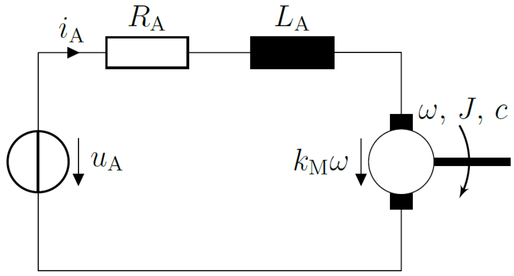

Fig. 2 – Schematic representation of the electrical subsystem

The mathematical model is given by

$$R\, i(t) + L\, \frac{d\, i(t)}{dt} + k_m\, \omega(t) = u(t).$$

The motor current $i(t)$ represents a state of the system and the motor voltage $u(t)$ is the system input. The input can be rewritten as a puls width modulated signal with the duty cycle $s(t) \in [0, 1]$:

$$u(t) = V_0\, s(t)$$

Furthermore, the mechanical equation of the fan is represented by

$$J\, \dot{\omega}(t) = k_m\, i(t) + b\, \omega(t)\, ,$$

with the moment of inertia J, the motor constant k_m and the viscose friction b. The output equation is the fan speed $\omega(t)$. Since for a fan the inductance can be neglected, the system can be simplified to

$$\frac{J\, R}{k_m}\, \dot{\omega}(t) - \frac{b\, R + k_m^2}{k_m}\, \omega(t) = V_0\, s(t)$$

Task: Simplify the mathematical model

Simplify the system by summing up the constant coefficients. Use the following notation:

$$\ddot{z}(t) = \left(\alpha_1\, \omega(t) - \alpha_2\, \dot{z}(t)\right)^2 - \alpha_3$$

and

$$\tau_\omega\, \dot{\omega}(t) + \omega(t) = k_\omega\, u(t)\,.$$

Think about, how it is possible to get the coefficients α_i, τ_ω and k_ω.
Note: You can measure the fan speed and the height of the ball. You can also use a scale, a ruler or similar things to get parameters.

Control design

Task: Linearize the non-linear state-space model

Linearize the non-linear state-space model in order to enable a more straightforward PID controller design. Choose an appropriate operating point (e.g. $\bar{z} = 15$ cm). Calculate the transfer function of the LTI-system.
Note: Do not forget about the PT1-behavior of the fan!

Task: PID control design

Use the linearized model and design a PID-controller with the Matlab-tool "pidTuner" for controlling the ball height in the tube. Choose the parameters wisely, since the system has constrains in the tube length as well as the fan speed.

Task: Simulate closed loop system

Simulate your closed-loop system in Simulink by using the **non-linear state-space model incl. fan model** as plant. Use parameters defined in a script to easily modify all parameters in the lab.

- Add a **saturation** at the voltage input.
- Add a **noise** at the measured signal (Band-Limited White Noise) and use a gain-block to get a max. amplitude of the noise of approx. 2 mm.
- Use a **output limitation** at the integrator, since the ball height can never get below zero and over the tube length (use therefore 35 cm in the simulations).

Repeat the PID control design, if troubles occur due to the saturation or the signal noise. Use a PT1-filter, if necessary, to reduce the output noise (e.g. with $\tau_y = 1$ s).
Hint: You did a linearization, so be aware of adding the operating point at the input. Furthermore, the initial states of the integrator can be set correctly. Before starting controlling the system, check whether the steady state of the operating point is really a steady state. If not, fix the problems.

Task: Flatness based feed forward

With a little experience, it is obvious that the height of the ball $z(t)$ is a flat output of the nonlinear system - explain why.
Calculate the input by means of the flat output. How often must a trajectory be continuously differentiable so that it can be used as reference for the flat output?
Calculate a polynomial transition (as introduced in the exercises) for the flat output and calculate the reference trajectory as well as the feed-forward signal. Choose the step size and the transition time wisely. Add your trajectory to the simulation and observe the improvement.

Task: Prepare for lab

Read the instructions on what to do in the lab, because time in the lab is short.

- How to calibrate the time-of-flight sensor?
- How to get the system constants α_i, τ_ω and k_ω by measurements?
  Note that m, A_B, A_T, ρ_L and g are well known and can be used if necessary.
- Implement your control algorithm to the pre-defined Matlab/Simulink files for the lab. Don't change the input and output names, because they are used in the Beckhoff environment.

Implementation in the lab

Implement the designed control system on the real hardware using Simulink Coder and the provided script for integration into Beckhoff. Test the behavior, if needed, get new parameters and redesign the controller / feed-forward.

Report

The report shall follow the usual MCI Guidelines adopted for the documentation of laboratory activities and shall be as short as possible, although including all the relevant information. The following is a proposed structure of the content of the report that is commonly adopted either in scientific papers as well as thesis and that can be easily customized and adapted to the specific laboratory activity:

1. Definition of the learning target of the laboratory experiment and description of the structure of the document
2. Summary of theoretical aspects addressed both in preparatory work as well as in class activities (among others: state linearization, parameter identification, . . .)
3. Description of laboratory setup

4. Simulations (Remark: this chapter contains mainly the block diagrams and relevant comments related to the implemented simulations)

5. Results and interpretation (Remark: in this chapter the measurements shall be compared with results out of the simulations possibly showing simulation and measurements on the same plots or with side-by-side figures)

6. Conclusions: summarize the main achievements and problems/solutions encountered during the laboratory experience

Please mark on the cover page at least the following information:

- Title of the activity
- Date of the lab
- Group members

Create a .zip-file containing the report in .pdf-format and a folder with simulations and measurements. Only this .zip-file has to be submitted in digital form on SAKAI Assignments (please upload one copy per group only) within the date reported on the announcements on SAKAI.