Advanced Control II

1. Exercise

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Crane Control

Objective

Develop a mathematical model of a crane system consisting of a linear drive and a pendulum-mounted load. The goal is to design a feed-forward control strategy to transition the load smoothly between steady-state positions while considering realistic system dynamics, measurement uncertainties, and parameter variations.

System Description

The crane model consists of:

- A linear drive elastically mounted to the inertial frame, exhibiting internal dynamics characterized by a mass-spring-damper system.
- A pendulum of length ℓ and mass m_{load} mounted on the moving cart.
- Measured variables: The angle of the pendulum $\theta(t)$ and the cart position $z(t)$ with respect to the base of the linear drive.

Tasks

- Develop a mathematical model of the system, incorporating the linear mass-spring-damper characteristics and the pendulum dynamics:

  Consider a linear drive with a total length of $z_{cart,max} = 2\,\mathrm{m}$. The mass of the linear drive is given by $m_{base} = 20\,\mathrm{kg}$. The cart mass is considered to be $m_{cart} = 2\,\mathrm{kg}$. Without pendulum, a acceleration of the cart of $\ddot{z}(t) = 10\,\mathrm{m\,s^{-2}}$ shows an amplitude on the base deflection of $z_{base} = 10\,\mathrm{mm}$. The internal dynamics follows a damped oscillation with a frequency of $f_{base} = 4\,\mathrm{Hz}$ that disappears within $T = 25\,\mathrm{s}$. The pendulum has a length of $\ell = 25\,\mathrm{cm}$ and a load with mass $m_{load} = 1\,\mathrm{kg}$.

- Design a feed-forward control strategy to transition the crane's load smoothly between steady-state positions.
- Implement an Extended Kalman Filter (EKF) for state estimation to compensate for noisy and imperfect measurements.
- Design and apply a Linear Quadratic Regulator (LQR) to control the system effectively.
- Simulate additional realistic factors, including:

- Friction in the linear drive mechanism. Choose a value wisely and explain why selected that friction.
- Noise in the measured signals. Position accuracy is given by 1 mm. Angle noise with an amplitude of $2\,\frac{360^\circ}{1024}$.
- Quantization effects from digital processing. Angle is given in discrete steps of $\frac{360^\circ}{1024}$.
- Parameter variations affecting system behavior. In detail, the mass and damping ratio of the base are not well known, thus a variation of up to $\pm 10\%$ should be analyzed.

Expected Outcomes

By the end of the project, students will have:

- A fully developed mathematical model describing the dynamic behavior of the crane system.
- An optimized control strategy that ensures smooth motion of the load.
- A simulation environment demonstrating the system's response to various real-world imperfections.
- Insights into control engineering techniques, sensor limitations, and system robustness.

Conclusion

This project integrates mathematical modeling, control design, and simulation to provide students with a hands-on learning experience in dynamic system analysis and control engineering.

Localization of a Mobile Robots

Objective

Develop a mathematical model of a differential wheeled mobile robot. The goal is to design a sensor fusion strategy to estimate the position of the mobile robot with a higher sampling rate than given by GNSS-localization while considering realistic system dynamics, measurement uncertainties, and parameter variations. Furthermore, the heading (orientation) of a mobile robot can not be measured precisely, but is necessary e.g. for docking or for driving meander lines. Thus, also the heading of the rover should be an output of the sensor-fusion algorithm.

System Description

The mobile robot consists of:

- Two separately actuated wheels (differential drive). As input parameters the two wheel speeds should be used.
- Measured variables: The gear-rate is measured by a gyroscope, the wheel-ticks of both wheels are measured by encoders and the robot position is measured by means of GNSS-RTK.

Tasks

- Explain, how to calculate the forward velocity $v(t)$ and the rotation speed ω by means of the wheel speed and geometric constraints.
- Develop a mathematical model of the system:

  Consider a mobile robot with wheel distance of $\ell = 50\,\mathrm{cm}$, wheel diameter of $d = 20\,\mathrm{cm}$ and a total robot mass of $m_{rob} = 1\,\mathrm{kg}$.
- Design a feed-forward control strategy to drive the robot in a straight line for $10\,\mathrm{m}$, stop, rotate the robot by $135°$, drive $5\,\mathrm{m}$ and finally make a full clockwise circle with radius of $2\,\mathrm{m}$.
- Simulate additional realistic factors, including:
  - Slip / dirt on wheels $\rightarrow$ this can be simulated by a change of the wheel diameter.
  - Gyroscope bias
  - Noise in the measured signals. Position accuracy by means of the GNSS-STK system is given by $5\,\mathrm{cm}$ in a sampling rate of $f_{GNSS} = 4\,\mathrm{Hz}$. Gyro noise with an amplitude of $0.1\,{}^\circ\,\mathrm{s}^{-1}$.
  - Quantization effects from digital processing. Wheel-ticks are given in discrete steps of $\frac{360°}{4096}$.
- Implement an Extended Kalman Filter (EKF) for state estimation and sensor fusion to compensate for noisy and imperfect measurements. The heading should be known as fast as possible.

Expected Outcomes

By the end of the project, students will have:

- A fully developed mathematical model describing the dynamic behavior of a mobile robot.
- A sensor fusion to get the heading of the rover as well as a detailed localization estimation.
- A simulation environment demonstrating the system's response to various real-world imperfections.

Conclusion

This project integrates mathematical modeling, control design, and simulation to provide students with a hands-on learning experience in dynamic system analysis and control engineering.

SoC Estimation of a Li-Ion Battery Module

Objective

Develop a mathematical model of a Li-Ion Battery Cell. The goal is to design an observer to estimate the SoC-level of the battery by means of an extended Kalman-filter while considering realistic system dynamics, measurement uncertainties, and parameter variations.

System Description

The Li-Ion battery cell model consists of:

- A inner resistor R_0
- Two parallel RC-elements in series, one representing the diffusion (dif) and one the charge transfer (ct)
- A nonlinear behavior of the open circuit voltage due to the State of charge (SoC).
- Input is the current, output the voltage
- Measured variables: The output voltage as well as the input current.

Tasks

- Develop a mathematical model of the system:

  Consider a Li-Ion battery module given by 4 parallel cells with a rated capacity value of $Q_{bat} = 2.6\,\mathrm{A\,h}$, e.g. in e-bikes 10 of those modules are used in series which is also called 10s4p structure. The resistors are given by $R_{ct} = 1.6\,\mathrm{m\Omega}$, $R_{dif} = 7.7\,\mathrm{m\Omega}$, the time constants are given by $\tau_{ct} = R_{ct}\,C_{ct} = 3.68\,\mathrm{s}$, $\tau_{dif} = R_{dif}\,C_{dif} = 84.34\,\mathrm{s}$ and the inner resistor is given by $R_0 = 6\,\mathrm{m\Omega}$.

- The open circuit voltage V_{OCV} is given as a function of the SoC and and should show a similar curve to Fig. 1.
- Implement an Extended Kalman Filter (EKF) for state and parameter estimation to compensate for noisy and imperfect measurements. Parameters of interest are the battery cypacity Q_{bat}, the SoC as well as the inner resistor R_0.
- Simulate additional realistic factors, including:
  - Noise in the measured signals. Voltage accuracy is given by $5\,\mathrm{mV}$.
  - Parameter variations affecting system behavior. In detail, the resistors are not well known, thus a variation of up to $\pm 10\%$ should be analyzed.

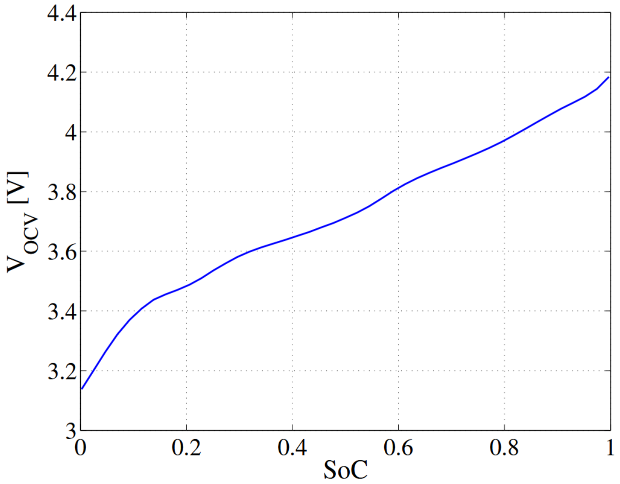

Fig. 1 – Open circuit voltage with respect to state-of-charge.

Expected Outcomes

By the end of the project, students will have:

- A fully developed mathematical model describing the dynamic behavior of a Li-Ion battery module.
- An optimized observer to get the information of the inner resistor, the battery module load as well as the state-of-charge.

Conclusion

This project integrates mathematical modeling, control design, and simulation to provide students with a hands-on learning experience in dynamic system analysis and control engineering.