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|---|-----------------------------|------------------------|---------------------------------------|
| <b>MODULE TITLE</b>                                   | <b>Stochastic Processes</b> | <b>CREDIT VALUE</b>    | <b>15</b>                             |
| <b>MODULE CODE</b>                                    | <b>MTH3024</b>              | <b>MODULE CONVENER</b> | <b>Dr Kyle Wedgwood (Coordinator)</b> |
| <b>DURATION: TERM</b>                                 | <b>1</b>                    | <b>2</b>               | <b>3</b>                              |
| <b>DURATION: WEEKS</b>                                | <b>0</b>                    | <b>11 weeks</b>        | <b>0</b>                              |
| <b>Number of Students Taking Module (anticipated)</b> |                             | <b>100</b>             |                                       |

#### DESCRIPTION - summary of the module content

A stochastic process is one that involves random variables. A large number of practical systems within industry, commerce, finance, biology, nuclear physics and epidemiology can be described as stochastic and analysed using the techniques developed in this module. The systems considered may exist in any one of a finite or possibly countably infinite, number of states. The state of a system may be examined continuously through time or at fixed and regular intervals of time.

You will study processes whose changes of state through time are governed by probabilistic laws, and you will learn how models of such processes can be applied in practice. Module MTH1004 Probability, Statistics and Data is an essential prerequisite, while MTH2006 Statistical Modelling & Inference is desirable.

Prerequisite module: MTH1004 Probability, Statistics and Data or equivalent.

#### AIMS - intentions of the module

The probability models considered in this module have a common thread running through them: that the behaviour of the system under consideration depends only on the state of the system at a particular point in time and a probabilistic description of how the state of the system may change from one point in time to the next. The systems considered may exist in any one of a finite (or possibly countably infinite) number of possible states and the state of the system may be examined continuously through time or at fixed (and regular) intervals of time. A large number of practical systems within industry, commerce, finance, biology, nuclear physics and epidemiology, can be described and analysed using the techniques developed in this module.

#### INTENDED LEARNING OUTCOMES (ILOs) (see assessment section below for how ILOs will be assessed)

On successful completion of this module you should be able to:

##### Module Specific Skills and Knowledge:

1. Demonstrate enhanced methodologies for tackling probabilistic problems;
2. Show awareness of a number of processes and systems whose behaviour through time are governed by probabilistic laws;
3. Construct and apply models describing that behaviour.

##### Discipline Specific Skills and Knowledge

4. Exhibit familiarity with the concept of random behaviour and the facility to analyse queues - skills which will be applied in later modules;
5. Display enhanced facility with the fundamental mathematical techniques of finite and infinite summation, and of differential and integral calculus.

##### Personal and Key Transferable / Employment Skills and Knowledge:

6. Reveal enhanced analytical skills, numerical skills, reasoning skills, problem-solving skills, time-management skills and facility to understand complex and abstract ideas.

#### SYLLABUS PLAN - summary of the structure and academic content of the module

- Probability generating functions (PGFs): definition, basic properties and illustrative examples of PGFs;
- moments of random sums of random variables;
- branching processes: definition, PGF and moments of the population in generation  $n$  of a branching process;
- probability of ultimate extinction;
- stochastic size of original population;
- Poisson processes: definition;
- memoryless property;
- Erlang distribution of time to the  $n$ th event;
- Poisson distribution of number of events in a given period of time;
- binomial distribution of number  $r$  of events in  $t$  given  $n$  in  $T$ ;
- beta distribution of time  $t$  to  $r$ th event given  $n$  events in  $T$ ;
- combining and decomposing independent Poisson processes;
- queueing theory: differential equations for the transient behaviour of models with state-dependent Markov arrival and departure processes;
- derivation of the steady state behaviour of this model;
- existence conditions for steady state;
- specific queueing models: fixed arrival rate, finite source population, customer baulking behaviour, one or more servers, finite system capacity, non-queueing systems which can be modelled as queues;
- mean number of customers in the system/queueing;
- mean time spent in the system/queueing;
- statement and proof of Little's formula;
- distribution of time spent in system/queueing given first come first served;
- Markov processes: Markov property;
- time homogeneity;
- stochastic matrices;
- Chapman-Kolmogorov equations;
- classification of states: accessible, communicating, transient, recurrent, periodic, aperiodic;
- Ergodic Markov chains;
- renewal theorem;
- mean recurrence time;
- necessary/sufficient conditions for the system to tend to a steady state;
- random walks: definition of a random walk with absorbing/reflecting/elastic barriers;
- statement of, solution for and mean time to finish for the Gambler's Ruin problem.

#### LEARNING AND TEACHING

##### LEARNING ACTIVITIES AND TEACHING METHODS (given in hours of study time)

|   |              |                                 |               |                                 |
|---|--------------|---------------------------------|---------------|---------------------------------|
| <b>Scheduled Learning &amp; Teaching Activities</b> | <b>33.00</b> | <b>Guided Independent Study</b> | <b>117.00</b> | <b>Placement / Study Abroad</b> |
|---|--------------|---------------------------------|---------------|---------------------------------|

##### DETAILS OF LEARNING ACTIVITIES AND TEACHING METHODS

| Category | Hours of study time | Description |
|----------|---------------------|-------------|
|----------|---------------------|-------------|

Category  
Scheduled learning and teaching activities  
Guided independent study

Hours of study time  
33  
117

Description  
Lectures/example classes  
Guided independent study

## ASSESSMENT

### FORMATIVE ASSESSMENT - for feedback and development purposes; does not count towards module grade

| Form of Assessment          | Size of Assessment (e.g. duration/length) | ILOs Assessed | Feedback Method   |
|-----------------------------|---|---------------|---|
| Coursework – example sheets |   | All           | Tutorial sessions during lectures/office hours, written feedback on work. |

### SUMMATIVE ASSESSMENT (% of credit)

|            |    |               |    |                 |
|------------|----|---------------|----|-----------------|
| Coursework | 20 | Written Exams | 80 | Practical Exams |
|------------|----|---------------|----|-----------------|

### DETAILS OF SUMMATIVE ASSESSMENT

| Form of Assessment          | % of Credit | Size of Assessment (e.g. duration/length) | ILOs Assessed | Feedback Method           |
|-----------------------------|-------------|---|---------------|---------------------------|
| Coursework – problem sheets | 20          |   | All           | Written                   |
| Written exam – closed book  | 80          | 2 hours                                   | All           | Written/verbal on request |

### DETAILS OF RE-ASSESSMENT (where required by referral or deferral)

| Original Form of Assessment | Form of Re-assessment | ILOs Re-assessed | Time Scale for Re-reassessment |
|-----------------------------|-----------------------|------------------|--------------------------------|
| All above                   | Written exam (100%)   | All              | August Ref/Def period          |

### RE-ASSESSMENT NOTES

If a module is normally assessed entirely by coursework, all referred/deferred assessments will normally be by assignment.

If a module is normally assessed by examination or examination plus coursework, referred and deferred assessment will normally be by examination. For referrals, only the examination will count, a mark of 40% being awarded if the examination is passed. For deferrals, candidates will be awarded the higher of the deferred examination mark or the deferred examination mark combined with the original coursework mark.

## RESOURCES

### INDICATIVE LEARNING RESOURCES - The following list is offered as an indication of the type & level of information that you are expected to consult. Further guidance will be provided by the Module Convener

ELE – <http://vle.exeter.ac.uk>

#### Reading list for this module:

| Type | Author                  | Title  | Edition | Publisher | Year | ISBN              | Search                    |
|------|-------------------------|--|---------|-----------|------|-------------------|---------------------------|
| Set  | Jones P.W. and Smith P. | Stochastic Processes: methods and applications |         | Arnold    | 2001 | 000-0-340-80654-0 | <a href="#">[Library]</a> |
| Set  | Ross, Sheldon M         | Introduction to Probability Models             | 10th    | Elsevier  | 2010 | 978-0123756862    | <a href="#">[Library]</a> |

|              |    |            |     |
|--------------|----|------------|-----|
| CREDIT VALUE | 15 | ECTS VALUE | 7.5 |
|--------------|----|------------|-----|

|                       |         |
|-----------------------|---------|
| PRE-REQUISITE MODULES | MTH1004 |
|-----------------------|---------|

#### CO-REQUISITE MODULES

|                  |   |                                |    |
|------------------|---|--------------------------------|----|
| NQF LEVEL (FHEQ) | 6 | AVAILABLE AS DISTANCE LEARNING | No |
|------------------|---|--------------------------------|----|

|             |                      |                    |                           |
|-------------|----------------------|--------------------|---------------------------|
| ORIGIN DATE | Tuesday 10 July 2018 | LAST REVISION DATE | Thursday 16 December 2021 |
|-------------|----------------------|--------------------|---------------------------|

|                  |   |
|------------------|---|
| KEY WORDS SEARCH | Scochastic processes; probability models; Markov process. |
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