LISTA 01

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Questão 1.

Integral numérica.

$$\mathcal{I} = \int_2^3 (\cos(x) - \ln(x))\, dx$$

(a) Integração direta (método das fitas).

Método:

$$\begin{aligned} &N_i = I_{\max} + 1 \\ &i \in \mathbb{Z} \\ &i \in [0, I_{\max}] \\ &\Delta x = \frac{b-a}{N_i} \end{aligned}$$

$$\int_a^b f(x)\, dx \approx \Delta x \sum_{i=0}^{I_{\max}} f(a + (i + \tfrac{1}{2})\Delta x)$$

Resolução:
$N_i = 5$; $I_{\max} = 4$
$a = 2$; $b = 3$
$\Delta x = \frac{3-2}{5} = \frac{1}{5} = \mathbf{0.2}$
$f(x) = \cos(x) - \ln(x)$

$$\begin{aligned} \mathcal{I} \approx &0.2 \cdot [f(2 + (\tfrac{1}{2}) \cdot 0.2) + f(2 + (1\tfrac{1}{2}) \cdot 0.2) + f(2 + (2\tfrac{1}{2}) \cdot 0.2) + \\ &f(2 + (3\tfrac{1}{2}) \cdot 0.2) + f(2 + (4\tfrac{1}{2}) \cdot 0.2)] \\ \\ \approx &0.2 \cdot [f(2.1) + f(2.3) + f(2.5) + f(2.7) + f(2.9)] \\ \approx &0.2 \cdot [-1.24678 - 1.49919 - 1.71743 - 1.89732 - 2.03567] \\ \mathcal{I} \approx &-1.679278 \end{aligned}$$

(b) Integração por parábolas, intervalo $2 \leq x \leq 3$, 6 pontos.

Método:

$$\begin{aligned} &N_i = I_{\max} \\ &i \in \mathbb{Z} \\ &i \in [0, I_{\max}] \\ &\Delta x = \frac{b-a}{N_i} \end{aligned}$$

$$f(x) \approx y = Ax^2 + Bx + C \therefore \begin{cases} Ax_0^2 + Bx_0 + C = f(x_0) \\ Ax_1^2 + Bx_1 + C = f(x_1) \\ Ax_2^2 + Bx_2 + C = f(x_2) \end{cases}$$

$$\int_{x_0}^{x_2} f(x)\,dx \approx \int_{x_0}^{x_2} y\,dx = \left[A \cdot \frac{x^3}{3} + B \cdot \frac{x^2}{2} + C \cdot x\right]_{x_0}^{x_2}$$

Resolução:
$N_i = 5$; $I_{\max} = 5$
$a = 2$; $b = 3$
$\Delta x = \frac{3-2}{5} = \frac{1}{5} = \mathbf{0.2}$
$f(x) = \cos(x) - \ln(x)$

$$\begin{aligned} x_0 &= 2.0 & f(x_0) &= -1.10929 \\ x_1 &= 2.2 & f(x_1) &= -1.37695 \\ x_2 &= 2.4 & f(x_2) &= -1.61286 \\ x_3 &= 2.6 & f(x_3) &= -1.81240 \\ x_4 &= 2.8 & f(x_4) &= -1.97184 \\ x_5 &= 3.0 & f(x_5) &= -2.08860 \end{aligned}$$

$$\begin{cases} A_1x_0^2 + B_1x_0 + C_1 = f(x_0) \\ A_1x_1^2 + B_1x_1 + C_1 = f(x_1) \\ A_1x_2^2 + B_1x_2 + C_1 = f(x_2) \end{cases} \rightarrow \begin{cases} A_1(2.0)^2 + B_1(2.0) + C_1 = -1.10929 \\ A_1(2.2)^2 + B_1(2.2) + C_1 = -1.37695 \\ A_1(2.4)^2 + B_1(2.4) + C_1 = -1.61286 \end{cases} \rightarrow \begin{cases} A_1 &= 0.39701 \\ B_1 &= -3.00575 \\ C_1 &= 3.31418 \end{cases}$$

$$\begin{cases} A_2x_2^2 + B_2x_2 + C_2 = f(x_2) \\ A_2x_3^2 + B_2x_3 + C_2 = f(x_3) \\ A_2x_4^2 + B_2x_4 + C_2 = f(x_4) \end{cases} \rightarrow \begin{cases} A_2(2.4)^2 + B_2(2.4) + C_2 = -1.61286 \\ A_2(2.6)^2 + B_2(2.6) + C_2 = -1.81240 \\ A_2(2.8)^2 + B_2(2.8) + C_2 = -1.97184 \end{cases} \rightarrow \begin{cases} A_2 &= 0.50120 \\ B_2 &= -3.50370 \\ C_2 &= 3.90909 \end{cases}$$

$$\begin{cases} A_3x_3^2 + B_3x_3 + C_3 = f(x_3) \\ A_3x_4^2 + B_3x_4 + C_3 = f(x_4) \\ A_3x_5^2 + B_3x_5 + C_3 = f(x_5) \end{cases} \rightarrow \begin{cases} A_3(2.6)^2 + B_3(2.6) + C_3 = -1.81240 \\ A_3(2.8)^2 + B_3(2.8) + C_3 = -1.97184 \\ A_3(3.0)^2 + B_3(3.0) + C_3 = -1.61286 \end{cases} \rightarrow \begin{cases} A_3 & = 0.53348 \\ B_3 & = -3.67801 \\ C_3 & = 4.14409 \end{cases}$$

$$\boldsymbol{\mathcal{I}} \approx \left[A_1\frac{x^3}{3} + B_1\frac{x^2}{2} + C_1x\right]_{x_0}^{x_2} + \left[A_2\frac{x^3}{3} + B_2\frac{x^2}{2} + C_2x\right]_{x_2}^{x_4} + \left[A_3\frac{x^3}{3} + B_3\frac{x^2}{2} + C_3x\right]_{x_4}^{x_5}$$

$$\begin{aligned} &\approx [1.126875812964030 - 1.67554183832941] + \\ &\quad [0.878419503878535 - 1.60070650416725] + \\ &\quad [0.682555129628827 - 1.08931109280790] \\ \boldsymbol{\mathcal{I}} &\approx -1.67770898883318 \end{aligned}$$

Questão 2.

(a) Utilize série de Taylor para deduzir a aproximação:

$$\frac{df(x)}{dx} \approx \frac{Af(x+2\Delta x)+Bf(x+\Delta x)+Cf(x)+Df(x-1\Delta x)+Ef(x-2\Delta x)}{\Delta x}$$

Resolução:

$$\begin{aligned}
f(x+2\Delta x) &= f(x)+2\Delta x f^{(1)}(x)+4\Delta x^2\frac{f^{(2)}(x)}{2}+8\Delta x^3\frac{f^{(3)}(x)}{6}+16\Delta x^4\frac{f^{(4)}(x)}{24}+[\sim 0]\\
f(x+\Delta x) &= f(x)+\Delta x f^{(1)}(x)+\Delta x^2\frac{f^{(2)}(x)}{2}+\Delta x^3\frac{f^{(3)}(x)}{6}+\Delta x^4\frac{f^{(4)}(x)}{24}+[\sim 0]\\
f(x) &= f(x)\\
f(x-\Delta x) &= f(x)-\Delta x f^{(1)}(x)+\Delta x^2\frac{f^{(2)}(x)}{2}-\Delta x^3\frac{f^{(3)}(x)}{6}+\Delta x^4\frac{f^{(4)}(x)}{24}+[\sim 0]\\
f(x-2\Delta x) &= f(x)-2\Delta x f^{(1)}(x)+4\Delta x^2\frac{f^{(2)}(x)}{2}-8\Delta x^3\frac{f^{(3)}(x)}{6}+16\Delta x^4\frac{f^{(4)}(x)}{24}+[\sim 0]
\end{aligned}$$

Substituindo na equação:

$$\begin{aligned}
\Delta x\frac{df(x)}{dx} \approx\ & f(x)(A+B+C+D+E) & +\\
& +\Delta x f^{(1)}(x)(2A+B-D-2E) & +\\
& +\Delta x^2\frac{f^{(2)}(x)}{2}(4A+B+D+4E) & +\\
& +\Delta x^3\frac{f^{(3)}(x)}{6}(8A+B-D-8E) & +\\
& +\Delta x^4\frac{f^{(4)}(x)}{24}(16A+B+D+16E) & +
\end{aligned}$$

Para que isso seja verdadeiro o sistema deve ser satisfeito:

$$\begin{cases}
A+B+C+D+E &= 0\\
2A+B-D-2E &= 1\\
4A+B+D+4E &= 0\\
8A+B-D-8E &= 0\\
16A+B+D+16E &= 0
\end{cases}
\rightarrow
\begin{cases}
A &= -\frac{1}{12}\\
B &= \frac{8}{12}\\
C &= 0\\
D &= -\frac{8}{12}\\
E &= \frac{1}{12}
\end{cases}$$

A aproximação é:

$$\frac{df(x)}{dx} \approx \frac{-f(x+2\Delta x) + 8f(x+\Delta x) - 8f(x-1\Delta x) + f(x-2\Delta x)}{12\Delta x}$$

(b) Qual é a ordem da aproximação?

Quarta ordem pois a expansão da série de Taylor usou 5 termos.

Questão 3.

Calcular a série de Taylor para $f(x) = 1/x$, em torno de $x = 1$.

Fórmula:

$$f(x) = \sum_{n=0}^{\infty} \left[\frac{f^{(n)}(a)}{n!}(x-a)^n \right] \tag{1}$$

Resolução:

$$\begin{aligned}
f(x) =& \frac{f^{(0)}(1)}{0!}(x-1)^0 + \frac{f^{(1)}(1)}{1!}(x-1)^1 + \frac{f^{(2)}(1)}{2!}(x-1)^2 + \frac{f^{(3)}(1)}{3!}(x-1)^3 + \frac{f^{(4)}(1)}{4!}(x-1)^4 + \cdots \\
=& f^{(0)}(1)\frac{(x-1)^0}{0!} + f^{(1)}(1)\frac{(x-1)^1}{1!} + f^{(2)}(1)\frac{(x-1)^2}{2!} + f^{(3)}(1)\frac{(x-1)^3}{3!} + f^{(4)}(1)\frac{(x-1)^4}{4!} + \cdots \\
=& [x^{-1}]_{x=1}\frac{1}{1} + [(-1)x^{-2}]_{x=1}\frac{(x-1)^1}{1!} + [(-1)(-2)x^{-3}]_{x=1}\frac{(x-1)^2}{2!} + \\
&+ [(-1)(-2)(-3)x^{-4}]_{x=1}\frac{(x-1)^3}{3!} + [(-1)(-2)(-3)(-4)x^{-5}]_{x=1}\frac{(x-1)^4}{4!} + \cdots \\
=& [x^{-1}]_{x=1} + (1!)[-x^{-2}]_{x=1}\frac{(x-1)^1}{1!} + (2!)[x^{-3}]_{x=1}\frac{(x-1)^2}{2!} + \\
&+ (3!)[-x^{-4}]_{x=1}\frac{(x-1)^3}{3!} + (4!)[x^{-5}]_{x=1}\frac{(x-1)^4}{4!} + \cdots \\
=& [1] + [-1](x-1)^1 + [1](x-1)^2 + [-1](x-1)^3 + [1](x-1)^4 + \cdots \\
f(x) =& 1 + -(x-1)^1 + (x-1)^2 + -(x-1)^3 + (x-1)^4 + \cdots
\end{aligned}$$