Rigid Tether Modeling for Underwater Environment Simulation

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Abstract—Simulation in the field of robotics is a powerful tool. Indeed, it allows to easily and quickly test the robot in different conditions and to have a reproducibility of the results. Then it let us be able to create situations that would be difficult to find in reality, in order to make sure of the robot's behavior. It should be noticed that the simulation of robots does not replace tests in real conditions, but it remains practical during the development phase. However, the simulation of robots in the maritime environment is a field that still has shortcomings, especially when we want to simulate the tethers of submarine robots. Indeed, it is not easy to find an analytical way to simulate a tether that does not require large resources, especially when simulation environments become complex. This is why we will try to suggest an approach of simulation by discretization of the tether by proposing a behavioral model of force between each tether element. The results seem satisfactory and with a correct initialization of the position of the tether elements, the behavior of the tether seems quite right. However, the tether stiffness phenomena are not taken into account in this modeling, the tether is of fixed length, and both ends have been fixed so the system has not been tested dynamically.

Index Terms—Modeling, Tether, Remotely Operated Underwater Vehicle, Simulation.

I. INTRODUCTION

In the world of marine and underwater robotics we can indentify two categories of elements: mobile marine objects (MMO) and flexible tethers (FT). Mobile marine objects include surface vessels, submarines and remotely operated vehicles. Flexible tethers can represent umbilical cables, traction cables and anchor chains in the marine environment. They constitute all the necessary elements to achieve a mission in this environment which is sometimes quite difficult to explore.

The simulation of moving marine objects is something well known. We are now able to know from state equations the behavior of robots in their environment, and these equations are known for ships, submarines, sailboats, etc...

On the other hand, determining the behavior of a flexible tether becomes more complicated. Indeed, the equation of motion of these objects involves non-linear partial differential equations and the motion between the different objects in the environment are dynamically dependent. It is well illustrated that if the boat moves, it will induce a motion in the flexible tether that will modify the trajectory of the remotely operated vehicle. This is why it is difficult to find a way to model flexible tethers in underwater environments.

II. FORMALISM

The idea proposed in the scientific paper [1] is to solve this problem using finite element simulation. This implies that we
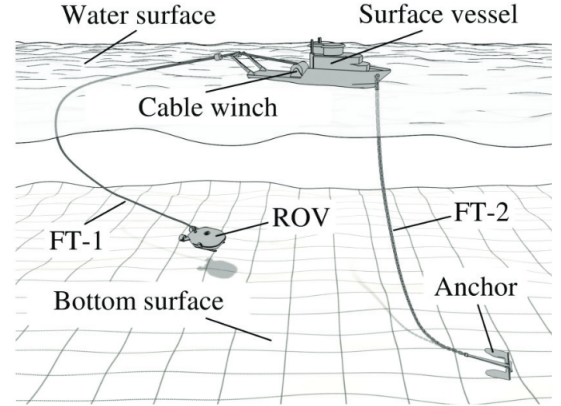

Fig. 1. Example of a complex underwater environment

need to discretize the tether in order to simulate its global behavior.

Suppose we want to simulate a tether of length L. We will then divide it into a finite number n of nodes connected by links. These links should be of length $l = \frac{L}{n-1}$ as the two nodes at the ends of the tether will not be connected to any other links.

We will focus here on the case where the first node and the last node are immobile, because otherwise we would have to simulate a mobile marine object that would be attached at the end, which is not the goal of our study.

Next, it is necessary to make a balance of the forces that apply to each tether element. For this simulation, we will take into account the weight, noted F_g, the buoyancy, noted F_b, and the force exerted by the previous element on the considered element, noted $F_{t,p}$, as well as that of the next element, noted $F_{t,n}$.

These forces will allow us to simply describe the behavior of the tether in its environment. Moreover we could then improve the quality of the simulation by adding other forces such as drag forces.

A. Weight F_g

Considering that each element has a mass m, we are able to express the weight that applies to this node :

$$\overrightarrow{F_g} = \begin{bmatrix} 0 \\ 0 \\ -m.g \end{bmatrix}$$

B. Buoyancy F_b

If we note the volume of each element V and ρ the density of the fluid in which the tether is immersed, we can also express the buoyancy force of this node:

$$\overrightarrow{F_b} = \begin{bmatrix} 0 \\ 0 \\ \rho.V.g \end{bmatrix}$$

C. Tether force $F_{t,p}$ and $F_{t,n}$

It is difficult to find an analytical form to describe these two forces. Therefore, we must find a way to describe them. This is why we will use a behavioral model here. We know that each node will have to be at a distance l from each of its neighbors. We can assume that the system behaves here as a three-dimensional damped mass-spring system and we will then consider that these forces are like elastic spring forces and viscous frictionnal forces.

By noting then p_p the position of the previous node and p_c the position of the current node, by introducing three coefficients K_p, K_d and K_i allowing to express the stiffness with which a node will correct its position with respect to its neighbors, we are able to express the behavioral model of these two forces:

$$\overrightarrow{F_t} = -\left(K_p \cdot e + K_d \cdot \dot{e} + K_i \cdot \int_0^t e(\tau) \cdot d\tau \right) \cdot \overrightarrow{u}$$

In these expressions, it is assumed that $\overrightarrow{u}$ is the unitary vector oriented from the current node to the neighboring node, $e(t)$ is the error of position between two nodes, $\dot{e}$ is the derivative of this error and $\int_0^t e(\tau) \cdot d\tau$ is the integral of this error. Both derivative and integral part will be estimated numerically respectively using the Euler's method and the rectangles method. We have therefore :

$$\overrightarrow{u} = \frac{\overrightarrow{p_c - p_p}}{\|p_c - p_p\|} \qquad e(t) = \frac{\|p_c - p_p\| - l}{\|p_c - p_p\|}$$

These two forces $F_{t,p}$ and $F_{t,n}$ can therefore be expressed using the expression of F_t, taking care to take the correct current and previous element for each case.

III. IMPLEMENTATION

The Python 3 implementation of this simulation is available in a GitHub repository. The goal of this simulator is to study the viability of such a system, in particular to validate the performance of the tether with this behavioral model.

So we will create a class *TetherElement* which will represent a node. It will have to contain its mass, volume and distance information from its neighbors, but also its position, velocity and acceleration, as well as a pointer to each of its two neighbors. Finally it is necessary to have each coefficient K_p, K_d and K_i to compute $F_{t,p}$ and $F_{t,n}$.

This will allow us later on to implement a *Tether* class to be able to simulate a tether. This object must have a length, a number of elements and a list containing the different *TetherElement* that compose it. It must also know the mass,
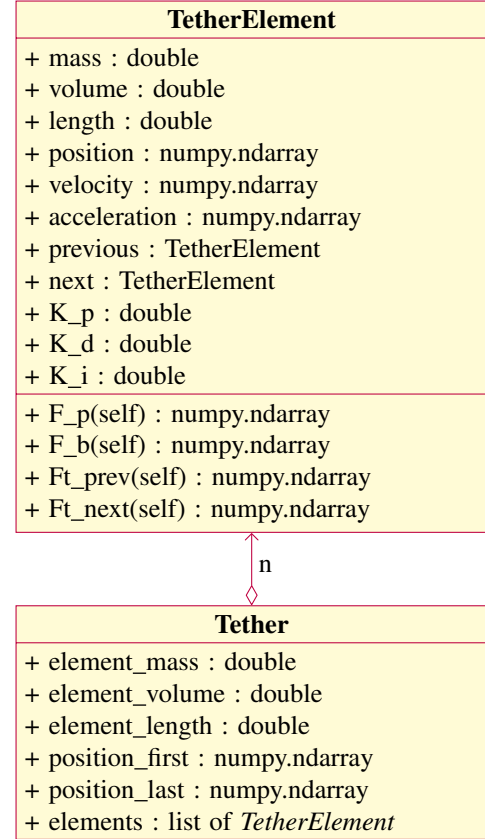

Fig. 2. UML diagram of the implementation

the volume of each node in order to correctly instantiate the *TetherElement*, but also the length of each link between nodes.

A diagram of these two classes is visible on the FIGURE 2. It respects the UML format and allows to see the different class variables and methods associated to each class.

IV. RESULTS

Cette section va prsenter les rsultats de la simulation. Pour rgler les diffrents coefficients de la simulation, nous allons avoir besoin de construire quelques outils nous permettant d'valuer les performances d'un tel modle.

A. Length of different links

It is useful to trace the length of the links over time in order to verify that the Tether's behavior is correct.

The FIGURE 3 shows in gray the plot of the length of each link for a simulation containing 15 nodes. In crimson is plotted the average of these lengths and in yellow the target length. Finally, the blue area shows the 95 % confidence interval.

As we can see, after setting the coefficients we are able to have a coherent behavior a few seconds after initialization. On average the link lengths converge towards the set length and the standard deviation is not too high. However, there are still some oscillations and not all links are exactly at the target length.
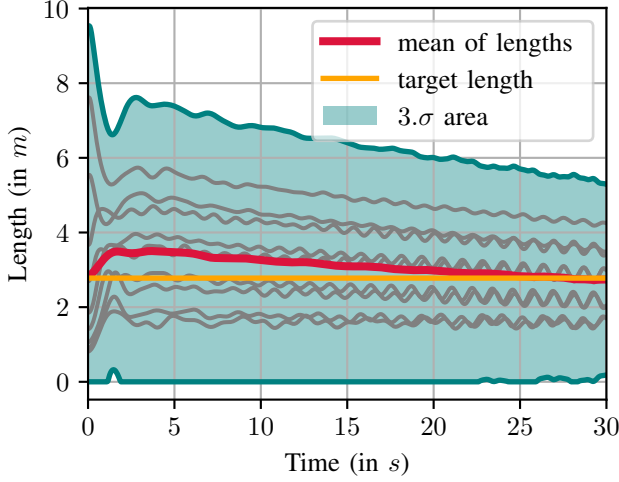
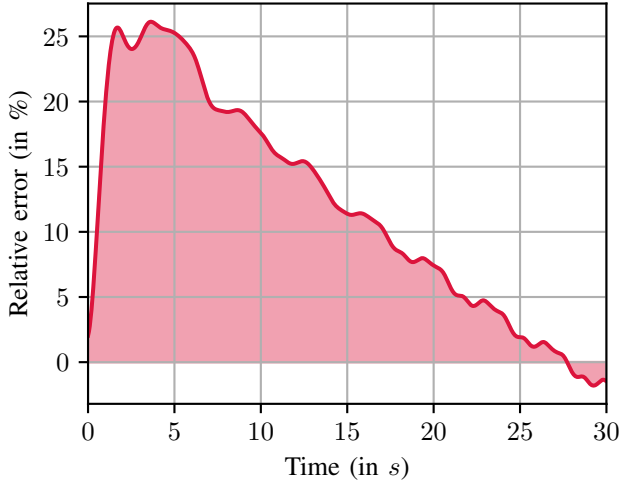Fig. 3. Length of the links, average length and confidence interval

Fig. 5. Kinetic energy of the system

Fig. 4. Relative error between the target length and the mean length of links

B. Relative error

The relative error is also very interesting because it makes it possible to be truly aware of the error but also to see more clearly when setting the coefficients of the behavioral model.

The FIGURE 4 shows us the average relative length error of the different links compared to the target length.

C. Energetical approach

Finally, the energy approach is the most important to validate the simulation, because it carries the physical meaning of the system. The system must not have a divergent energy, which would be a physical counter-sense, but here, as there is no energy source, the overall energy must decrease over time, as soon as there is some fluid friction due to the drag force.

1) Kinetic Energy: The global kinetic energy of the Tether is calculated simply by summing the kinetic energies of the different TetherElements.
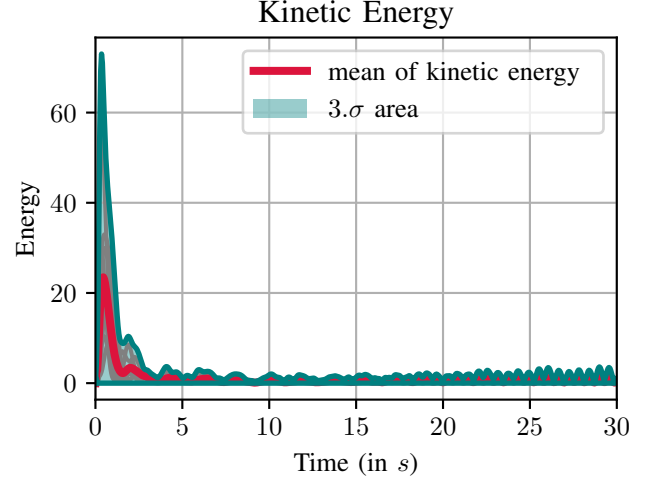
$$E_k = \sum_{i=0}^{N} \left(\frac{1}{2} \cdot m \cdot v_i^2 \right)$$

Where v_i is the velocity of the ith element.

2) Potential Energy: To calculate the potential energy related to the application of a force on a solid, it is necessary to return to the definition of potential energy. Indeed we know that by noting dt the time step of the simulation :

$$\delta W(\overrightarrow{F}) = \overrightarrow{F} \cdot \overrightarrow{dOM} = \overrightarrow{F} \cdot \overrightarrow{v} \cdot dt$$

$$E_p = \int_0^t \sum_{F_{ext}} \delta W(\overrightarrow{F_{ext}}) + cste$$

Thus we are able to calculate the potential energy by calculating the sum of the elementary work of each force on the system, i.e. the forces that apply to each node, and then by integrating this quantity over time. We then have an expression of the potential energy of the system over time for the Tether which is defined to within a constant. This constant is initially set to 0 in our case.

The FIGURE 6 shows us the evolution of potential energy over time. In gray is plotted the potential energy of each node, in crimson is the average potential energy and the blue area represents the 95 % confidence interval. We see that the potential energy of the system does not diverge, and that the system tends to position itself in a minimum of potential energy. This is something that has a strong physical meaning because all real systems tend to put themselves in positions that minimize their potential energy. This seems to validate the behavior of our modeling on this point.

3) Mechanical Energy: Finally, by summing the kinetic and potential energies previously calculated, we are able to define the mechanical energy of the system. This will give us information about the non-conservative forces that are included in this system. Indeed, the drag force will cause the
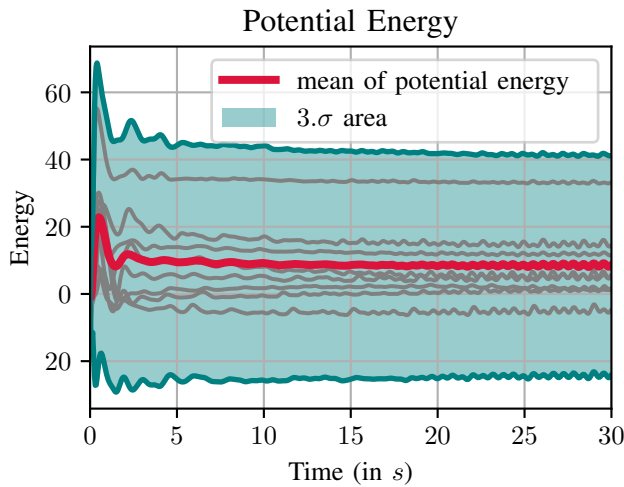
Potential Energy

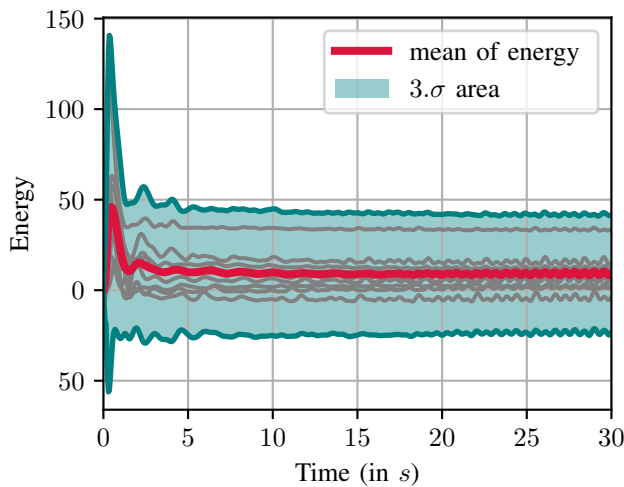Fig. 6. Potential energy of the system over the time

Fig. 7. Energy of the system

system to lose energy since this energy will not be transformed into another form that can be used by the system, but will be dissipated as heat.

The FIGURE 7 shows us the evolution of the mechanical energy over time. In gray is plotted the mechanical energy of each node, in crimson is the average mechanical energy and the blue area represents the 95 % confidence interval.

We can see that the energy of the system does not diverge, which seems to support the idea that the modeling is correct. Then, we notice that the system tends to minimize its energy, which is exactly the behavior expected for any system.

V. CONCLUSION

The conclusion goes here.

APPENDIX A
PROOF OF THE FIRST ZONKLAR EQUATION

Appendix one text here.

APPENDIX B

Appendix two text goes here.

ACKNOWLEDGMENT

REFERENCES

[1] H. Kopka and P. W. Daly, *A Guide to LaTeX*, 3rd ed.   Harlow, England: Addison-Wesley, 1999.

Michael Shell Biography text here.

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John Doe Biography text here.

Jane Doe Biography text here.