BLUP Best Linear Unbiased Prediction-Estimation

References

Searle, S.R. 1971 Linear Models, Wiley

Schaefer, L.R., Linear Models and Computer Strategies in Animal Breeding

Lynch and Walsh Chapter 26

OLS Independently and Identically Distributed Errors with Mean 0 and variance σ^2

$$V(\varepsilon) = E\left[\varepsilon - E(\varepsilon)\right]^2$$

The residual distribution from which each observation is sampled is the same

Inbreeding Changes These

$$V(\varepsilon) = \begin{bmatrix} \sigma_e^2 & 0 & 0 & 0 \\ 0 & \sigma_e^2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \sigma_e^2 \end{bmatrix}$$

Related Individuals Cause these to be nonzero

$$V(\varepsilon) = \mathbf{I}\sigma_e^2$$

Residuals independent

Solutions

- GLS
  - Fixes problem with changing variances and correlations in the data
- What about fixed effects?
  - How does one correct for
    - Environmental trend without a control
    - Herd effects
    - Year effects
    - Hatch effects

Confounding of data

- Herd effects
  - Balanced design no problem
  - Require sample of every family in every herd
  - Old solution was within herd deviations
  - What if better herds have better genetics
- Fixed effects must be adjusted for genetic differences
- Random effects must be adjusted for fixed effects

Simultaneous Adjustment of Fixed and Random effects

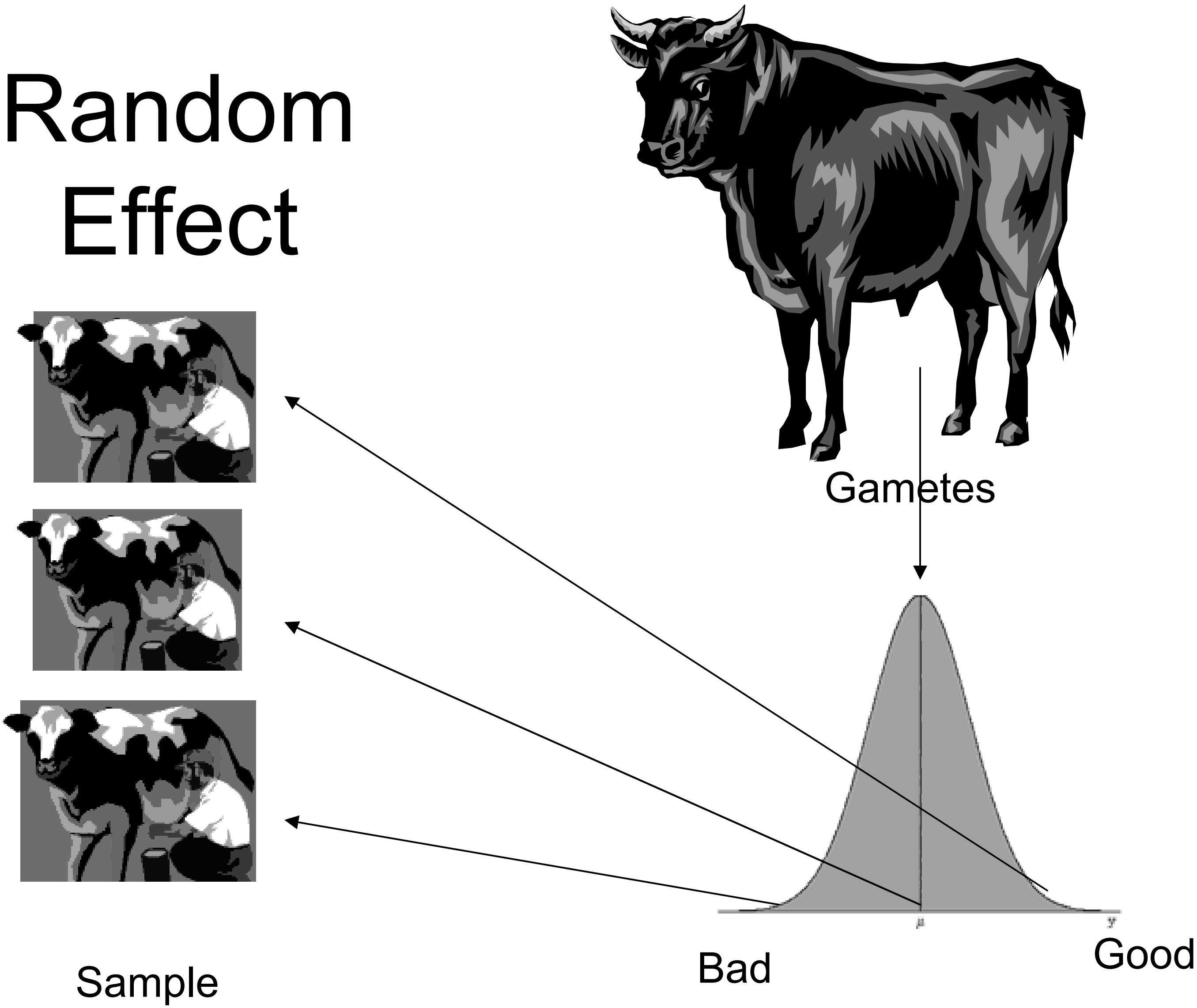
- Separate Independent variable into those that are
  - Fixed $\mathbf{Xb}$
  - Random $\mathbf{Zu}$

$$\mathbf{Y} = \mathbf{Xb} + \mathbf{Zu} + \mathbf{e}$$

Fixed and Random Effects

- Fixed Effect
  - Inference Space only to those levels
  - Herd, Year, Season, Parity, and Sex effect
- Random Effect
  - Effect Sampled From A Distribution Of Effects
  - Inference Space To The Population From Which The Random Effect Was Sampled

Random Effect



Inference is to the genetic worth of the bull

Variances In Mixed Models

$$\mathbf{Y} = \mathbf{X}\mathbf{b} + \mathbf{Z}\mathbf{u} + \mathbf{e}$$

$$V(\mathbf{b}) = \mathbf{0}$$

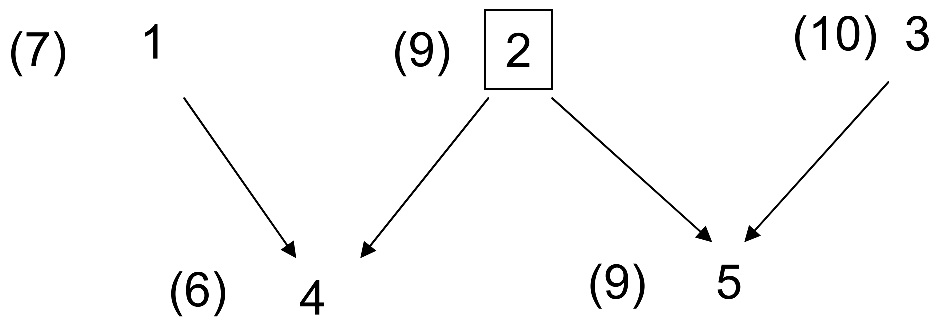
$$V(\mathbf{u}) = E(\mathbf{u}\mathbf{u}') = \mathbf{G}$$

$$V(\mathbf{e}) = E(\mathbf{e}\mathbf{e}') = \mathbf{R}$$

$$V(\mathbf{Y}) = V(\mathbf{X}\mathbf{b} + \mathbf{Z}\mathbf{u} + \mathbf{e}) = \mathbf{Z}\mathbf{G}\mathbf{Z}' + \mathbf{R}$$

Example 1

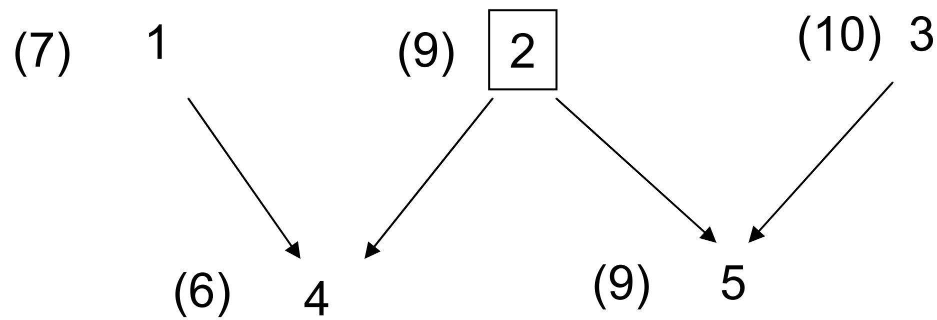
$$\mathbf{Y} = \begin{bmatrix} 7 \\ 9 \\ 10 \\ 6 \\ 9 \end{bmatrix}$$



$$b = [\mu]$$

$$\mathbf{X} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad Z = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{U} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \qquad \mathbf{e} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix}$$

Example 1



$$\begin{bmatrix} 7 \\ 9 \\ 10 \\ 6 \\ 9 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} [\mu] + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix}$$

$$\mathbf{Y} = \mathbf{Xb} + \mathbf{Zu} + \mathbf{e}$$

ML Derivation of BLUP

Joint density of **y** and **u** $\qquad f(\mathbf{y},\mathbf{u}) = g(\mathbf{y}/\mathbf{u})h(\mathbf{u})$

$$g(\mathbf{y}/\mathbf{u}) = g(\mathbf{e})$$

$$g(\mathbf{e}) = \frac{1}{(2\pi)^{\frac{1}{2}N} V(e)^{\frac{1}{2}}} e^{-\frac{1}{2}\mathbf{e}'V(\mathbf{e})^{-1}\mathbf{e}} \qquad h(\mathbf{u}) = \frac{1}{(2\pi)^{\frac{1}{2}N} V(\mathbf{u})^{\frac{1}{2}}} e^{-\frac{1}{2}\mathbf{u}'V(\mathbf{u})^{-1}\mathbf{u}}$$

$$f(\mathbf{y},\mathbf{u}) = \frac{1}{(2\pi)^{\frac{1}{2}N} \mathbf{R}^{\frac{1}{2}}} e^{-\frac{1}{2}\mathbf{e}'\mathbf{R}^{-1}\mathbf{e}} \frac{1}{(2\pi)^{\frac{1}{2}N} \mathbf{G}^{\frac{1}{2}}} e^{-\frac{1}{2}\mathbf{u}'\mathbf{G}^{-1}\mathbf{u}}$$

$$f(\mathbf{y},\mathbf{u}) = c_1 e^{-\frac{1}{2}\mathbf{e}'\mathbf{R}^{-1}\mathbf{e}} c_2 e^{-\frac{1}{2}\mathbf{u}'\mathbf{G}^{-1}\mathbf{u}}$$

$$f(\mathbf{y},\mathbf{u}) = c e^{-\frac{1}{2}\mathbf{e}'\mathbf{R}^{-1}\mathbf{e}} e^{-\frac{1}{2}\mathbf{u}'\mathbf{G}^{-1}\mathbf{u}}$$

$$f(\mathbf{y},\mathbf{u}) = L = ce^{-\frac{1}{2}\mathbf{e}'\mathbf{R}^{-1}\mathbf{e}} e^{-\frac{1}{2}\mathbf{u}'\mathbf{G}^{-1}\mathbf{u}}$$

Maximize w.r.t **b and u**

$$\ln(L) = \ln(c) - \tfrac{1}{2}\mathbf{e}'\mathbf{R}^{-1}\mathbf{e} - \tfrac{1}{2}\mathbf{u}'\mathbf{G}^{-1}\mathbf{u}$$

$$\mathbf{e} = \mathbf{Y} - \mathbf{Xb} - \mathbf{Zu}$$

$$\ln(L) = \ln(c) - \tfrac{1}{2}(\mathbf{Y} - \mathbf{Xb} - \mathbf{Zu})'\mathbf{R}^{-1}(\mathbf{Y} - \mathbf{Xb} - \mathbf{Zu}) - \tfrac{1}{2}\mathbf{u}'\mathbf{G}^{-1}\mathbf{u}$$

Take Derivative w.r.t **b**

$$(\mathbf{Y}-\mathbf{Xb}-\mathbf{Zu})'\mathbf{R}^{-1}(\mathbf{Y}-\mathbf{Xb}-\mathbf{Zu})+\mathbf{u}'\mathbf{G}^{-1}\mathbf{u}$$
$$=\left[\mathbf{Y}'-(\mathbf{Xb})'-(\mathbf{Zu})'\right]\mathbf{R}^{-1}(\mathbf{Y}-\mathbf{Xb}-\mathbf{Zu})+\mathbf{u}'\mathbf{G}^{-1}\mathbf{u}$$

$$=\mathbf{Y}'\mathbf{R}^{-1}\mathbf{Y}-\mathbf{Y}'\mathbf{R}^{-1}\mathbf{Xb}-\mathbf{Y}'\mathbf{R}^{-1}\mathbf{Zu}$$
$$-(\mathbf{Xb})'\mathbf{R}^{-1}\mathbf{Y}+(\mathbf{Xb})'\mathbf{R}^{-1}\mathbf{Xb}+(\mathbf{Xb})'\mathbf{R}^{-1}\mathbf{Zu}$$
$$-(\mathbf{Zu})'\mathbf{R}^{-1}\mathbf{Y}+(\mathbf{Zu})'\mathbf{R}^{-1}\mathbf{Xb}+(\mathbf{Zu})'\mathbf{R}^{-1}\mathbf{Zu}+\mathbf{u}'\mathbf{G}^{-1}\mathbf{u}$$

$$\frac{\partial(\ln L)}{\partial \mathbf{b}}=0 \qquad \begin{array}{l} -\mathbf{Y}'\mathbf{R}^{-1}\mathbf{X}-\mathbf{X}'\mathbf{R}^{-1}\mathbf{Y}+(\mathbf{Xb})'\mathbf{R}^{-1}\mathbf{X}+ \\ \mathbf{X}'\mathbf{R}^{-1}\mathbf{Xb}+\mathbf{X}'\mathbf{R}^{-1}\mathbf{Zu}+(\mathbf{Zu})'\mathbf{R}^{-1}\mathbf{X}=0 \end{array}$$

$$-2\mathbf{X}'\mathbf{R}^{-1}\mathbf{Y}+2\mathbf{X}'\mathbf{R}^{-1}\mathbf{Xb}+2\mathbf{X}'\mathbf{R}^{-1}\mathbf{Zu}=0$$
$$\mathbf{X}'\mathbf{R}^{-1}\mathbf{Xb}+\mathbf{X}'\mathbf{R}^{-1}\mathbf{Zu}=\mathbf{X}'\mathbf{R}^{-1}\mathbf{Y}$$

Take Derivative w.r.t **u**

$$\left(\mathbf{Y}-\mathbf{Xb}-\mathbf{Zu}\right)'\mathbf{R}^{-1}\left(\mathbf{Y}-\mathbf{Xb}-\mathbf{Zu}\right)+\mathbf{u}'\mathbf{G}^{-1}\mathbf{u}$$

$$\begin{aligned}&=\mathbf{Y}'\mathbf{R}^{-1}\mathbf{Y}-\mathbf{Y}'\mathbf{R}^{-1}\mathbf{Xb}-\mathbf{Y}'\mathbf{R}^{-1}\mathbf{Zu}\\&-\left(\mathbf{Xb}\right)'\mathbf{R}^{-1}\mathbf{Y}+\left(\mathbf{Xb}\right)'\mathbf{R}^{-1}\mathbf{Xb}+\left(\mathbf{Xb}\right)'\mathbf{R}^{-1}\mathbf{Zu}\\&-\left(\mathbf{Zu}\right)'\mathbf{R}^{-1}\mathbf{Y}+\left(\mathbf{Zu}\right)'\mathbf{R}^{-1}\mathbf{Xb}+\left(\mathbf{Zu}\right)'\mathbf{R}^{-1}\mathbf{Zu}+\mathbf{u}'\mathbf{G}^{-1}\mathbf{u}\end{aligned}$$

$$\frac{\partial\left(\ln L\right)}{\partial\mathbf{u}}=0 \qquad \begin{aligned}&-\mathbf{Y}'\mathbf{R}^{-1}\mathbf{Z}+\left(\mathbf{Xb}\right)'\mathbf{R}^{-1}\mathbf{Z}-\left(\mathbf{Z}\right)'\mathbf{R}^{-1}\mathbf{Y}+\left(\mathbf{Z}\right)'\mathbf{R}^{-1}\mathbf{Xb}\\&+\left(\mathbf{Z}\right)'\mathbf{R}^{-1}\mathbf{Zu}+\left(\mathbf{Zu}\right)'\mathbf{R}^{-1}\mathbf{Z}+2\mathbf{G}^{-1}\mathbf{u}=0\end{aligned}$$

$$-2\mathbf{Y}'\mathbf{R}^{-1}\mathbf{Z}+2\mathbf{Z}'\mathbf{R}^{-1}\mathbf{Xb}+2\mathbf{Z}'\mathbf{R}^{-1}\mathbf{Zu}+2\mathbf{G}^{-1}\mathbf{u}=0$$

$$\mathbf{Z}'\mathbf{R}^{-1}\mathbf{Xb}+\mathbf{Z}'\mathbf{R}^{-1}\mathbf{Zu}+\mathbf{G}^{-1}\mathbf{u}=\mathbf{Y}'\mathbf{R}^{-1}\mathbf{Z}$$

Mixed Model Equations

$$\mathbf{X}'\mathbf{R}^{-1}\mathbf{X}\mathbf{b} + \mathbf{X}'\mathbf{R}^{-1}\mathbf{Z}\mathbf{u} = \mathbf{X}'\mathbf{R}^{-1}\mathbf{Y}$$

$$\mathbf{Z}'\mathbf{R}^{-1}\mathbf{X}\mathbf{b} + \mathbf{Z}'\mathbf{R}^{-1}\mathbf{Z}\mathbf{u} + \mathbf{G}^{-1}\mathbf{u} = \mathbf{Y}'\mathbf{R}^{-1}\mathbf{Z}$$

$$\begin{bmatrix} \mathbf{X}'\mathbf{R}^{-1}\mathbf{X} & \mathbf{X}'\mathbf{R}^{-1}\mathbf{Z} \\ \mathbf{Z}'\mathbf{R}^{-1}\mathbf{X} & \mathbf{Z}'\mathbf{R}^{-1}\mathbf{Z} + \mathbf{G}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{b} \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{R}^{-1}\mathbf{Y} \\ \mathbf{Y}'\mathbf{R}^{-1}\mathbf{Z} \end{bmatrix}$$

Simplifications If $\mathbf{R} = \mathbf{I}\sigma_e^2$

$$\begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{Z} \\ \mathbf{Z}'\mathbf{X} & \mathbf{Z}'\mathbf{Z} + \sigma_e^2\mathbf{G}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{b} \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{Y} \\ \mathbf{Y}'\mathbf{Z} \end{bmatrix}$$

Normal Distribution of Y Not Necessary

It is possible to show with alternative BLUE estimation techniques that the Same Results would be obtained without assuming normality

See Schaffer Notes

Simplifications

$$\begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{Z} \\ \mathbf{Z}'\mathbf{X} & \mathbf{Z}'\mathbf{Z} + \sigma_e^2 \mathbf{G}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{b} \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{Y} \\ \mathbf{Y}'\mathbf{Z} \end{bmatrix}$$

Assuming Additivity

$$\mathbf{G} = \mathbf{A}\sigma_a^2$$

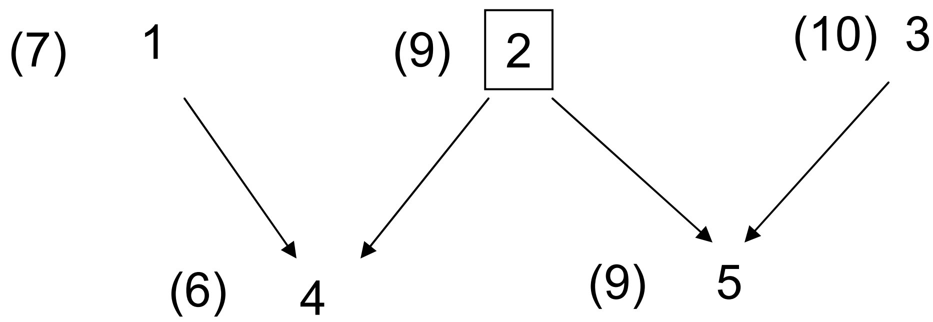
$$\begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{Z} \\ \mathbf{Z}'\mathbf{X} & \mathbf{Z}'\mathbf{Z} + \frac{\sigma_e^2}{\sigma_a^2} \mathbf{A}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{b} \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{Y} \\ \mathbf{Y}'\mathbf{Z} \end{bmatrix}$$

Only Estimate of Ratio is Needed

Only inverse is needed

Example 1

$$\mathbf{Y} = \begin{bmatrix} 7 \\ 9 \\ 10 \\ 6 \\ 9 \end{bmatrix}$$



$$b = [\mu]$$

$$\mathbf{X} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad Z = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \qquad \mathbf{e} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{Z} \\ \mathbf{Z}'\mathbf{X} & \mathbf{Z}'\mathbf{Z} + \frac{\sigma_e^2}{\sigma_a^2}\mathbf{A}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{b} \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{Y} \\ \mathbf{Y}'\mathbf{Z} \end{bmatrix}$$

$$\mathbf{X'X} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\mathbf{X'X} = 5$$

$$\mathbf{X'Z} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

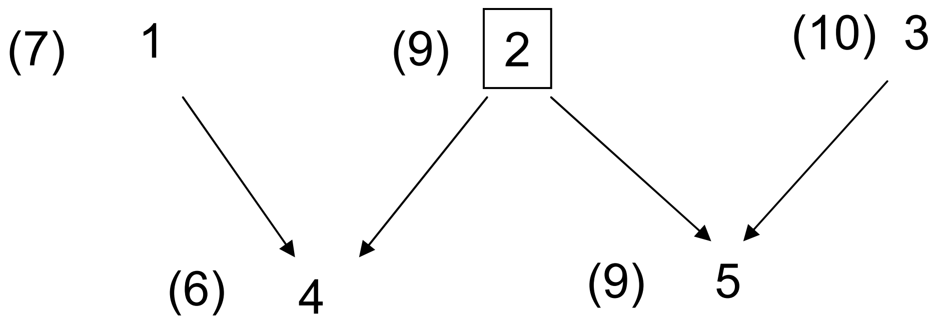
$$\mathbf{X'Z} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{Z} \\ \mathbf{Z}'\mathbf{X} & \mathbf{Z}'\mathbf{Z} + \frac{\sigma_e^2}{\sigma_a^2}\mathbf{A}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{b} \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{Y} \\ \mathbf{Y}'\mathbf{Z} \end{bmatrix}$$

$$\mathbf{Z}'\mathbf{X} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\mathbf{Z}'\mathbf{Z} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{Z}'\mathbf{X} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$



$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 & 1 & \frac{1}{4} \\ 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & 1 \end{bmatrix}$$

Assume heritability=.5

$$\frac{\sigma_e^2}{\sigma_a^2} = 1$$

$$\mathbf{Z}'\mathbf{Z} + \frac{\sigma_e^2}{\sigma_a^2}\mathbf{A}^{-1} = \begin{bmatrix} \frac{5}{2} & \frac{1}{2} & 0 & -1 & 0 \\ \frac{1}{2} & 3 & \frac{1}{2} & -1 & -1 \\ 0 & \frac{1}{2} & \frac{5}{2} & 0 & -1 \\ -1 & -1 & 0 & 3 & 0 \\ 0 & -1 & -1 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{Z} \\ \mathbf{Z}'\mathbf{X} & \mathbf{Z}'\mathbf{Z} + \frac{\sigma_e^2}{\sigma_a^2}\mathbf{A}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{b} \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{Y} \\ \mathbf{Y}'\mathbf{Z} \end{bmatrix}$$

$$\mathbf{X'Y} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 9 \\ 10 \\ 6 \\ 9 \end{bmatrix}$$

$$\mathbf{Y'Z} = \begin{bmatrix} 7 \\ 9 \\ 10 \\ 6 \\ 9 \end{bmatrix}$$

$$\mathbf{X'Y} = \begin{bmatrix} 41 \end{bmatrix}$$

MME

$$
\begin{bmatrix}
5 & 1 & 1 & 1 & 1 & 1 \\
1 & \frac{5}{2} & \frac{1}{2} & 0 & -1 & 0 \\
1 & \frac{1}{2} & 3 & \frac{1}{2} & -1 & -1 \\
1 & 0 & \frac{1}{2} & \frac{5}{2} & 0 & -1 \\
1 & -1 & -1 & 0 & 3 & 0 \\
1 & 0 & -1 & -1 & 0 & 3
\end{bmatrix}
\begin{bmatrix}
\mu \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5
\end{bmatrix}
=
\begin{bmatrix}
41 \\ 7 \\ 9 \\ 10 \\ 6 \\ 9
\end{bmatrix}
$$

Estimation of Error Variance if the Ratio is Known

$$\lambda = \frac{\sigma_e^2}{\sigma_a^2}$$

$$\hat{\sigma}_e^2 = MSE = \left[\frac{\mathbf{Y'Y} - \hat{\mathbf{b}}\mathbf{X} - \hat{\mathbf{u}}\mathbf{Z}}{N - R(X)} \right]$$

$$\hat{\sigma}_a^2 = \frac{\hat{\sigma}_e^2}{\lambda}$$

```
proc iml;
start main;

y={ 7,
    9,
   10,
    6,
    9};

 X={1,
    1,
    1,
    1,
    1};

A={1  0  0 .5   0,
   0  1  0 .5  .5,
   0  0  1  0  .5,
  .5 .5  0  1  .25,
   0 .5 .5 .25  1};

lam=1;

Z={1 0 0 0 0,
   0 1 0 0 0,
   0 0 1 0 0,
   0 0 0 1 0,
   0 0 0 0 1};

LHS=((X`*X)||(X`*Z))//((Z`*X)||(Z`*
   Z+INV(A)#LAM));

RHS=(X`*Y)//(Z`*Y);
C=INV(HS);

BU=C*RHS;
print C BU;

finish main;
run;
quit;
```

Estimates

BU

$$b = [\hat{\mu}]$$

$$\begin{array}{r} 8.3018868 \\ \left.\begin{array}{r} -0.960813 \\ 0.0754717 \\ 0.8853411 \\ -1.062409 \\ 0.5529753 \end{array}\right\} \end{array} \quad \mathbf{U} = \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \\ \hat{a}_4 \\ \hat{a}_5 \end{bmatrix}$$

Variance of the Estimates

$$\begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{Z} \\ \mathbf{Z}'\mathbf{X} & \mathbf{Z}'\mathbf{Z} + \frac{\sigma_e^2}{\sigma_a^2}\mathbf{A}^{-1} \end{bmatrix}^{-1}$$

$$V(\hat{\mathbf{b}}) = \mathbf{C}_{11}\sigma_e^2$$

$$V(\hat{\mathbf{u}} - \mathbf{u}) = C_{22}\sigma_e^2$$

Prediction Error Variance

$$V(\hat{\mathbf{u}}) = \mathbf{A}\sigma_a^2 + \mathbf{C}_{22}\sigma_e^2$$

Prediction Error Variance
Including Drift Variance

Kennedy and Sorensen *Quantitative Genetics*

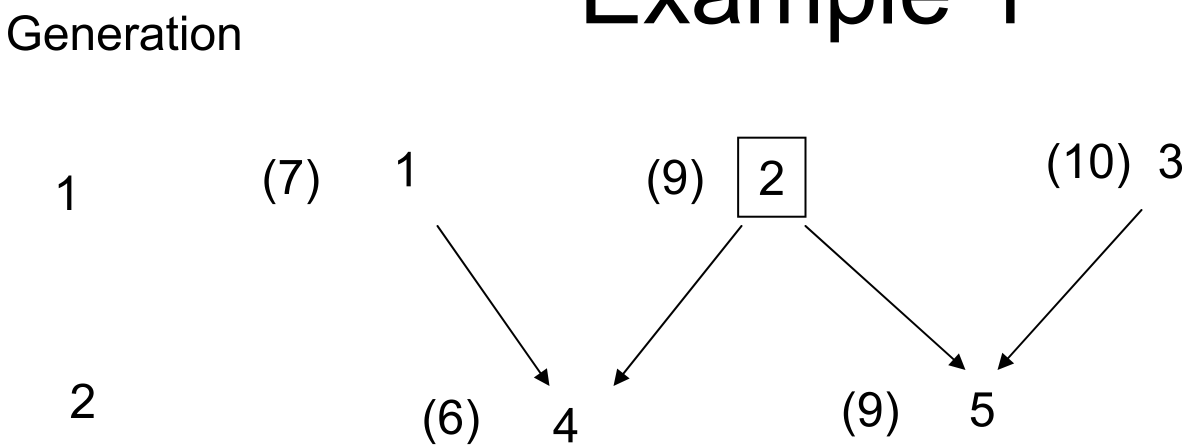
PEV

1.12236	0.29509	0.32030	0.65827	0.39093
0.29509	1.14758	0.29509	0.68854	0.68854
0.32030	0.29509	1.12236	0.39093	0.65827
0.65827	0.68854	0.39093	1.2686	0.60026
0.39093	0.68854	0.65827	0.60026	1.2686

EV

2.86014	0.29509	0.32030	1.52716	0.39093
0.29509	2.88536	0.29509	1.5574	1.5574
0.32030	0.29509	2.86014	0.39093	1.52716
1.52716	1.5574	0.39093	3.00643	1.03471
0.39093	1.5574	1.52716	1.03471	3.00643

Example 1



Generation	Mean	Variance
1	8.66	2.860
2	7.5	3.006

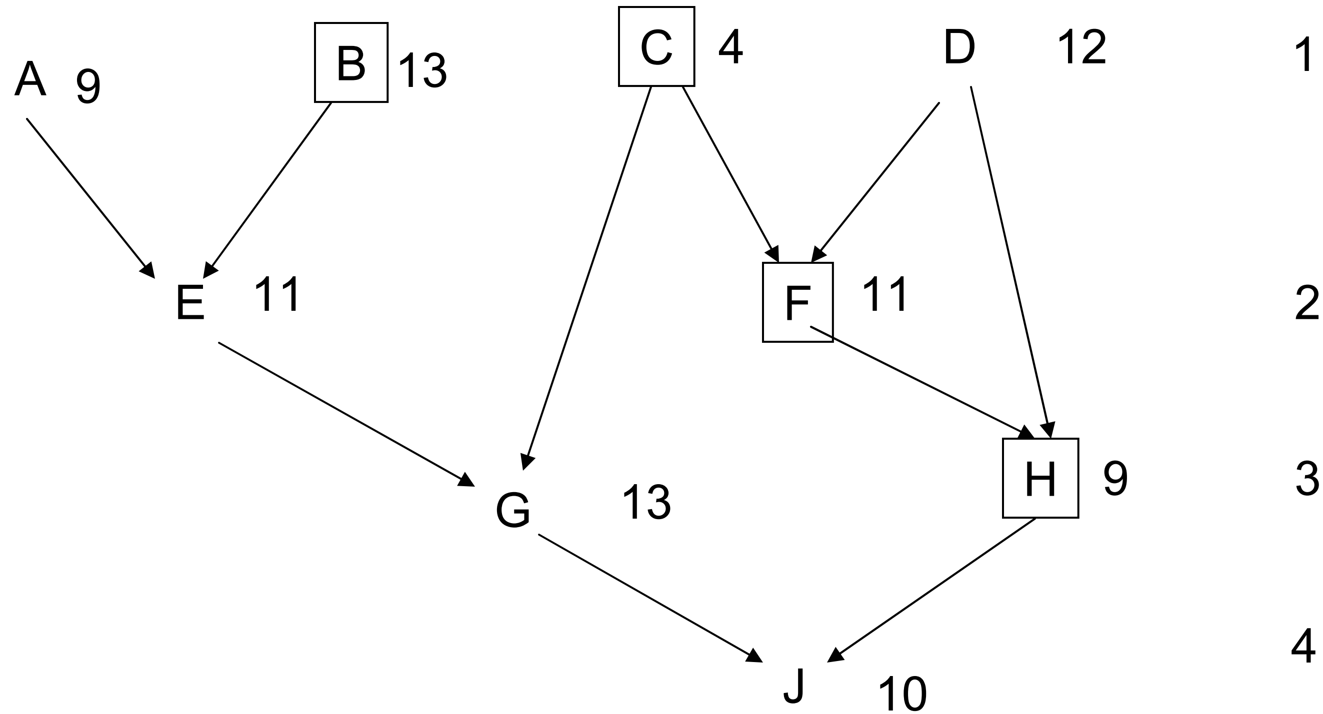
Lab Problem 6.1

- How does changing the heritability affect the estimates and PEV and PE variance? Set to each of the following and compare results

$$\frac{\sigma_e^2}{\sigma_a^2} = 100 \qquad\qquad \frac{\sigma_e^2}{\sigma_a^2} = .1$$

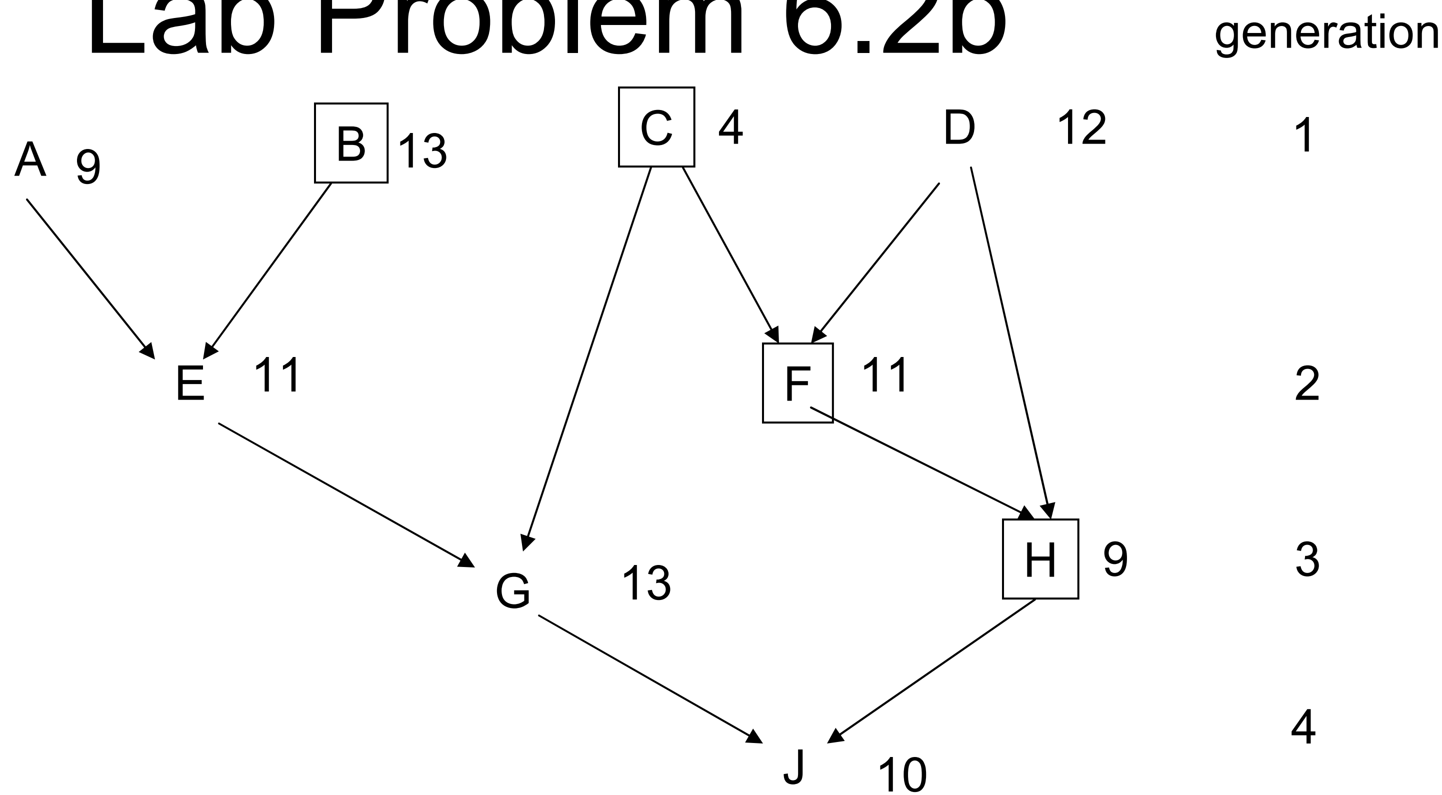
Interpret the results

Lab Problem 6.2a



Find the best estimate of the genetic worth of each animal, additive and error variance, PEV, and PV. Assume a heritability of .5

Lab Problem 6.2b



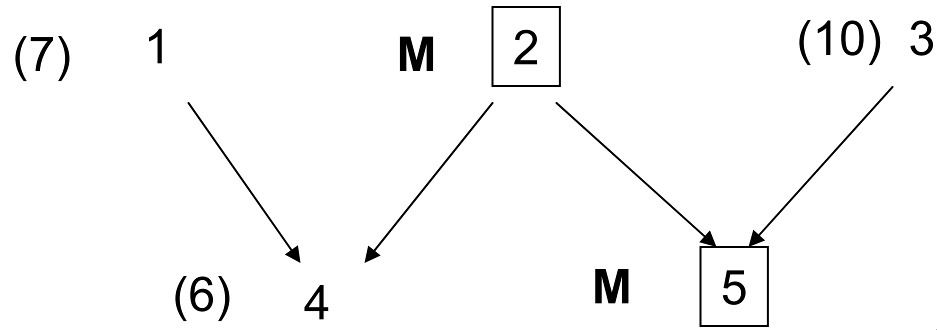
Environmental trend can be found by fitting generation number as a covariate. Genetic trend is found by taking the average of all EBV's in that generation and fitting the means to a linear regression. What are the genetic and environmental trend for this data

Missing Values (Sex Limited Traits)

Generation

1

2



$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 & 1 & \frac{1}{4} \\ 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & 1 \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} 7 \\ 10 \\ 6 \end{bmatrix} \qquad \mathbf{e} = \begin{bmatrix} e_1 \\ e_3 \\ e_5 \end{bmatrix} \qquad b = [\mu]$$

$$\mathbf{X} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad Z = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{U} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}$$

```
proc iml;
start main;

y={ 7,
   10,
    6};

 X={1,
     1,
     1};

A={1  0  0 .5   0,
    0  1  0 .5  .5,
    0  0  1  0  .5,
   .5 .5  0  1  .25,
    0 .5 .5 .25  1};

lam=1;

Z={1 0 0 0 0,
    0 0 1 0 0,
    0 0 0 0 1};

LHS=((X`*X)||(X`*Z))//((Z`*X)||(Z`*
    Z+INV(A)#LAM));

RHS=(X`*Y)//(Z`*Y);
C=INV(HS);

BU=C*RHS;
print C BU;

finish main;
run;
quit;
```

Estimates

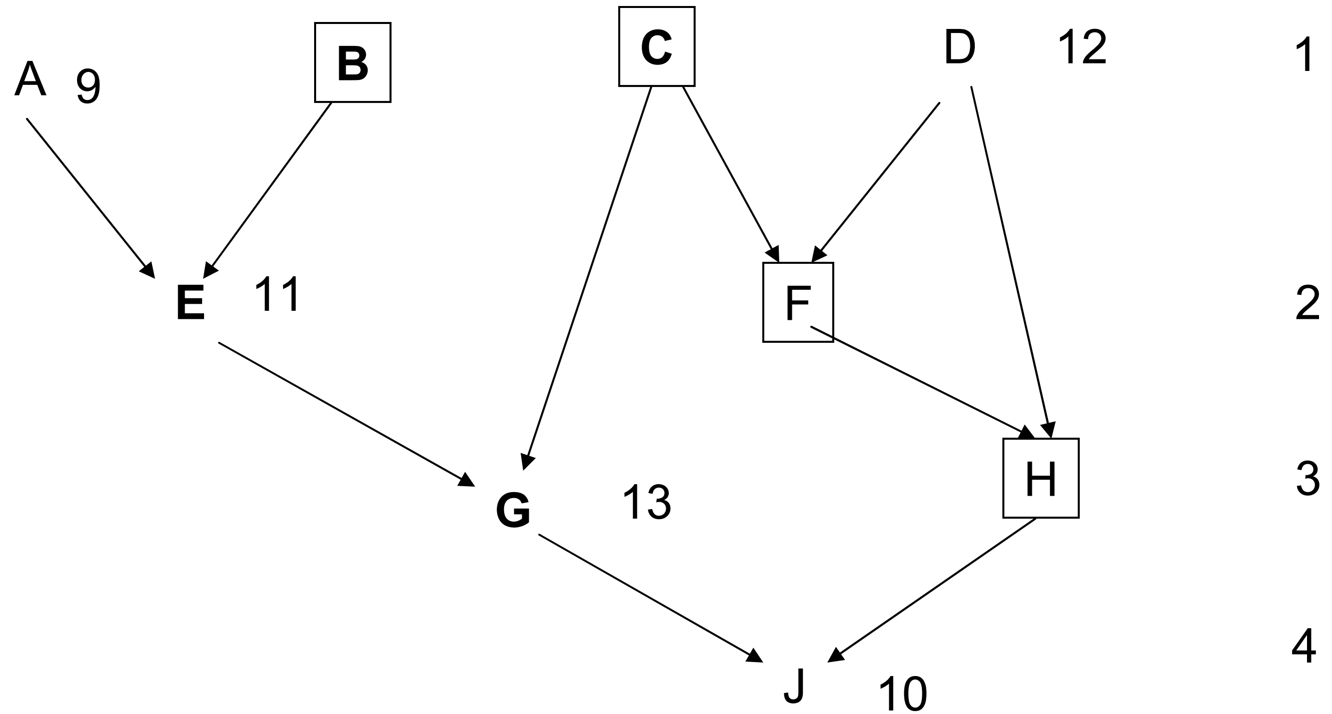
$$b = \left[\hat{\mu}\right]$$

7.6153846

-0.307692
-0.589744
0.8974359
-0.448718
-0.435897

$$\mathbf{U} = \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \\ \hat{a}_4 \\ \hat{a}_5 \end{bmatrix}$$

Lab Problem 6.3: Sex Limited Trait



Estimate breeding values for the males

$$\frac{\sigma_e^2}{\sigma_a^2} = 1$$

Extensions of Model

- Inclusion of Dominance and Epistasis
  - Dominance relationship needed
    - Reflect the probability that animals have the same pair of alleles in common
    - Epistatic genetic effects are the result of interactions of among additive and dominance genetic effects
  - Useful to determine in crossbreeding programs but generally not useful in pure breeding programs
    - An individual does not pass on a dominance or epistatic effect, it is the results of both parents

Limitations

- Based on infinitesimal model
  - Does not work for traits determined by small number of loci
  - Genetic variance is assumed constant except Bulmer effect (reduction in variance due to disequilibrium)
- Model needs to be correct
  - Garbage In Garbage Out (GIGO)
  - Typical Animal Model Assumes Additivity and Independence of residuals