Newton’s Divided Difference Interpolation Method

Newton’s Divided Difference Interpolation Method is a numerical technique used to construct an interpolation polynomial when the given data points are not necessarily equally spaced. This method generalizes Newton’s interpolation approach and is suitable for arbitrary data distributions.

Basic Concept

Suppose the values of a function $f(x)$ are known at n distinct points $x_0, x_1, x_2, ..., x_n$. Newton’s divided difference method constructs a polynomial that passes through all these points using divided differences instead of finite differences.

Divided Differences

The first divided difference is defined as:
$f[x_i, x_{i+1}] = (f(x_{i+1}) – f(x_i)) / (x_{i+1} – x_i)$

Higher-order divided differences are defined recursively as:
$f[x_i, x_{i+1}, ..., x_{i+k}] = (f[x_{i+1}, ..., x_{i+k}] – f[x_i, ..., x_{i+k–1}]) / (x_{i+k} – x_i)$

Newton’s Divided Difference Formula

The interpolation polynomial is given by:
$P(x) = f[x_0] + (x – x_0)f[x_0, x_1] + (x – x_0)(x – x_1)f[x_0, x_1, x_2] + ...$

Advantages

- Works for unequally spaced data - Easy to extend when new data points are added - Computationally efficient

Applications

Newton’s Divided Difference Interpolation Method is widely used in numerical analysis, data fitting, engineering computations, and scientific modeling where data points are irregularly spaced.