Newton’s Forward Interpolation Method

Newton’s Forward Interpolation Method is a numerical technique used to estimate the value of a function at a given point when the values of the function are known at equally spaced points. This method is particularly suitable when the interpolation point lies near the beginning of the data set.

Basic Concept

Assume that the values of a function f(x) are known at n equally spaced points x0, x1, x2, ..., xn. The spacing between consecutive x-values is constant and is denoted by h. Using these values, a polynomial is constructed that passes through all the given points. This polynomial is then used to estimate intermediate values of the function.

Forward Differences

Forward differences are used to build the interpolation polynomial. The first forward difference is defined as:
$\Delta y(i) = y(i+1) - y(i)$

Higher-order forward differences are defined recursively as:
$\Delta^2 y(i) = \Delta y(i+1) - \Delta y(i)$

Newton’s Forward Interpolation Formula

The Newton forward interpolation polynomial is given by:
$f(x) = y0 + u\Delta y0 + [u(u - 1) / 2!]\Delta^2 y0 + [u(u - 1)(u - 2) / 3!]\Delta^3 y0 + ...$

where the parameter u is defined as:
$u = (x - x0) / h$

Advantages

- Simple and systematic method - Efficient for equally spaced data - Easy to implement in computer programs

Applications

Newton’s Forward Interpolation Method is widely used in engineering, scientific computations, numerical analysis, and data approximation problems where intermediate values need to be estimated accurately.