Newton’s Backward Interpolation Method

Newton’s Backward Interpolation Method is a numerical technique used to estimate the value of a function when the known data points are equally spaced and the interpolation point lies near the end of the data set. It is particularly useful when forward interpolation becomes less accurate.

Basic Concept

Let the values of a function f(x) be known at equally spaced points x0, x1, x2, ..., xn with a constant spacing h. Newton’s backward interpolation constructs a polynomial using backward differences, which are calculated from the end of the data table.

Backward Differences

Backward differences are defined as follows:
$\nabla y(i) = y(i) - y(i-1)$

Higher-order backward differences are defined recursively as:
$\nabla^2 y(i) = \nabla y(i) - \nabla y(i-1)$

Newton’s Backward Interpolation Formula

The Newton backward interpolation polynomial is given by:
$f(x) = y(n) + u\nabla y(n) + [u(u + 1) / 2!]\nabla^2 y(n) + [u(u + 1)(u + 2) / 3!]\nabla^3 y(n) + ...$

where the parameter u is defined as:
$u = (x - x(n)) / h$

Advantages

- Suitable when interpolation point is near the end of the data set - Accurate for equally spaced data
- Easy to implement computationally

Applications

Newton’s Backward Interpolation Method is widely used in numerical analysis, engineering problems, scientific computations, and data approximation tasks where estimation near the end of tabulated data is required.