False Position Method (Regula Falsi)

The False Position Method, also known as the Regula Falsi method, is a numerical technique used to find the root of a continuous function. It combines the reliability of the Bisection Method with the idea of linear interpolation to obtain faster convergence in many cases.

Basic Concept

If a function f(x) is continuous on an interval [a, b] and f(a)·f(b) < 0, then at least one root exists in this interval. Instead of choosing the midpoint as in the bisection method, the False Position Method finds the point where the straight line joining (a, f(a)) and (b, f(b)) intersects the x-axis.

Formula

The approximation of the root is calculated using the formula:
$c = (a \cdot f(b) - b \cdot f(a)) / (f(b) - f(a))$

Algorithm

1. Choose initial values a and b such that f(a)·f(b) < 0.
2. Compute the approximation c using the False Position formula.
3. Evaluate f(c).
4. If |f(c)| is smaller than the given tolerance, stop; c is the root.
5. If f(a)·f(c) < 0, set b = c; otherwise set a = c.
6. Repeat the above steps until the required accuracy is achieved or the maximum number of iterations is reached.

Stopping Criteria

The iteration process is stopped when the absolute value of f(c) becomes smaller than a predefined tolerance or when the maximum number of iterations is reached.

Advantages

- Guaranteed convergence for continuous functions with a sign change
- Faster convergence than the Bisection Method in many cases
- Does not require derivatives

Applications

The False Position Method is commonly used in numerical analysis, engineering problems, and scientific computations where a reliable and relatively fast root-finding method is required.