Polytopes

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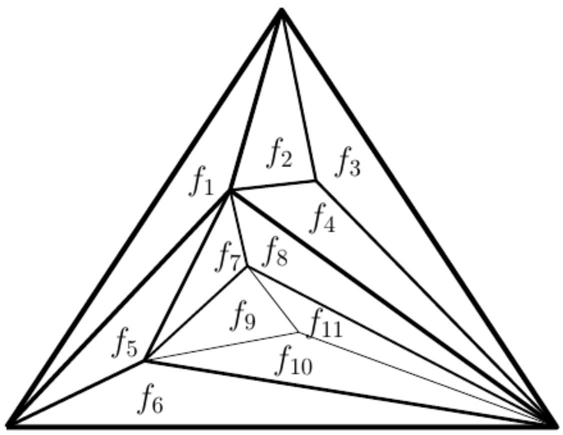
Algorithm based on the article "Embedding Stacked Polytopes on a Polynomial-Size Grid".

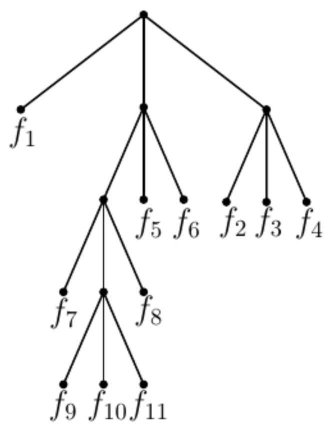
The algorithm can be divided in:

1. Receive the entry.
2. Heavy caterpillar decomposition.
3. Balance the tree.
4. Get the coefficients.
5. Lift the graph to a polytope.

1 The Entry of the algorithm

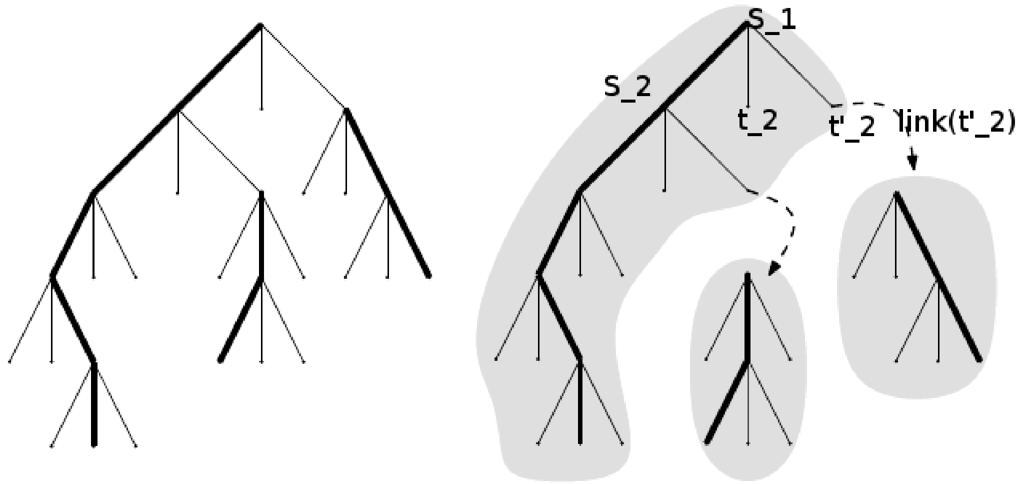
It is given to the algorithm a 3 connected planar graph and a tree representation of the graph, as showed below.





2 Heavy caterpillar decomposition

The tree representation is decomposed in *heavy paths*. Resulting sub-trees called *caterpillars*. When a node lies on a heavy path it is called a *spine node*, otherwise is a *tree node*. The spine nodes are labelled by s_1 (root), s_2, ..., s_i,...,$s_\perp$. The children from s_i are s_{i+1}, t_{i+1} and t'_{i+1}. In t are stored a pointer link(t) to a other caterpillar.



3 Balance the tree

function BALANCE(C)

Input: A caterpillar C from the heavy caterpillar decomposition of $\mathcal{T}(G)$. All weights are equal 1.

Output: Weights for the nodes of $\mathcal{T}(G)$.

for all t_i, t'_i in C **do**

BALANCE(link(t_i))

BALANCE(link(t'_i))

if $w(t_i) > w(t'_i)$ **then**

relabel $t_i \leftrightarrow t'_i$

end if

$w(t_i) = w(t'_i)$

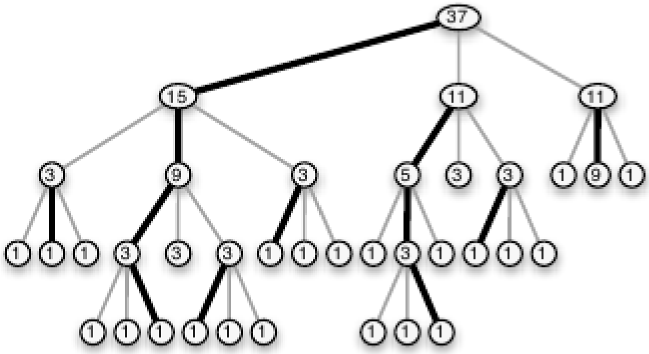
add $(w(t_i) - w(t'_i))$ to the weight of $s_\perp$ in link (t_i)

add $(w(t'_i) - w(t_i))$ to the weight of $\text{link}^{-1}(C)$

end for

end function

A balanced Tree:



4 Get the coefficients

If v_i is the vertex stacked on the face $v_j v_k v_l$ then:

$$\alpha_{ijkl} = \frac{1}{w(t_u)}$$

Where $w(t_u)$ is the weight of the subface $v_j v_k v_l$ obtained by the function BALANCE.

The weights are now updated with the coordinate values. First the coordinates from the vertices are rounded down. $r_i = (\lfloor x_i \rfloor, \lfloor y_i \rfloor)$.

The weights are:

$$\dot{w}_{ij} = \sum_{\{i,j,k,l\} \in S} \lfloor Y \alpha_{ijkl} \rfloor w_{ij}^{kl}$$

The coefficient α_{ijkl} is scaled by multiplying it with $Y = 4n^2$, n is the number of vertices.

$$w_{ij}^{kl} = [i,k,l][j,k,l]$$

$$[i,j,k] = \det \begin{pmatrix} x_i & x_j & x_k \\ y_i & y_j & y_k \\ 1 & 1 & 1 \end{pmatrix}$$

5 The lifting

Pages 133 to 139 of REALIZATION SPACES OF POLYTOPES