Troy Daneel Oster due: 6 September 2019

CSCI 532 Problem 1-1

Collaborators: *Dana Parker*

1. Write this homework in LaTex. (You can use this document as a starting point!)
2. Update your photo on D2L to be a recognizable headshot of you.
   Done!

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CSCI 532 Problem 1-2

Collaborators: *Dana Parker*

Consider the third function defined in EPI, Section 13.1 (Compute the Intersection of Two Sorted Arrays).

1. When we design an algorithm, we design the algorithm to solve a problem or answer a question. What is the problem that this algorithm solves?

   This while loop solves the problem of finding the intersection of two arrays A, B. That is to say, that a new array "intersection_A_B" is created and contains the elements present in both array A and array B

2. Prove that the while loop terminates using a decrementing function.

   Proof. Let there be a decrementing function $D(x)$ such that x equals a possible state of the while loop. Let $x = (len(A) + len(B)) - (i + j)$

   Note: the function $D(x)$ is a valid decrementing function for this while loop because it includes the conditions for termination present within the while loop, $i < len(A)$ and $j < len(B)$. therefore, if i, j increment, the function $D(x)$ decrements. If x reaches 0 it must mean that either i or j or both have increased to a point where they are greater than $len(A)$ or $len(B)$.

   Now, we must show that either i or j, or both, increment during each iteration of the loop.

   If we look at the while loop. We notice that there are 3 cases that can occur. An "if", an "elif", and an "else".

   Case 1: If the "if" statement is triggered, $i = i + 1$ (i increments) and $j = j + 1$ (j increments).

   Case 2: If the "elif" statement is triggered, $i = i + 1$ (i increments)

   Case 3: If the "else" statement is triggered, $j = j + 1$ (j increments)

   Therefore, since all possible cases within the while loop increment either i or j or both, the decrementing function must reach 0. Thus, the while loop must terminate. □

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	CSCI 532 Problem 1-3	
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Prove the following statement: Every tree with one or more nodes/vertices has exactly $n-1$ edges.

Proof. Let n be the number of vertices in a tree (T).

Base Case: If n = 1, then the number of edges is 0, because there are no other nodes to connect to.

Assume: Assume that n=k holds: A tree with k nodes has k - 1 edges.

Show: We must show that n = k+1 holds: A tree k+1 nodes has k edges

Now, let V_1 and V_2 be vertices of a tree (M) that has k and let E be the edge connecting them. Since M is a tree, then there must be only one path, or edge, connecting vertices V_1, V_2. Thus, if edge E is deleted, there are now two independent trees containing the vertices V_1, V_2 respectively. Let these two trees be G_1, G_2 respectively. It can be assumed that G_1, G_2 have no cycle, since M had no cycles.

Let k_1 and k_2 be the number of vertices in G_1, G_2 respectively such that $k_1 + k_2 = n$. therefore, because of the properties of addition $k_1 < k$ and $k_2 < k$ Thus, based on the inductive hypothesis, G_1, G_2 must have $k_1 - 1$ and $k_2 - 1$ nodes respectively.

therefore, the number of edges in $M = k$

$$
\begin{aligned}
&= (k_1 - 1) + (k_2 - 1) + 1 \\
&= k_1 + k_2 - 1 && k_1 + k_2 = k \text{ by substitution} \\
&= k - 1
\end{aligned}
$$

□

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CSCI 532 Problem 1-4

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Consider the following statement: If a and b are both odd numbers, then ab is an odd number.

1. What is the definition of an odd number?
   An odd number can be defined as k = 2n + 1 where $n \in \mathbb{Z}$
2. What is the definition of an even number?
   An even number can be defined as k = 2n where $n \in \mathbb{Z}$
3. What is the contrapositive of this statement?
   The contrapositive of this statement is, if ab is an even number, then a and b are not both odd.
4. Prove this statement, using the contrapositive.

   Proof. Notice that there are 3 possible cases for this statement

   Case 1: a, b are both even, where $a, b \in \mathbb{Z}$

   $$\begin{aligned} \text{Let } a &= 2n && \text{by definition of an even integer} \\ b &= 2m && \text{by definition of an even integer} \end{aligned}$$

   $$\begin{aligned} ab &= (2n)(2m) \\ &= 2(mn) && \text{Let } k = mn, \text{ where } k \text{ is an integer by the multiplicative properties of integers} \\ &= 2k \end{aligned}$$

   ab can be written in the form $2k$ where k is an integer. Therefore, by the definition of an even integer, ab is even. Thus, the statement, if ab is an even number, then a and b are not both odd, is true for this case.

   Case 2: a or b is even, but not both. where $a, b \in \mathbb{Z}$

   $$\begin{aligned} \text{Let } a &= 2n && \text{by definition of an even integer} \\ b &= 2m + 1 && \text{by definition of an odd integer} \end{aligned}$$

   $$\begin{aligned} ab &= (2n)(2m + 1) \\ &= 4(mn) + 2n \\ &= 2(2mn + n) && \text{Let } k = 2mn + n, \text{ where } k \text{ is an integer by the multiplicative and additive properties of intege} \\ &= 2k \end{aligned}$$

ab can be written in the form $2k$ where k is an integer. a, b are arbitrary integers, so this proof is valid for the case where a is odd and b is even, and the case where a is even and b is odd. Therefore, by the definition of an even integer, ab is even. Thus, the statement, if ab is an even number, then a and b are not both odd, is true for this case.

Case 3: a, b are both odd

$$\begin{aligned} \text{Let } a &= 2n+1 && \text{by definition of an odd integer} \\ b &= 2m+1 && \text{by definition of an odd integer} \end{aligned}$$

$$\begin{aligned} ab &= (2n+1)(2m+1) \\ &= 4mn + 2n + 2m + 1 \\ &= 2(2mn + n + m) + 1 && \text{Let } k = 2mn + n + m\text{, where } k \text{ is an integer by the multiplicative and additive propert} \\ &= 2k + 1 \end{aligned}$$

ab can be written in the form $2k+1$ where k is an integer. Therefore, by the definition of an odd integer, ab is odd. Thus, the statement, if ab is an even number, then a and b are not both odd, is false for this case.

Having shown that for every case, except where a and b are both odd, that ab is even, we have shown that the contrapositive is true. Therefore, by proof by contraposition, the original statement, if a and b are both odd numbers, then ab is an odd number, is true.

$\square$