Production Line Scheduling - Integer Program Model

Summary

Describe the problem briefly

Problem Statement

The above problem can be formulated mathematically as a linear programming problem using the following model.

Inputs

Symbol	Description
N	number of batches
D_i	deadline of batch i
T_{pi}	time needed to produce batch i
D_s	time to start scheduling
D_l	last deadline, $\texttt{max}(D_i)$
T_r	total time available, $D_l - D_s$

Variables

Symbol	Description
T_{si}	start time of batch i
P_{ij}	$\begin{cases} 1 & T_{si} < T_{sj} \\ 0 & T_{si} > T_{sj} \end{cases}$
T_f	finish time of all batches $\texttt{max}(T_{si} + T_{pi})$

Constraints

Deadline and Overlapping

This ensures that the deadline is met for batch i.

$$T_{si} + T_{pi} \leq D_i$$

$$0 \leq T_{si} \leq T_r$$

This ensures that any two batches i and j are not separated by more than the total time available for production.

$$-T_r \times P_{ij} \leq T_{si} - T_{sj} \leq T_r \times (1 - P_{ij})$$

This ensures that the production times of any two batches i and j do not overlap.

$$T_{sj} - (T_{si} + T_{pi}) \geq T_r \times (P_{ij} - 1)$$

$$(T_{sj} + T_{pj}) - T_{si} \leq T_r \times P_{ij}$$

Objective

minimize T_f