Production Line Scheduling - Integer Program Model

Summary

Describe the problem briefly

Problem Statement

The above problem can be formulated mathematically as a linear programming problem using the following model.

Inputs

Symbol	Description
n	number of batches
D_i	deadline of batch i
$T_{p,i}$	time needed to produce batch i
D_s	time to start scheduling
D_l	last deadline, $\texttt{max}(D_i)$
T_r	total time available, $D_l - D_s$
A_{ij}	changeover time between batches i, j due to change in allergens
K_{ij}	changeover time between batches i, j due to kosher status switch
C_{ij}	additional changeover time between batches i, j

Variables

Symbol	Description
$T_{s,i}$	start time of batch i
P_{ij}	$\begin{cases} 1 & T_{s,i} < T_{s,j} \\ 0 & T_{s,i} > T_{s,j} \end{cases}$
d_i	day on which batch i is scheduled
t_f	finish time of all batches $\texttt{max}(T_{s,i} + T_{p,i})$

Constraints

Deadline and Overlapping

This ensures that the deadline is met for batch i.

$$T_{s,i} + T_{p,i} \leq D_i$$

$$0 \leq T_{s,i} \leq T_r$$

This ensures that any two batches i and j are not separated by more than the total time available for production.

$$-T_r \times P_{ij} \leq T_{s,i} - T_{s,j} \leq T_r \times (1 - P_{ij})$$

This ensures that the production times of any two batches i and j do not overlap.

$T_{s,j} - (T_{s,i} + T_{p,i}) \geq T_r \times (P_{ij} - 1)$

$(T_{s,j} + T_{p,j}) - T_{s,i} \leq T_r \times P_{ij}$

Changeover Period

This ensures that batch j starts only after the various changeover periods of batch i.

$(T_{s,j} - (T_{s,i} + T_{p,i})) + T_r \times (1 - P_{ij}) \geq C_{ij} + A_{ij} + K_{ij}$

Shifts

This ensures that no start or finish time lies outside the workday or during the weekends. Here, d_i can only take on values which correspond to valid workdays. For example, if you begin scheduling today (say, a Tuesday) and end on Monday the following week, $d_i \in \{0, 1, 2, 3, 6\}$.

$24 \cdot d_i + 8 \leq T_{s,i} \leq 24 \cdot d_i + 16$

$24 \cdot d_i + 8 \leq T_{s,i} + T_{p,i} \leq 24 \cdot d_i + 16$

Objective

minimize t_f