SBP-SAT approach for horizontal approximation of atmospheric dynamics equations on grids with local space refinement

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Introduction

Methods of increasing detalization of atmosphere state forecast:

1. Increasing resolution globally
   + Improving effective resolution
   – Rapid growth of required computing resources
2. Local refinement of computational grid
   + Saving computing resources
   + Increasing the effective resolution in the refinement region
   – Numerical errors at domains boundaries

Shallow water equations

Vector-invariant form of shallow water equations:

$$\begin{cases} \frac{\partial \mathbf{v}}{\partial t} = -(\xi + f)\mathbf{k} \times \mathbf{v} - \nabla(gh + K) \\ \frac{\partial h}{\partial t} = -\nabla \cdot (\mathbf{v}h) \end{cases}$$

$\mathbf{v}-$ horizontal velocity vector, h - thickness of the fluid layer, $\mathbf{k}$ - vertical unit vector, $\xi = \mathbf{k} \cdot \nabla \times \mathbf{v}$ - relative vorticity, f - Coriolis parameter, $K = \mathbf{v} \cdot \mathbf{v}/2$ - kinetic energy density.

Computational grid

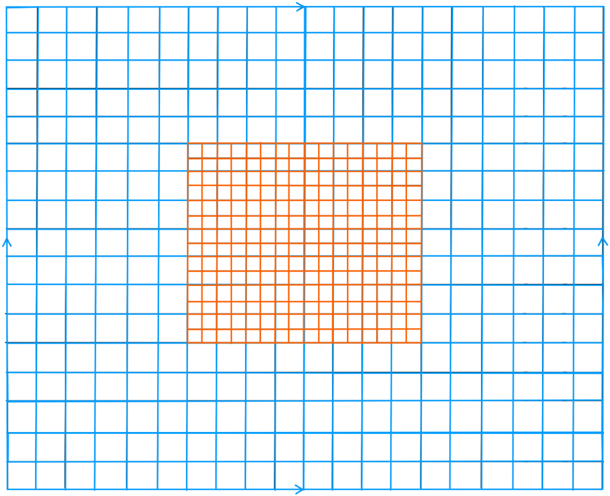

Рис.: Possible configuration of the computational grid: biperiodic boundary conditions, a block with a local refinement of 2 times in the middle.

(SBP) Summation-by-parts finite differences

$$\frac{\partial U}{\partial x} \approx D\mathbf{u}$$

Summation-by-parts (SBP):

$$\mathbf{u}^T HD\mathbf{v} = (u_N v_N - u_0 v_0) - (D\mathbf{u})^T H\mathbf{v}$$

Discrete analogue of integration-by-parts (IBP):

$$\int_a^b U(x)\frac{\partial V(x)}{\partial x} = U(b)V(b) - U(a)V(a) - \int_a^b V(x)\frac{\partial U(x)}{\partial x}$$

1. Discrete analogues of mass and energy conservation laws performed.
2. Stability theorem for linearized problem.

(SAT) Simultaneous Approximation Terms

Weak form of continuity conditions SAT-terms:

$$\frac{\partial U}{\partial x} \approx D\mathbf{u} + SAT_0 + SAT_N,$$

$$SAT_0 = \sigma_0(\Delta u^{left}),\ SAT_N = \sigma_N(\Delta u^{right})$$

Weights σ_0, σ_N are selected based on global SBP property requirement (global energy conservation).

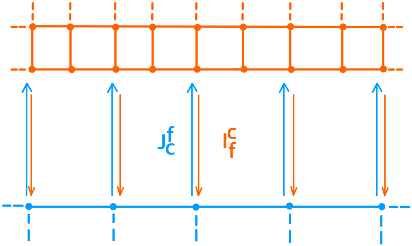

Рис.: Interpolation between blocks with different resolutions

Kelvin-Helmholtz instability

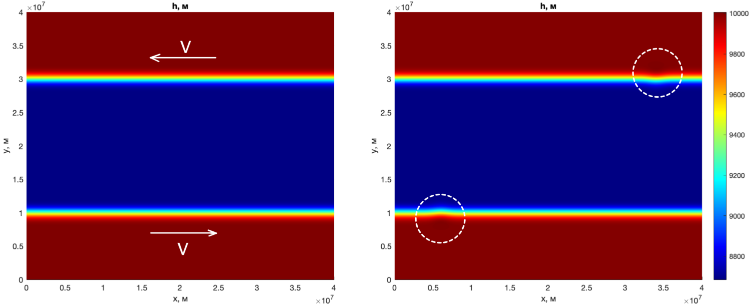

Рис.: Initial conditions for fluid layer thickness field – h

Kelvin-Helmholtz instability

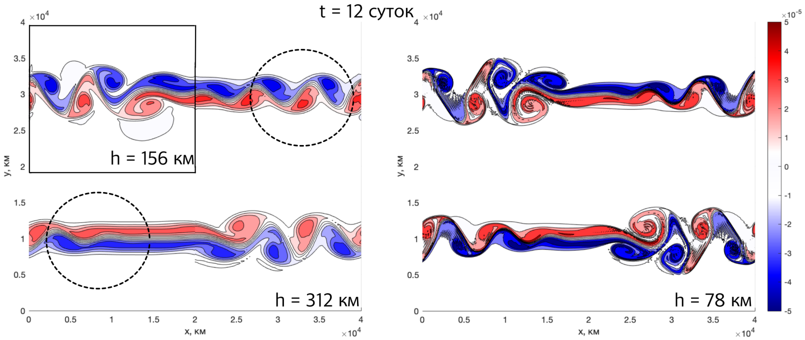

Рис.: Relative vorticity field after 12 days

Kelvin-Helmholtz instability

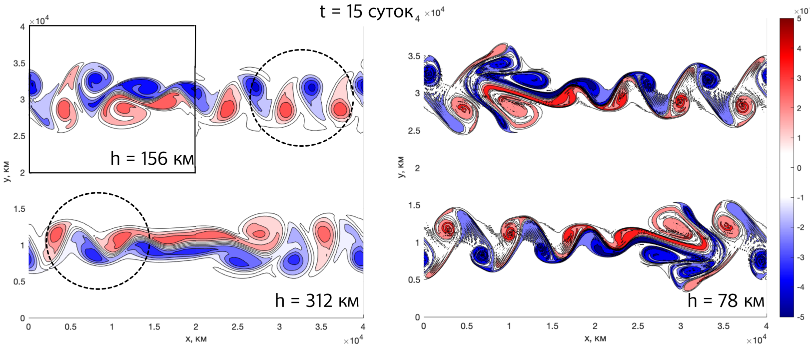

Рис.: Relative vorticity field after 15 days

Conclusion

An approach based on the SAP-SAT method for approximating shallow water equations on grids with local refinement is proposed.

Approach advantages

1. Mass and energy conservation
2. Stability, a proven theorem
3. High-order of approximation
4. Local refinement has a positive impact on entire solution as a whole

Approach disadvantages

1. Approximation orders reduction near block boundaries

Further research

1. Approach implementation within spherical geometry
2. Approach implementation for three-dimensional equations of atmospheric dynamics
3. Comparison with other methods

Thanks for your attention