Continuous Probability Distributions

- Uniform, Normal

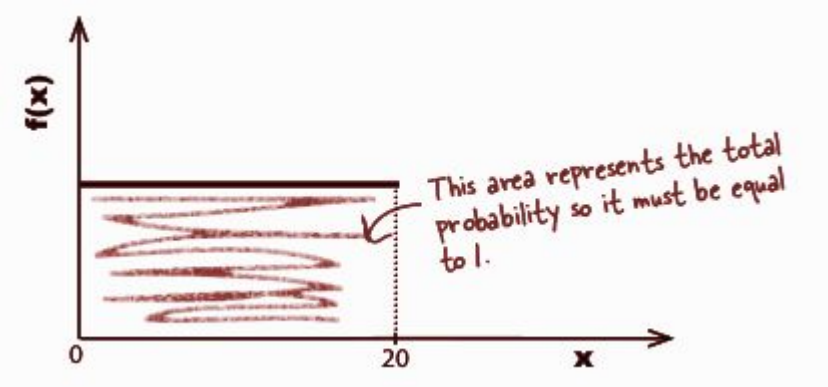
Finding the probability

Q) You are waiting at a bus-stop for a bus which can come anytime within the next 20 minutes. What is the probability that you will have to wait for more than 5 minutes?

Finding the probability

Q) You are waiting at a bus-stop for a bus which can come anytime within the next 20 minutes. What is the probability that you will have to wait for more than 5 minutes?

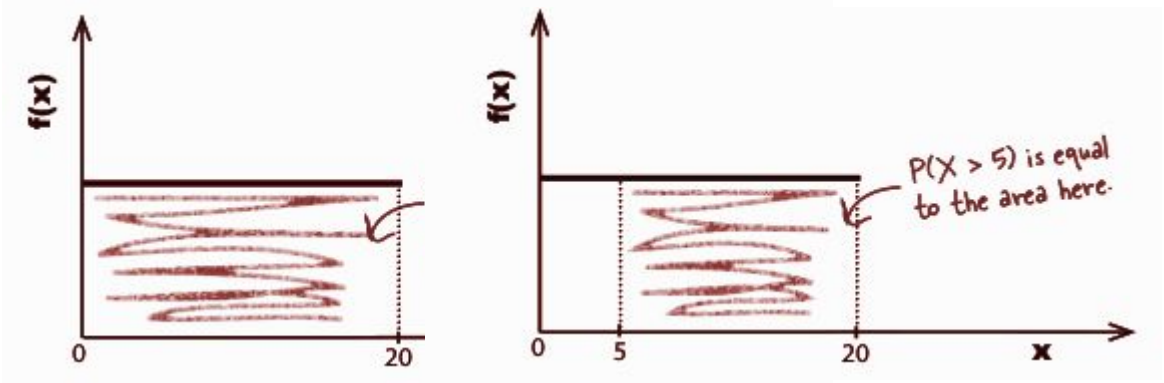
Sol:



Finding the probability

Q) You are waiting at a bus-stop for a bus which can come anytime within the next 20 minutes. What is the probability that you will have to wait for more than 5 minutes?

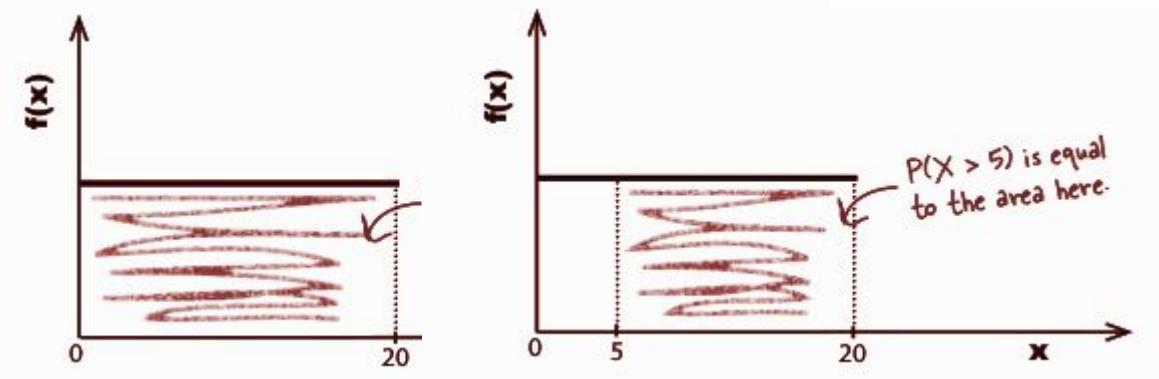
Sol:



Finding the probability

Q) You are waiting at a bus-stop for a bus which can come anytime within the next 20 minutes. What is the probability that you will have to wait for more than 5 minutes?

Sol:



f(x) x 20 = 1 => f(x)= 1/20 = 0.05

P(X>5) = (20 - 5) x 0.05 = 0.75

This is the **Uniform distribution**

Discrete vs Continuous

- Probability distribution function vs Probability density function

- For discrete probability distributions, we look at the probability of getting a particular value; for continuous probability distributions, we look at the probability of getting a particular range.

Height of male population

Q) What would be the height distribution of male population in India?

Height of male population

Q) What would be the height distribution of male population in India?

Ans: 1) Height is a continuous variable

Height of male population

Q) What would be the height distribution of male population in India?

Ans: 1) Height is a continuous variable

2)Most men will fall in the 5 feet to 6 feet range

Height of male population

Q) What would be the height distribution of male population in India?

Ans: 1) Height is a continuous variable

2)Most men will fall in the 5 feet to 6 feet range

3)There will be fewer shorter guys and fewer taller guys

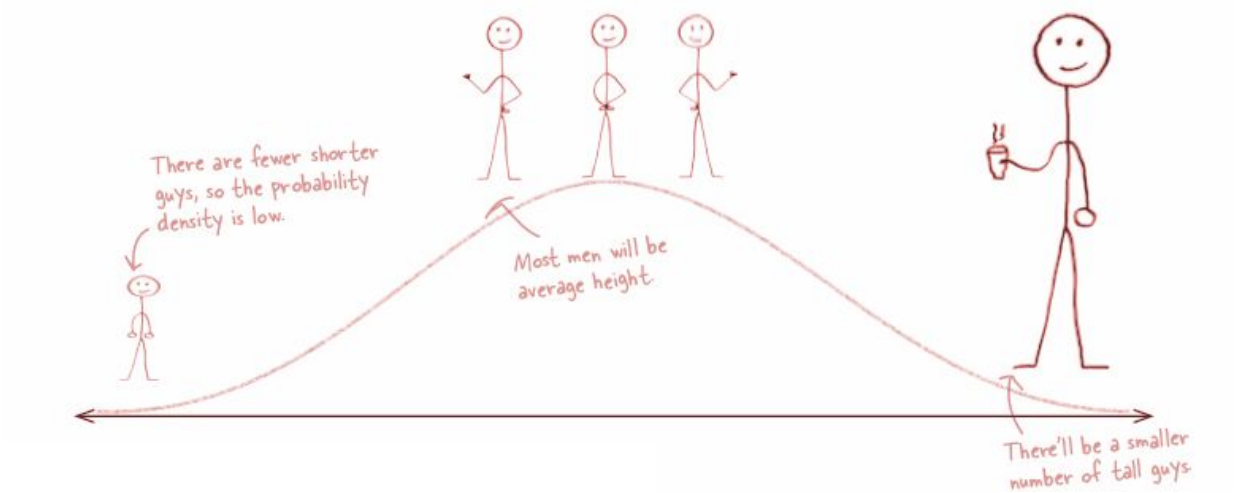
Height of male population

Q) What would be the height distribution of male population in India?

Ans: 1) Height is a continuous variable

2)Most men will fall in the 5 feet to 6 feet range

3)There will be fewer shorter guys and fewer taller guys



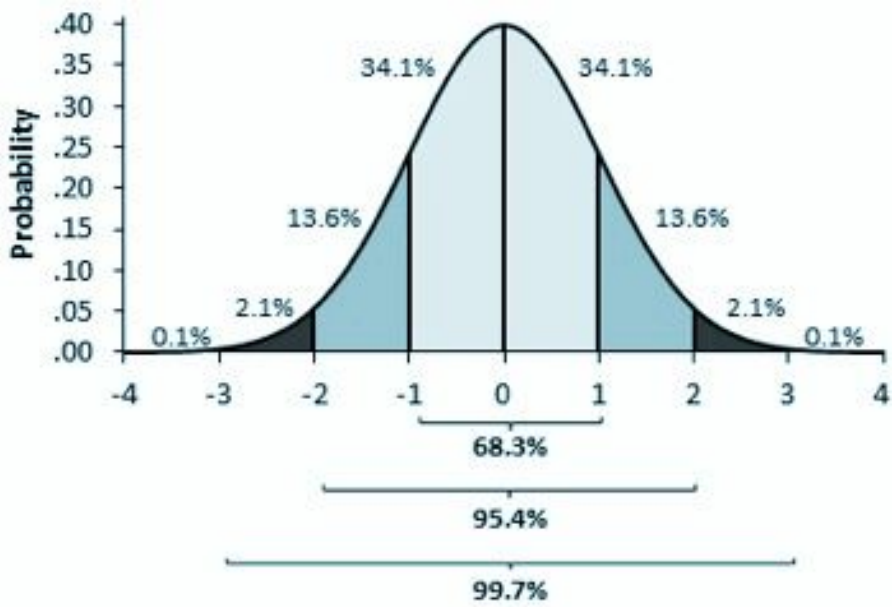
Normal Distribution

- The commonest and the most useful continuous distribution.
- A symmetrical probability distribution where most results are located in the middle and few are spread on both sides.
- It has the shape of a bell.
- Can entirely be described by its mean and standard deviation

Normal Distribution

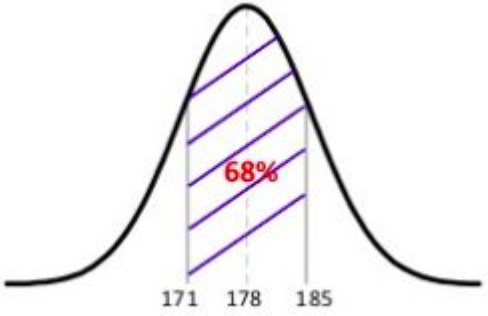
- Can be found practically everywhere
- The body temperature for healthy humans.
- The heights and weights of adults.
- The thickness and dimensions of a product.
- IQ and standardized test scores.
- Quality control test results.

Normal Distribution



Height of male population

- Suppose that the mean height is 178 cm and standard deviation is 7 cm
- We can say that around 68% of population are between 171 cm and 185 cm
- This might be a generalization, but it’s true if the data is normally distributed



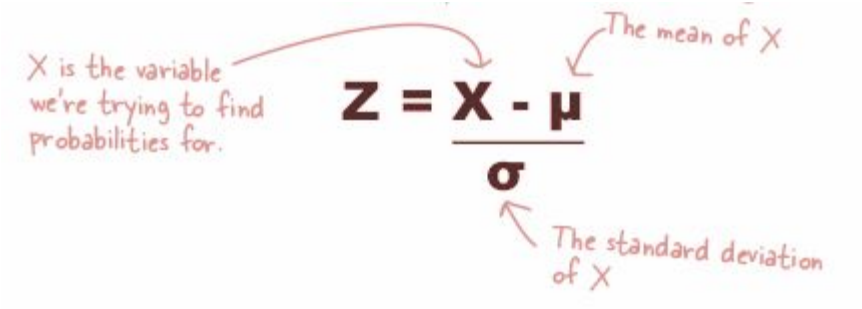
INTERNSHIPSTUDIO

Standard Normal Distribution

- A common practice to convert any normal distribution to the standardized form and then use the standard normal table to find probabilities.
- The Standard Normal Distribution (Z distribution) is a way of standardizing the normal distribution.
- It always has a mean of 0 and a standard deviation of 1

Normal to Standard Normal

- Converting helps to calculate probabilities



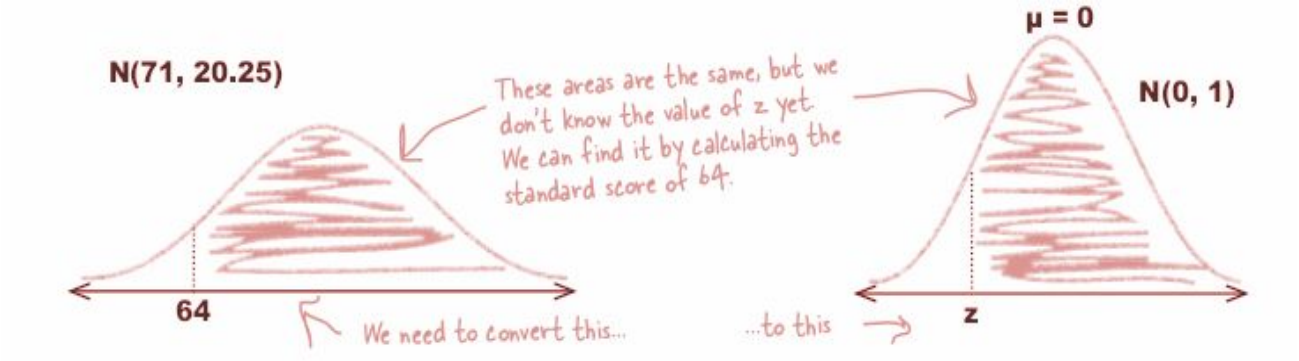
Find the probability

Q) X follows N(71, 20.25). Find z score for x = 64 and also P(X<64)

Find the probability

Q) X follows N(71, 20.25). Find z score for x = 64 and also P(X<64)

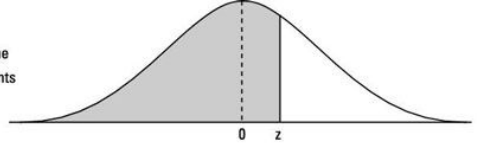
Sol:



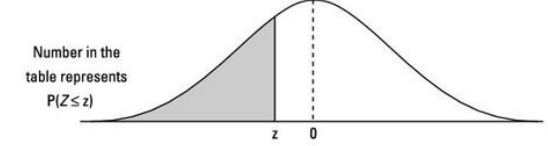
$\mu = 71$, $\boldsymbol{\sigma}^2 = 20.25$

$z = (x - \mu)/\boldsymbol{\sigma} = (64 - 71)/4.5 = -1.56$

Number in the table represents $P(Z \leq z)$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

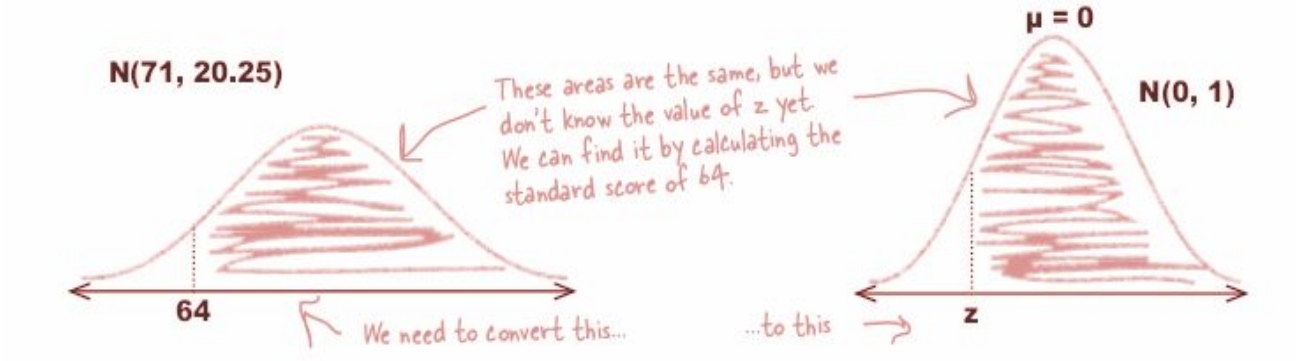


z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
–3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
–3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
–3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
–3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
–3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
–3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
–3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
–2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
–2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
–2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
–2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
–2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
–2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
–2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
–2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
–2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
–2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
–1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
–1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
–1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
–1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
–1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
–1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
–1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
–1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
–1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
–1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
–0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
–0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
–0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
–0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
–0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
–0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
–0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
–0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
–0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
–0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Find the probability

Q) X follows N(71, 20.25). Find z score for x = 64 and also P(X<64)

Sol:



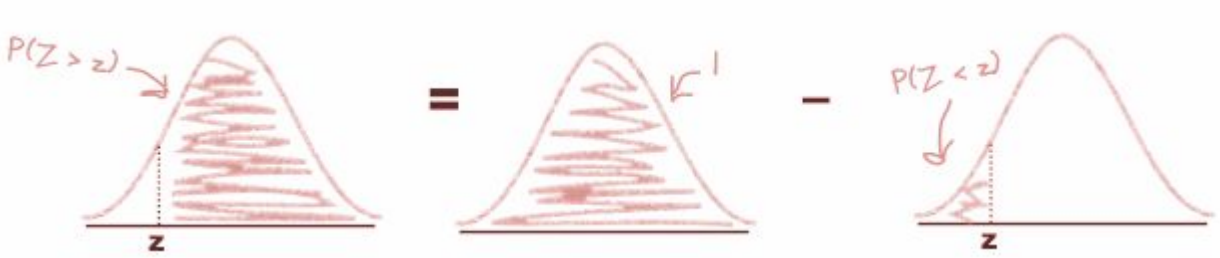
$\mu = 71$, $\sigma^2 = 20.25$

$z = (x - \mu)/\sigma = (64 - 71)/4.5 = -1.56$

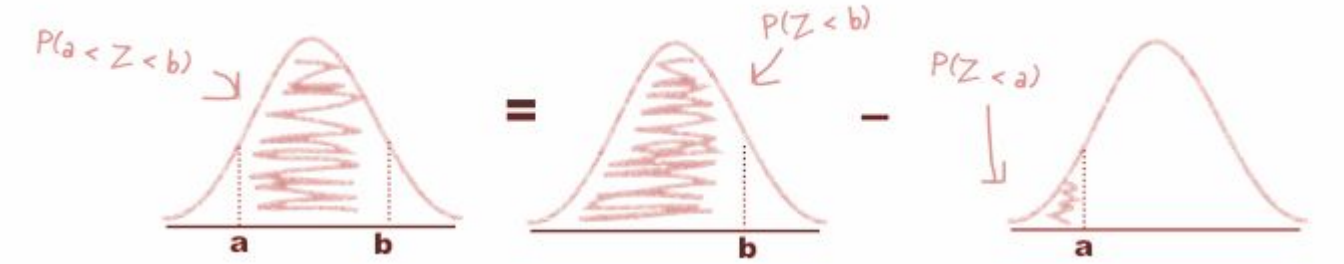
$P(X<64) = P(Z < -1.56) = 0.0594$

Tips for calculating probability

- Finding P(Z>z) => P(Z > z) = 1 - P(Z < z)



- Finding P(a < Z < b) => P(a < Z < b) = P(Z < b) - P(Z < a)



Practice Problem

Q) Find $P(Z < 1.42)$, $P(-0.15 < Z < 0.5)$ and if $P(Z > z) = 0.1423$ What's z?

Practice Problem

Q) Find a) $P(Z < 1.42)$, b)$P(-0.15 < Z < 0.5)$ and c) if $P(Z > z) = 0.1423$ What's z?

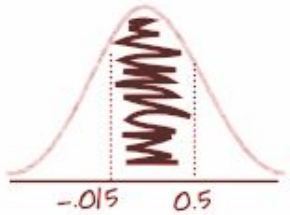
Sol: a)$P(Z < 1.42) = 0.9222$

Practice Problem

Q) Find a) P(Z < 1.42), b)P(-0.15 < Z < 0.5) and c) if P(Z > z) = 0.1423 What's z?

Sol: a)P(Z < 1.42) = 0.9222

b) P(-0.15 < Z < 0.5 = P(Z < 0.5) - P(Z < -0.15)

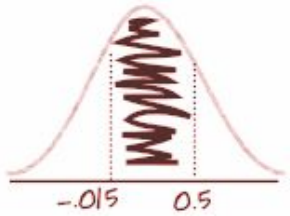


Practice Problem

Q) Find a) P(Z < 1.42), b)P(-0.15 < Z < 0.5) and c) if P(Z > z) = 0.1423 What's z?

Sol: a)P(Z < 1.42) = 0.9222

b) P(-0.15 < Z < 0.5 = P(Z < 0.5) - P(Z < -0.15)



c) P(Z > z) = 0.1423, which means that

P(Z < z) = 1 - 0.1423 = 0.8577

The next thing to do is find which value of z has a probability of 0.8577. Looking this up in the probability tables gives us z = 1.07 i.e. P(Z > 1.07) = 0.1423