Permutations and Combinations

Counting things

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Arranging things

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There are 6 possible results of the race

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Alternate:

There are 3 possible results for the first place

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There are 6 possible results of the race

Alternate:

There are 3 possible results for the first place

There are 2 possible results for the second place if first place is filled

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Arranging things

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There are 3 possible results for the first place

There are 2 possible results for the second place if first place is filled

There is 1 possible result for the third place if top two places are filled

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Hence total no of ways = 3x2x1 = 6 ways

Number of arrangements

If we had n horses, then ...

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Number of arrangements

If we had n horses, then we have nx(n-1)x(n-2)x...3x2x1 ways

i.e. We have n! ways

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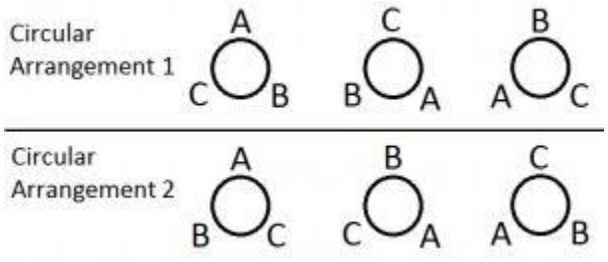
Circular arrangements



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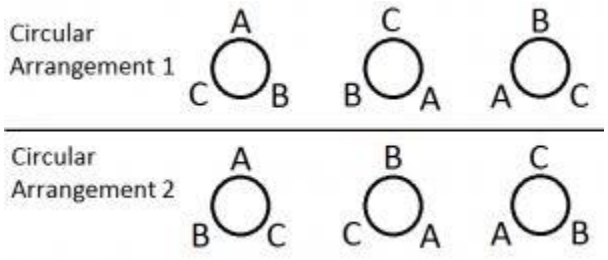

Number of arrangements here = (n-1)!

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Sol: There are 6! ways if the animals were different. But all 3 horses are all alike and all 3 zebras are considered alike

There are 3! ways of arranging the three zebras, and the result 6! includes each of these 3! arrangements. But since we're not concerned about which individual zebra goes where, these arrangements are all the same. So, to eliminate these repetitions, we can just divide the total number of arrangements by 3!

Same is the case with horses

Duplicates

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Sol: There are 6! ways if the animals were different. But all 3 horses are all alike and all 3 zebras are considered alike

Number of possible results = 6!/ 3!3! = 720/6x6 = 20

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Duplicates

If you want to arrange n objects where j of one type are alike, k of another type are alike, so are m of another type

and so on, the number of arrangements is given by

$$\frac{n!}{j!k!m!\ldots}$$

Top few arrangements

Q) We have 20 horses. We are only concerned with the top 3 positions. What are the number of possible results?

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Sol: Number of possible results for 1st place = 20

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Top few arrangements

Q) We have 20 horses. We are only concerned with the top 3 positions. What are the number of possible results?

Sol: Number of possible results for 1st place = 20

Number of possible results for 2nd place = 19

Number of possible results for 3rd place = 18

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Top few arrangements

Q) We have 20 horses. We are only concerned with the top 3 positions. What are the number of possible results?

Sol: Number of possible results for 1st place = 20

Number of possible results for 2nd place = 19

Number of possible results for 3rd place = 18

Total no of ways = 20x19x18 = 20!/17! = 6840

Permutations

In general, the number of permutations of r objects taken from n is the number of possible way in which each set of r objects can be ordered.

$$ {}^{n}P_{r} = \frac{n!}{(n-r)!} $$

Order doesn’t matter

Q) In the previous example, if you are not concerned with the precise order in which top 3 horses finished the race. Then what are the number of possibilities?

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No of ways = 20!/17!3! = 6840/3! = 1140

Combinations

In general, the number of combinations is the number of ways of choosing r objects from n, without needing to know the exact order of the objects.

Combination formula :

$$^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

Permutation formula :

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$