Discrete Probability Distributions

-Geometric, Binomial & Poisson

Find the probability

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P(X = 1) = P(Success in trial 1) = $\frac{1}{6}$

Find the probability

Q)You are playing a game of ludo, a game of dice and you can only start the game if you roll a 6. What's the probability that you will need at least 2 dice rolls

Sol: If we say X is the number of trials needed, then

$P(X = 1) = P(\text{Success in trial 1}) = \frac{1}{6}$

$P(X = 2) = P(\text{Success in trial 2} \cap \text{Failure in trial 1}) = \frac{1}{6} \times \frac{5}{6}$

Find the probability

Q)You are playing a game of ludo, a game of dice and you can only start the game if you roll a 6. What's the probability that you will need exactly two dice rolls and for at least two dice rolls?

Sol: If we say X is the number of trials, then

$P(X = 1) = P(\text{Success in trial 1}) = \frac{1}{6}$

$P(X = 2) = P(\text{Success in trial 2} \cap \text{Failure in trial 1}) = \frac{1}{6} \times \frac{5}{6} = 5/36$

$P(X \leq 2) = P(X = 1) + P(X = 2) = \frac{1}{6} + 5/36 = 11/36$

INTERNSHIPSTUDIO

Find the probability distribution

What if you needed to look at the probability of needing fewer than 10 attempts or even 20 or 100?

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What if you needed to look at the probability of needing fewer than 10 attempts or even 20 or 100?

-> If we have to work out every single probability, it would take forever

->Even though it's never ending, there's still a way of figuring out as there is a pattern to this type of probability distribution

Find the probability distribution

x	P(X = x)
1	$\frac{1}{6}$
2	$\frac{5}{6} \times \frac{1}{6}$
3	$\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$
4	$\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$

Generalising the distribution

$P(X = r) = \frac{5}{6} \wedge (r - 1) \times \frac{1}{6}$

$P(X = r) = q \wedge (r - 1)\, p$

(r - 1) failures and 1 success

Generalising the distribution

$P(X = r) = \frac{5}{6} \wedge (r - 1) \times \frac{1}{6}$

$P(X = r) = q \wedge (r - 1)\, p$

(r - 1) failures and 1 success

Here, $q = 1 - p$

p -> probability of success

q -> probability of failure

Generalising the distribution

$P(X = r) = \frac{5}{6}$ ^ $(r - 1) \times \frac{1}{6}$

$P(X = r) = q$ ^ $(r - 1)\ p$

$(r - 1)$ failures and 1 success

Here, $q = 1 - p$

p -> probability of success

q -> probability of failure

This is the **Geometric distribution**

Geometric distribution

- $P(X = r) = p\, q^{r-1}$
- $P(X > r) = q^r$
- $P(X \leq r) = 1 - q^r$

Represented as $X \sim Geo(p)$

Geometric distribution

- $P(X = r) = p\, q \wedge {}_{r-1}$
- $P(X > r) = q$
- $P(X \leq r) = 1 - q \wedge {}_{r}$

Represented as $X \sim Geo(p)$

Mean = Expectation(E(X)) = $\sum xP(X = x)$ = 1/p

Geometric distribution

- $P(X = r) = p\, q \wedge {r - 1}$
- $P(X > r) = q$
- $P(X \leq r) = 1 - q \wedge r$

Represented as $X \sim Geo(p)$

Mean = Expectation(E(X)) = $\sum xP(X = x)$ = 1/p

$Var(X) = E(X \wedge 2) - E \wedge 2 (X) = q/p\wedge 2$

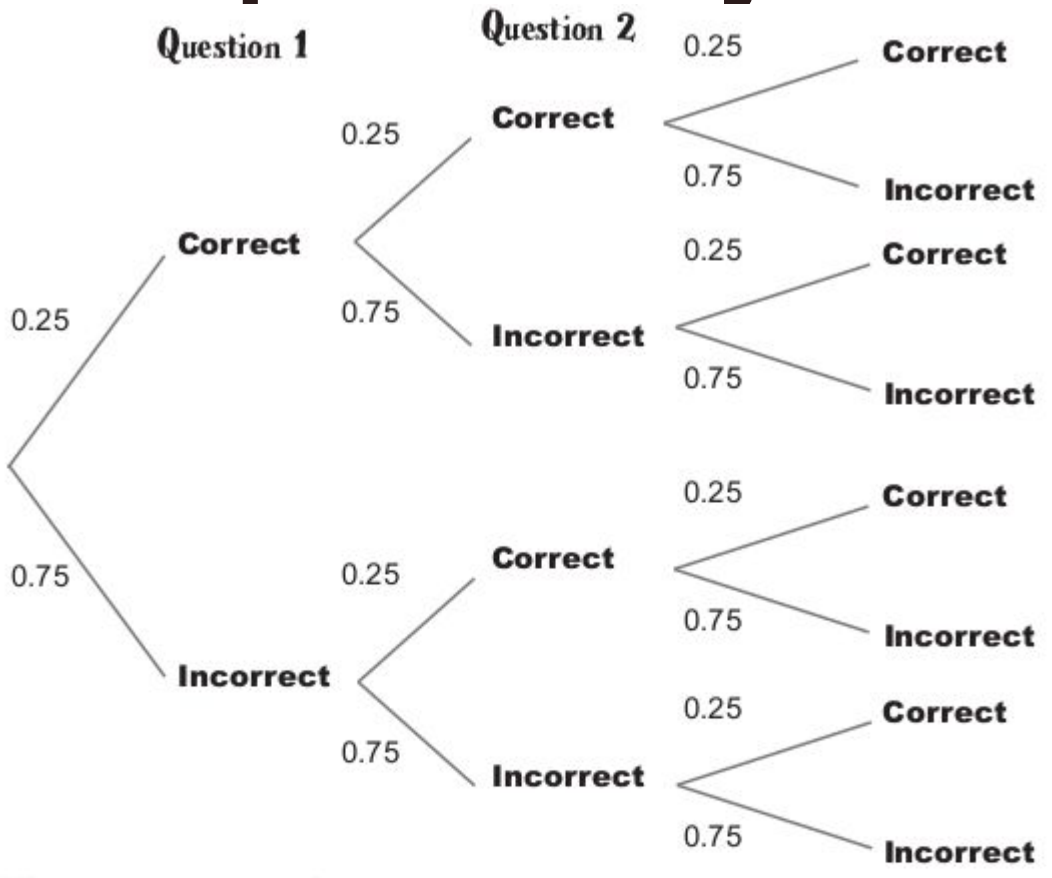
Find the probability

Q) You are attempting an exam which has MCQs. You do not know the answer to three questions and have to guess. Find the probability of getting 0 questions right, 1 question right, 2 right and 3 right if you have 4 options in a question?

INTERNSHIPSTUDIO

Find the probability

Sol:



Find the probability

x	P(X = x)	Power of 0.75	Power of 0.25
0	0.75^3	3	0
1	3x0.75^2x0.25	2	1
2	3x0.75x0.25^2	1	2
3	0.25^3	0	3

INTERNSHIPSTUDIO

Generalising the distribution

- $P(X = r) = {}^{n}C_{r} \times p^{r} \times q^{n-r}$
- $X \sim B(n, p)$
- $E(X) = np$
- $Var(X) = npq$

This is the **Binomial distribution**

Finding the probability

Q) There is a vending machine and it has been observed that it doesn't work properly on an average of 3.4 times per week. What's the probability of the machine not malfunctioning next week? Also, find the probability of malfunctioning three times next week

Poisson distribution

- Individual events occur at random and independently in a given interval. This can be an interval of time or space—for example, during a week, or per km

- You know the mean number of occurrences in the interval or the rate of occurrences, and it’s finite. The mean number of occurrences is represented by the Greek letter λ (lambda)

Poisson distribution

- Represented as X ~ Po(λ)
- $$P(X) = \frac{\lambda^x e^{-\lambda}}{X!}$$
- E(X) = λ
- Var(X) = λ

Finding the probability

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Q) There is a vending machine and it has been observed that it doesn't work properly on an average of 3.4 times per week. What's the probability of the machine not malfunctioning next week? Also, find the probability of malfunctioning three times next week

Sol: If there are no malfunctions, then X must be 0.

Substitute x = 0 and λ = 3.4 in $P(X) = \dfrac{\lambda^x e^{-\lambda}}{X!}$

We get P(X = 0) = 0.033

Similarly for P(X = 3), we get 0.216

INTERNSHIPSTUDIO