

Probability

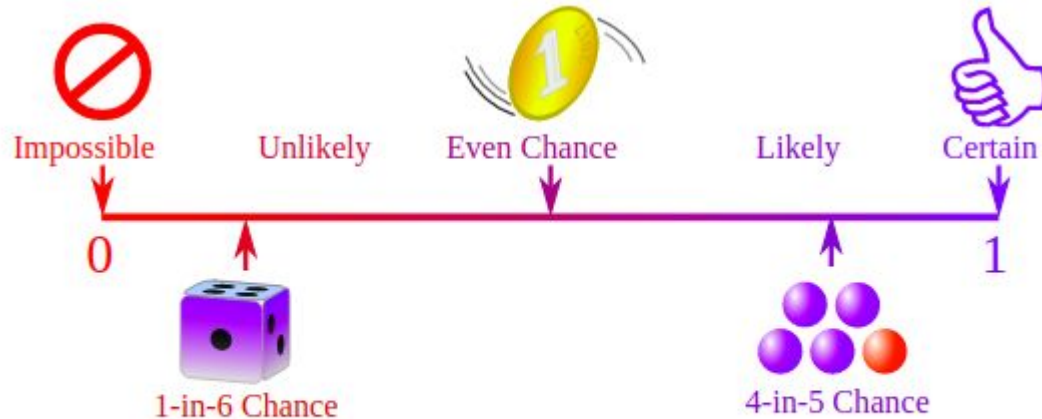
Taking chances

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Probability is always between 0 and 1

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Total number of ways = $2 \times 2 \times 2 = 8$. Fav. Cases = 7

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$$P(A) = \frac{7}{8}$$

Events

- Complementary Events
- Mutually Exclusive Events
- Independent Events
- Dependent Events

Complementary Event

$P(A)$ means "Probability of Event A"

$P(A')$ means "Probability of the complement of Event A"

The two probabilities always add to 1

$$P(A) + P(A') = 1$$

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$$P(A) = \frac{7}{8}$$

OR

$$P(\text{of getting at least one head}) = 1 - P(\text{no head}) \Rightarrow 1 - (1/8) = 7/8$$

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- A or B is the sum of A and B: $P(A \text{ or } B) = P(A) + P(B)$
- $P(\text{King and Queen}) = 0$
- $P(\text{King or Queen}) = (1/13) + (1/13) = 2/13$

Not Mutually Exclusive

- A or B is the sum of A and B minus A and B:
 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
OR $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

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OR $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Probability of choosing a king or a heart
 $P(A \cup B) = 13/52 + 4/52 - 1/52 = 16/52$

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$$P(A \text{ and } B \text{ and } C) = P(A) \times P(B) \times P(C)$$

Practice Question

Q) What is the probability of getting 3 heads in a row?

Sol: Sample space = [HHH, HHT, HTH, THH, TTH, THT, HTT, TTT]

$$P(A) = \frac{1}{8}$$

$$\begin{aligned} P(A) &= P(\text{H in 1st toss and H in 2nd toss and H in 3rd toss}) \\ &= P(H_1) \times P(H_2) \times P(H_3) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

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- Picking a card until you get a King
- $P(A) = 4/52$,
- If successful in first attempt $P(B) = 3/51$
- Represented as $P(B/A) = 3/51$

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$$\begin{aligned}P(A \text{ and } B) &= P(A) \times P(B/A) \\&= 4/52 \times 3/51 \\&= 12/2652 \\&= 1/221\end{aligned}$$

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$P(A|E2)$ = Probability that man reports four and it is not a four = $1/3$

$$P(E1|A) = P(E1)P(A|E1)/P(A)$$

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$P(A) = P(E1)P(A|E1) + P(E2)P(A|E2) = \frac{1}{6} \times \frac{2}{3} + \frac{5}{6} \times \frac{1}{3} = 7/18$

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$P(E1|A) = P(E1)P(A|E1)/P(A)$

$P(E1)P(A|E1) + P(E2)P(A|E2) = \frac{1}{6} \times \frac{2}{3} + \frac{5}{6} \times \frac{1}{3} = 7/18$

$P(E1|A) = P(E1)P(A|E1)/P(A) = \frac{1}{6} \times \frac{2}{3} / 7/18 = 2/7$



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