Seat No.: ________ Enrolment No.______________

GUJARAT TECHNOLOGICAL UNIVERSITY

MSC - INTEGRATED– SEMESTER - I - EXAMINATION- SUMMER-2023

Subject Code: 1310502 **Date: 26/06/2023**

Subject Name: Mathematics-I

Time: 02:30 PM TO 05:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make Suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Use of simple calculators and non-programmable scientific calculators are permitted.**

			Marks
Q.1	**(a)**	Evaluate: $\lim_{x\to 1}\frac{x\log x-(x-1)}{(x-1)\log x}$	**03**
	(b)	Solve the system of linear equations by Gauss elimination method: $x+2y-z=1, x+y+2z=9, 2x+y-z=2$	**04**
	(c)	Find the inverse of the matrix $\begin{bmatrix}8&4&3\\2&1&1\\1&2&1\end{bmatrix}$ by Gauss -Jordan method.	**07**
Q.2	**(a)**	Find the sum and product of the eigen values of the matrix $\begin{bmatrix}1&2&5\\2&3&4\\3&6&7\end{bmatrix}$	**03**
	(b)	Find the eigen values of the matrix $\begin{bmatrix}1&-6&-4\\0&4&2\\0&-6&-3\end{bmatrix}$.	**04**
	(c)	Check whether the matrix $\begin{bmatrix}2&1&0\\0&2&1\\0&0&2\end{bmatrix}$ is diagonalizable or not.	**07**
		OR	
	(c)	Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}$ and hence find A^{-1}.	**07**
Q.3	**(a)**	Expand $7+(x+2)+3(x+2)^3+(x+2)^4$ in powers of x.	**03**
	(b)	If $x=r\cos\theta, y=r\sin\theta$ then prove that $\left(\frac{\partial r}{\partial x}\right)^2+\left(\frac{\partial r}{\partial y}\right)^2=1$	**04**
	(c)	Find the extreme values of the function $x^3+y^3-3x-12y+20$.	**07**
		OR	
Q.3	**(a)**	Prove that $a^x=1+x\log a+\frac{x^2}{2!}(\log a)^2+.....$ by Maclaurin's series.	**03**

	(b)	If $u=\log\left(x^2+y^2+z^2\right)$, Show that $x\dfrac{\partial^2 u}{\partial y\partial z}-y\dfrac{\partial^2 u}{\partial z\partial x}=0$	**04**
	(c)	Find the minimum values of x^2yz^3 subject to the condition $2x+y+3z=6$ using Lagrange's method of undetermined multipliers	**07**
Q.4	**(a)**	Check whether the differential equation $\left(x^4-2xy^2+y^4\right)dx-\left(2x^2y-4xy^3+\sin y\right)dy=0$ is exact or not. If yes, then find the general solution.	**03**
	(b)	Solve the differential equation $\dfrac{dy}{dx}+\dfrac{y}{x}=\dfrac{y^2}{x^2}$.	**04**
	(c)	Solve the differential equation $(D^2-4D+4)y=\cos 2x$.	**07**

OR

Q.4	**(a)**	Solve the differential equation $\left(x^2+y^2+3\right)dx-2xydy=0$.	**03**
	(b)	Solve the differential equation $(x+1)\dfrac{dy}{dx}-y=e^{3x}\left(x+1\right)^2$.	**04**
	(c)	Solve the differential equation $y''-3y'+2y=e^x$ using the method of variation of parameters.	**07**
Q.5	**(a)**	Find the value of determinant $\begin{vmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \\ -2 & 1 & 3 \end{vmatrix}$	**03**
	(b)	Show that $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}=(x-y)(y-z)(z-x)$	**04**
	(c)	Solve the differential equation $y''+6y=x+x^2$ using method of undetermined coefficients.	**07**

OR

Q.5	**(a)**	Verify that $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}=\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$	**03**
	(b)	Use Cramer's rule to solve the system $-4x+2y-9z=2$ $3x+4y+z=5$ $x-3y+2z=8$	**04**
	(c)	Solve the differential equation $x^2\dfrac{d^2y}{dx^2}+2x\dfrac{dy}{dx}-12y=x^3$	**07**

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