Enrolment No. /Seat No.: ______________

GUJARAT TECHNOLOGICAL UNIVERSITY

M.SC(IT)- INTEGRATED – SEMESTER I- EXAMINATION –WINTER-2024

Subject Code: 1310502 **Date: 20/12/2024**

Subject Name: Mathematics-I

Time:10:30 AM TO 01:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make Suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Use of simple calculators and non-programmable scientific calculators are permitted.**

Q.1	**(a)**	Prove that $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$.	**03**
	(b)	Solve the following system of equations using Cramer's rule $2x - y = 5\,,\ x + y = 4$.	**04**
	(c)	Find the inverse of the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ by elementary operations.	**07**
Q.2	**(a)**	If $A = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & -4 \end{bmatrix}$, find the eigenvalues for the matrices (1) A^2 ,(2) A^{-1}.	**03**
	(b)	Verify Cayley-Hamilton theorem for matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.	**04**
	(c)	Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$.	**07**
		OR	
	(c)	Reduce the quadratic form $2x^2 + 2y^2 + 2z^2 - 2xz$ to canonical form by orthogonal transformations.	**07**
Q.3	**(a)**	Define Rank of matrix with example.	**03**
	(b)	Write the statement of Rolle's theorem. Verify Rolle's theorem for the function $\mathrm{f(x)} = \mathrm{x}^2 - 5\mathrm{x} + 4$ on [1,4].	**04**
	(c)	Solve $(D^2 + 1)y = cosec(x)$ using method of variation of parameters.	**07**
		OR	
Q.3	**(a)**	Define Normal form with example.	**03**
	(b)	Write the statement of Mean Value Theorem. Verify Mean Value Theorem for the function $\mathrm{f(x)} = \mathrm{x}^2 - 2\mathrm{x} + 4$ on [1,5].	**04**
	(c)	Solve $(D^2 + 9)y = \cos 4x + 2$.	**07**
Q.4	**(a)**	Evaluate $\lim_{x \to 0} \frac{1}{x}(1 - xcotx)$.	**03**
	(b)	If $u = \sin^{-1}\left(\frac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} + 3tanu = 0$.	**04**
	(c)	Find the extreme points of $f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$.	**07**
		OR	

Q.4 **(a)** Expand $\log_e x$ in powers of $(x-1)$ by Taylor's Theorem. **03**

(b) If $u = f(y-z, z-x, x-y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **04**

(c) Find the minimum values of x^2yz^3, subject to the condition $2x + y + 3z = a$. **07**

Q.5 **(a)** Solve: $\sinh x \cos y \, dx - \cosh x \sin y \, dy = 0$. **03**

(b) Solve $\frac{dy}{dx} + (tanx)\, y = sin2x$. **04**

(c) Solve $y'' + 4y = 8x^2$ using method of undetermined coefficients. **07**

OR

Q.5 **(a)** Solve $(D^3 - D^2 + 4D - 4)y = 0; D = \frac{d}{dx}$. **03**

(b) Solve $e^{-y}\frac{dy}{dx} + \frac{e^{-y}}{x} = \frac{1}{x^2}$. **04**

(c) Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \sin(logx)$. **07**
