Seat No.: ________ Enrolment No.______________

GUJARAT TECHNOLOGICAL UNIVERSITY

M.SC(IT)- INTEGRATED– SEMESTER I- EXAMINATION –WINTER-2023

Subject Code:1310502 **Date: 03/01/2024**

Subject Name: Mathematics-I

Time: 10:30 AM TO 01:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make Suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Use of simple calculators and non-programmable scientific calculators are permitted.**

			Marks
Q.1	**(a)**	Evaluate: $\lim_{x \to \frac{\pi}{2}} (\tan x)^{\sin 2x}$.	**03**
	(b)	Determine whether the following system of linear equations are consistent ? If yes then find its solution by using elementary row operations. $3x + y - z = 3; 2x - 8y + z = -5; x - 2y + 9z = 8$	**04**
	(c)	Find the inverse of $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ by using Gauss-Jordan method.	**07**
Q.2	**(a)**	If $A = \begin{bmatrix} 2+i & 3 & -1+3i \\ -5 & i & 4-2i \end{bmatrix}$, verify that A^*A is a Hermitian matrix, where A^* is the conjugate transpose of A.	**03**
	(b)	Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.	**04**
	(c)	Solve the differential equation by the method of variation paremeters $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$.	**07**
		OR	
	(c)	Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log x$.	**07**
Q.3	**(a)**	Verify Rolle's theorem for the function$f(x) = x^2 - 6x + 8$ in the interval $[2,4]$, and finc c if possible.	**03**
	(b)	Solve by Cramer's rule $x - 3y + z = 2; 3x + y + z = 6; 5x + y + 3z = 3$	**04**
	(c)	Solve $(D^2 - 4D + 3)y = e^x \cos 2x$.	**07**
		OR	
Q.3	**(a)**	Verify mean value theorem for the function $f(x) = x^3 - 6x^2 + 11x - 6$ in the interval $[0,4]$ and find c if possible.	**03**

	(b)	Show that $\begin{vmatrix} a & b & c \\ b+c & c+a & a+b \\ a^2 & b^2 & c^2 \end{vmatrix} = -(a-b)(b-c)(c-a)(a+b+c)$.	04
	(c)	Solve $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 6\frac{dy}{dx} = 1 + x^2$.	07
Q.4	(a)	Solve $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$.	03
	(b)	Solve $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$.	04
	(c)	If $u = sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{sinu\ cos2u}{4\ cos^3 u}$.	07
		OR	
Q.4	(a)	Solve $\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x^2}$.	03
	(b)	Solve $x^2\left(\frac{dy}{dx}\right)^2 + xy\left(\frac{dy}{dx}\right) - 6y^2 = 0$.	04
	(c)	Find the extreme values of the function $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.	07
Q.5	(a)	Prove that: $\begin{vmatrix} a+b & a & b \\ a & a+c & c \\ b & c & b+c \end{vmatrix} = 4abc$.	03
	(b)	If $u = xy + yz + zx, v = x^2 + y^2 + z^2, w = x + y + z$ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.	04
	(c)	Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and, hence, find the mtrix represantated by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$.	07
		OR	
Q.5	(a)	Without expanding, prove that the value of $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is zero.	03
	(b)	If $u = x^2tan^{-1}\frac{y}{x} - y^2tan^{-1}\frac{x}{y}$, find the value of $\frac{\partial^2 u}{\partial x \partial y}$.	04
	(c)	Show that the matrix $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ is diagonalizable. Hence find matrix P such that $P^{-1}AP$ is a diagonal matrix.	07

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