Enrolment No. /Seat No.: _____

GUJARAT TECHNOLOGICAL UNIVERSITY

M.SC(IT)- INTEGRATED– SEMESTER I- EXAMINATION –SUMMER-2024

Subject Code: 1310502 **Date: 13/05/2024**

Subject Name: Mathematics-I

Time: 02:30 PM TO 05:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make Suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Use of simple calculators and non-programmable scientific calculators are permitted.**

Q.1 **(a)** Evaluate $\lim_{x \to \frac{\pi}{2}} (\tan x)^{\sin 2x}$. **03**

(b) Solve the linear equation by Gauss Elimination Method: **04**
$x + y + 2z = 9$; $2x + 4y - 3z = 1$; $3x + 6y - 5z = 0$

(c) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$. **07**

Q.2 **(a)** Find the Eigen values of A and A^{-1} , where $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. **03**

(b) Verify Cayley –Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. **04**

(c) Is the matrix diagonalizable? $A = \begin{bmatrix} 0 & -2 & 2 \\ 1 & 2 & 0 \\ 3 & 2 & 0 \end{bmatrix}$ **07**

OR

(c) Solve the differential equation: $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 3y = x^2 \log x.$ **07**

Q.3 **(a)** Verify Rolle's theorem for the function $f(x) = x^2-4x+6$ in the interval $[2,4]$, and find c if possible. **03**

(b) Solve by Cramer's rule: $x - 3y + z = 2; 3x + y + z = 6; 5x + y + 3z = 3$ **04**

(c) Find the minimum values of x^2yz^3 subject to the condition $2x+y+3z = 6$ using Lagrange's method of undetermined multipliers. **07**

OR

Q.3 **(a)** Verify mean value theorem for the function $f(x) = x^3-9x^2+12x-6$ in the interval [0,4] and find c if possible. **03**

(b) Find the extreme values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ **04**

(c) Solve $(D^2-4D+3)y = e^x \cos 2x$ **07**

Q.4 **(a)** Solve $9yy' + 4x = 0$ **03**

(b) Solve $(x^3+ 3xy^2)dx + (3x^2y + y^3)dy = 0$ **04**

(c) Solve the differential equation $(D^2-4D+4)y = \cos 2x$ **07**

OR

Q.4 **(a)** Solve the differential equation $(x^2+y^2+3)dx-2xydy = 0$ **03**

(b) Solve $(4x^2D^2+16xD+9)y = 0$ **04**

(c) Solve the differential equation $y''-3y'+2y = e^x$ using the method of variation of parameters.

Q.5 **(a)** Find the value of the determinant $\begin{vmatrix} 1 & 2 & 7 \\ 5 & 0 & 2 \\ 3 & -4 & 6 \end{vmatrix}$ **03**

(b) Show that $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$ **04**

(c) Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence, find the matrix represented by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$. **07**

OR

Q.5 **(a)** Verify that $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ **03**

(b) Solve by Cramer's rule $\begin{aligned} 5x - 7y + z &= 11 \\ 6x - 8y - z &= 15 \\ 3x + 2y - 6z &= 7 \end{aligned}$ **04**

(c) Show that the matrix $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ is diagonalizable. Hence find matrix P such that $P^{-1}AP$ is a diagonal matrix. **07**

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