Ch8 Eigen Values and Vectors

Sunday, 16 March 2025 5:10 PM

Definition 8.1

Let A be $n \times n$ matrix.

A non-zero vector $\vec{x}$ is an EIGENVECTOR of A

if there is a real number scalar λ such $A\vec{x} = \lambda \vec{x}$.

eigenvalue.

Preposition 8.2

For an $n \times n$ matrix A, the set of eigenvectors of A corresponding to an eigenvalue λ is equal to the nonzero vectors in $Nul(A - \lambda I_n)$

$\vec{x} \in Nul(A - \lambda I_n)$

$(A - \lambda I_n)\vec{x} = 0$

$A\vec{x} = \lambda \vec{x}$

Activity 8.2

- Find the 2-eigenspace of A $\begin{pmatrix} 3 & 2 \\ 3 & 8 \end{pmatrix}$.

$\left(\begin{pmatrix} 3 & 2 \\ 3 & 8 \end{pmatrix} - \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\right)\vec{x} = 0$

$\begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0$

$x + 2y = 0$

$x = -2y$

$-\frac{x}{2} = y$.

$\therefore \{(t, -\frac{t}{2}) \mid t \in \mathbb{R}\}$

Definition 8.3.

- $Nul(A - \lambda I)$ is the λ-eigenspace of A
- $E_\lambda := Nul(A - \lambda I)$

Preposition 8.4

A real number λ is an eigenvalue of A if and only if $\det(A - \lambda I) = 0$

Definition 8.5

- Geometric multiplicity of λ

is defined to be the dimension of the λ-eigenspace E_λ.

8.2 The Characteristic Polynomial

Definition 8.6

For an nxn matrix A,

$$\chi_A(A) = \det(A - \chi I)$$

Strategies:

(1) Find all eigenvalues of A by solving polynomial equation $\chi_A(\chi) = 0$

(2) For each eigenvalue λ, calculate the λ-Eigenspace $E_\lambda = Nul(A - \lambda I)$

(3) Set of all eigenvalues is union of λ-Eigenspaces in (2)

Preposition 8.7

For any nxn matrix A

- The characteristic polynomial $\chi_A(x)$ is a polynomial of degree n.

Theorem 8.8 The fundamental theorem of Algebra

Let $f(x)$ be a polynomial of degree n with coefficients in $\mathbb{R}$.

$f(x) = (x - \alpha_1)^{m_1} \cdots (x - \alpha_k)^{m_k}$, where $\alpha_1, \dots, \alpha_k$ are distinct complex numbers and $m_i \geq 1$ are integers satisfying $m_1 + \cdots + m_k = n$

Definition 8.1

(1) We call equation above the factorization of f.

(2) The complex numbers $\alpha_1, \dots, \alpha_k$ are the roots of f.

(3) For each $i \in \{1, \dots, k\}$, the integer m_i is called the ALGEBRAIC MULTIPLICITY of α_i.