Ch9

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9.1 Integration by Substitution

If $\int f(x)\,dx = F(x) + C$

Then $\int f(g(x))\,g'(x)\,dx = F(g(x)) + C$

Change of variable for definite integrals.

- Let $a<b$
- Let g be function with continuous derivative on $[a,b]$
- Let f be a continuous function.
  - Assume range of g on $[a,b]$ is contained in f's domain.

Then $$\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du.$$

9.4. Integration by parts.

$$\int u\,dv$$
$$= uv - \int v\,du$$

9.5.
EG. $\int \arctan x\,dx.$

Let $u = \arctan x.$ $dv = dx$

$du = \frac{1}{1+x^2}$ $v = x.$

$= x \arctan x - \int \frac{x}{1+x^2}\, dx$ Let $t = 1+x^2$

$dt = 2x\, dx.$

$= x \arctan x - \frac{1}{2}\int \frac{1}{t}\, dt$

$= x \arctan x - \frac{1}{2} \ln|t| + C$

$= x \arctan x - \frac{1}{2} \ln|1+x^2| + C.$

9.6. $I = \int e^x \sin x\, dx. \ldots$

$= -e^x \cos x + e^x \sin x - I$

$2I = \ldots \ldots$

$I = \frac{\ldots}{2 \ldots}$

9.7. $\int \sin^n x \cos^n x\, dx.$

$\sin^n$ is odd $\Rightarrow$ make $\sin^n$ even

$\cos^n$ is odd $\Rightarrow$ make $\cos^n$ even

9.8. $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

9.9. Integral of sec.

$$I = \int \sec x \, dx$$

$$= \int \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x} dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \frac{d \sec x + \tan x}{\sec x \tan x + \sec^2 x}.$$

$$= \ln|\sec x + \tan x| + C$$

Method 2.

$$I = \int \sec x \, dx = \int \frac{1}{\cos x} dx$$

$$= \int \frac{\cos x}{\cos^2 x} dx$$

$$= \int \frac{1}{1 - \sin^2 x} d\sin x$$

$$= \int \frac{1}{1-u^2} du.$$

$$= \int \frac{1}{(1-u)(1+u)} du$$

$$\frac{1}{1-u^2} = \frac{A}{1-u} + \frac{B}{1+u}$$

$$1 = A + An + B - Bn$$

$$1 = (A-B)n + A+B$$

$A+B=1$

$A-B=0$

$2A=1.$

$A=\frac{1}{2}$

$B=\frac{1}{2}.$

$$\therefore = \frac{1}{2}\int \frac{1}{1-n} + \frac{1}{1+n}\, dn$$

$$= \frac{1}{2}\ln|1-n| + \frac{1}{2}\ln|1+n| + C$$

9.10 Partial Fraction.

$$\frac{ax^2+bx+c}{x(x-1)(x-2)} \Rightarrow \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2}$$

$\Rightarrow$ Must need $P(x)$ smaller degree for $\frac{P(x)}{Q(x)}$

E.g. $$\frac{3x^2+2x+1}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

9.12, EG 3.

$$\int \frac{1}{x^2+4}\,dx.$$

CANT PARTIAL FRACTION!

Let $\frac{x}{2} = u$.

$\frac{1}{2}\,dx = du$.

$$= \frac{1}{4}\int \frac{1}{\left(\frac{x}{2}\right)^2+1}\,dx$$

$$= \frac{1}{2}\int \frac{1}{u^2+1}\,du$$

$$= \frac{1}{2}(\arctan u) + C$$

$$= \frac{1}{2}\arctan\frac{x}{2} + C$$