

Ch4 Definitions

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Definition 4.1 Column space, null space

$$m \times n \text{ matrix } A = (\vec{v}_1 \ \dots \ \vec{v}_n)$$

$$\text{Column space } \text{Col}(A) := \text{Span}(\vec{v}_1, \dots, \vec{v}_n) \quad (\text{subspace of } \mathbb{R}^m)$$

$$\text{Null space } \text{Nul}(A) := \{ \vec{x} \in \mathbb{R}^n \mid A\vec{x} = \vec{0} \} \quad (\text{subspace of } \mathbb{R}^n)$$

Definition 4.3 Rank, nullity

$$\text{rank}(A) := \dim \text{Col}(A)$$

$$\text{Nullity}(A) := \dim \text{Nul}(A)$$

Definition 4.6 Row space

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 7 \\ 5 & 6 \end{pmatrix} \quad A^T = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 7 & 6 \end{pmatrix}$$

$$\text{Row}(A) = \text{Col}(A^T)$$

$$= \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 7 \end{pmatrix}, \begin{pmatrix} 5 \\ 6 \end{pmatrix} \right\}$$