Ch7 Definition

Saturday, 19 April 2025 5:55 PM

Definition 7.1 Unit Square

- a subset of $\mathbb{R}^2$

$S := \{\alpha_1\vec{e_1} + \alpha_2\vec{e_2} \mid 0 \le \alpha_1, \alpha_2 \le 1\}$

Definition 7.4 Determinants in $\mathbb{R}^2$

det(F) : oriented Area of F(S)

negatively oriented if e_1 cannot reach e_2 anticlockwise 180°.

parallelogram with $F(e_1)$, $F(e_2)$ as sides

Definition 7.6 Unit Cube

- subset of $\mathbb{R}^3$ by

$C := \{\alpha_1\vec{e_1} + \alpha_2\vec{e_2} + \alpha_3\vec{e_3} : 0 \le \alpha_1, \alpha_2, \alpha_3 \le 1\}$

Definition 7.8 Determinants in $\mathbb{R}^3$

- det(T) : oriented volume of F(C)

Definition 7.13 ij-minor

- $(n-1)\times(n-1)$ matrix A_{ij} (From $n\times n$ matrix A)
- i-th row and j-th column deleted

Definition 7.14 Determinants in $\mathbb{R}^n$ $\begin{pmatrix} + & - & + & - & + & - \\ \vdots & & & & & \end{pmatrix}$

$\det(A) := a_{11}\det(A_{11}) - a_{12}\det(A_{12}) + \dots + (-1)^{n+1} a_{1n}\det(A_{1n})$

Proposition 7.9

$$\det\begin{pmatrix} c & 0 & 0 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix} = c\det\begin{pmatrix} a_2 & a_3 \\ b_2 & b_3 \end{pmatrix}$$

Proposition 7.10

$$\det(v_1 \; v_2 \; v_3) = -\det(v_2 \; v_1 \; v_3) = \det(v_3 \; v_1 \; v_2)$$

Proposition 7.11

$$\det\begin{pmatrix} \alpha & \beta & \gamma \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} = \det\begin{pmatrix} \alpha & 0 & 0 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} + \det\begin{pmatrix} 0 & b & 0 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} + \det\begin{pmatrix} 0 & 0 & c \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

Example 7.12

$$A = \begin{pmatrix} a & b & c \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

or, just cofactor expansion (both row/column works)

$$\det(A) = \det\begin{pmatrix} a & 0 & 0 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} - \det\begin{pmatrix} b & 0 & 0 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} + \det\begin{pmatrix} c & 0 & 0 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

$$= a\begin{pmatrix} 2 & 3 \\ 5 & 6 \end{pmatrix} - b\begin{pmatrix} 1 & 3 \\ 4 & 6 \end{pmatrix} + c\det\begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix}$$

$$= -3a + 6b - 3c.$$

Theorem 7.18

- $\det(A) \neq 0 \iff$ invertible

Theorem 7.20

Changing two rows : $\det(B) = -\det(A)$; Add negative

Multiplying one row by c : $\det(B) = c\det(A)$; Add c per row

Add multiple of another row to any row : $\det(B) = \det(A)$; Add nothing

Lemma 7.22 $\det(EB) = \det(E)\det(B)$

Preposition 7.23 $\det(A) = \det(A^T)$

$\det(AB) = \det(A)\det(B)$