Ch10 Definition

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Definition 10.2 Defining matrix (with respect to bases B)

- Matrix A.
- $[F(\vec{x})]_B = A[\vec{x}]_B$
- $A_{F,B} = ([F(\vec{b_1})_B] \dots [F(\vec{b_n})_B])$

Definition 10.3 Geometric definition of matrix similarity

- $n \times n$ matrices B, C are similar if they represent the function but in different bases,
- there exist f s.t. $A_{F,B} = B$, $A_{F,C} = C$

Definition 10.4 Algebraic definition of matrix similarity

- there exist invertible $n \times n$ matrix P s.t.

$$B = P^{-1} C P$$

Definition 10.5 Diagonal matrix

- all nonzero entries in matrix appear in diagonal
- $D = \text{diag}(d_1, d_2, \dots, d_n)$

$$= \begin{pmatrix} d_1 & 0 & \cdots & 0 \\ 0 & d_2 & \cdots & 0 \\ 0 & 0 & 0 & d_n \end{pmatrix}$$

- Dilation Transformation T_D : transformation that stretch each coordinate by vector d.

$$D\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} d_1 x_1 \\ d_2 x_2 \\ \vdots \\ d_n x_n \end{pmatrix}$$

Definition 10.7 Diagonalizable matrix

- $n \times n$ matrix A diagonalizable if it is similar to a diagonal matrix

Corollary 10.10

- $n \times n$ matrix A has distinct n eigenvalues
  → A is diagonalizable

Lemma 10.12

GM of $\lambda \leq$ AM of λ.

Theorem 10.13 Diagonalization Theorem

- A is diagonalizable
  - Sum of GM of A $= n$
  - GM of every λ is equal to AM.

Definition 10.8 The Diagonalization Theorem

- how n linearly independent eigenvectors (for nxn matrix A)
  - Eigenvectors : $\vec{v_1}, \ldots, \vec{v_n}$
  - Eigenvalue : $\lambda_1, \ldots, \lambda_n$
  - $D = C^{-1}AC$

$$\begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_n \end{pmatrix} \qquad \begin{pmatrix} \vec{v_1} & \cdots & \vec{v_n} \end{pmatrix}$$

Definition 10.16 Eigendecomposition

- nxn diagonalizable matrix A.

$$A = CDC^{-1}$$

$C = (\vec{v_1} \cdots \vec{v_n})$, $D = \text{diag}(\lambda_1, \ldots, \lambda_n)$

C: Eigenvectors (must have n vectors!)

D: diagonal with λ!