Ch5 Definition

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Definition 5.1 Linear Transformation

- A function is linear if $F: \mathbb{R}^n \to \mathbb{R}^m$, for any $\vec{x}, \vec{y} \in \mathbb{R}^n$, $c \in \mathbb{R}$,

1. $F(\vec{x} + \vec{y}) = F(\vec{x}) + F(\vec{y})$
2. $F(c\vec{x}) = cF(\vec{x})$

Definition 5.2 Matrix Transformation

A: $m \times n$ matrix

Matrix Transformation:

$T_A(\vec{x}) := A\vec{x}$

Definition 5.5 Defining matrix

$F: \mathbb{R}^n \to \mathbb{R}^m$, $m \times n$ matrix A_F is defining matrix in transformation F.

no. of columns

$$A: \begin{pmatrix} F(e_1) & F(e_2) & \cdots & F(e_n) \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$

Definition 5.6 Matrix product

A: $m \times k$
B: $k \times n$ $\to$ C: $m \times n$

$T_A \circ T_B = T_C$, $C = AB$.

$AB = (A\vec{b_1} \quad A\vec{b_2} \quad \cdots \quad A\vec{b_m})$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}\begin{pmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}\begin{pmatrix} 7 \\ 9 \\ 11 \end{pmatrix}, \quad a_2 = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}\begin{pmatrix} 8 \\ 10 \\ 12 \end{pmatrix}$$

$$\begin{pmatrix} a_1 & a_2 \end{pmatrix}$$

Definition 5.10 Kernel and image.

$\text{im}(F) := \{ \vec{y} \in \mathbb{R}^m \mid F(\vec{x}) = \vec{y} \text{ for some } \vec{x} \in \mathbb{R}^n \}$

(Span of the product of transformation)

$\text{Ker}(F) := \{ \vec{x} \in \mathbb{R}^n \mid F(\vec{x}) = 0 \}$

(Vector $\vec{x}$ s.t. $A\vec{x} = 0$)