PAPER CODE	EXAMINER	DEPARTMENT	TEL
CPT 107	K.L. Man & G. Mogos	Discrete Mathematics and Statistics	1509

2024/25 SEMESTER 1 – Assessment II

BACHELOR DEGREE – Year 2

DISCRETE MATHEMATICS AND STATISTICS

DEADLINE: 8 December 2024 at 5 pm

LEARNING OUTCOMES

1. Understand, manipulate and reason about functions.
2. Represent statements in propositional logic and first-order predicate logic and to recognise, understand, and reason about formulas in propositional logic and first-order predicate logic.
3. Apply basic counting and enumeration methods as these arise in analysing permutations and combinations.
4. Perform simple calculation about discrete probability.

INSTRUCTIONS TO CANDIDATES

1. The Assessment should be done individually.
2. Total marks available are 100, accounting for 10% of the overall module marks.
3. The number in the column on the right indicates the marks for each question.
4. Answer all questions.
5. Answers should be written in English and in the answer script provided by student/candidate.
6. Relevant and clear steps should be included in your answers
7. Your solutions should be submitted electronically through the Learning Mall via the submission link.
8. The naming of Report (in pdf) is as follows: CPT107_StudentID_003.pdf (e.g CPT107_12345678_003.pdf)
9. Answers can also be handwritten, fully and clearly scanned or photographed for submission as one single PDF document through the Learning Mall via the submission link.

LATE SUBMISSIONS

Penalties will apply for any work submitted after the due date unless you have obtained a formal extension prior to the date for submission. The penalty applied will be 5% of the available marks for the assignment for each working day or part thereof that the assignment is late. The penalty will be capped at five working days (120 hours) from the assignment deadline. Work submitted after five working days will receive a grade of zero.

SPECIAL CONSIDERATION

Requests for an extension due to illness, misadventure, or other extenuating circumstances beyond your control will only be considered via a formal application for special consideration through e-Bridge.

PLAGIARISM AND ACADEMIC MISCONDUCT

It is assumed that you are thoroughly familiar with the policies of XJTLU regarding academic misconduct and plagiarism. Ignorance of the rules is not an acceptable defence against an allegation of academic misconduct. There are no excuses for engaging in plagiarism. Assignment answers will be checked for plagiarism. Impermissible similarities between student answers (current and former) can be detected by academic integrity software and by instructor and will be referred to the School's Examination Officer for investigation. Penalties will follow those of the University's Academic Integrity Policy on e-Bridge and can ranged from capped marks to expulsions from the university.

USE OF GENERATIVE ARTIFICAL INTELLIGENCE (AI) IS NOT PERMITTED

Notes:

- **To obtain full marks for each question, relevant and clear steps must be included in the answers.**
- **Partial marks may be awarded depending on the degree of completeness and clarity.**

QUESTION I: Functions and PHP (18 marks)

1). Let $f: \mathbb{Q} \to \mathbb{Q}$, check whether $f(x) = \frac{x+5}{3x-7}$ and $f^{-1}(x) = \frac{7x+5}{3x-1}$ are inverses.

(2 marks)

2). Let $S = \{x | x \in \mathbb{R} \ and\ x \geq 1\}$ and $T = \{x | x \in \mathbb{R}\ and\ 0 < x \leq 1\}$.

Find a function $f: S \to T$ that is a bijection. (4 marks)

3). Show that in a party there are always two persons who have shaken hands with the same number of persons. (2 marks)

4). Let $f: \mathbb{R} \to \mathbb{R}$ be given by $f(x) = 3x^2 + 2$ and $g: \mathbb{R} \to \mathbb{R}$ be given by $g(x) = 4x - 5$. Find $(f \circ g)(x)$, $(g \circ f)(x)$, $(f \circ f)(x)$ and $(g \circ g)(x)$.

(4 marks)

5). Find the composite function g ∘ f , where:

$$f: \mathbb{R} \to \mathbb{R}, \qquad f(x) = \begin{cases} x-2, & \text{if } x \geq 1 \\ x^3, & \text{if } x < 1 \end{cases}$$

$$g: \mathbb{R} \to \mathbb{R}, \qquad g(x) = \begin{cases} \dfrac{x+4}{3}, & \text{if } x \geq 0 \\ |x+1|, & \text{if } x < 0 \end{cases}$$

(6 marks)

QUESTION II: Logic (31 marks)

1). Consider the signature *S = {Cousin, Male, Female, jessie, carol, paul}* consisting of a binary predicate symbol *Cousin*, two unary predicate symbols *Male* and *Female*, and three constant symbols *jessie*, *carol* and *paul*. Assume that these symbols have the following meaning:

Cousin means "*is a cousin of*" (i.e., *Cousin (a, b)* states *a* is a cousin of *b*).

Male means "*is male*" (i.e., *Male(a)* states *a* is male).

Female means "*is female*" (i.e., *Female(a)* states *a* is female).

jessie, carol and *paul* refer to "Jessie", "Carol" and "Paul", respectively.

Translate the following sentences into S-formulae; that is, for each of the following sentences provide an S-formula that expresses the sentences:

(a) Jessie is a cousin of Carol. (3 marks)

(b) Paul has a female cousin. (3 marks)

(c) All cousins of Paul are also cousins of Carol. (3 marks)

(d) Jessie has at least 2 male cousins. (3 marks)

2). If statement *q* has the truth value 1 (TRUE), determine all truth value assignments for the statements *p, r*, and *s* for which the true value of the statement

$$(q \to ((\neg p \lor r) \land \neg s)) \land (\neg s \to (\neg r \land q))$$

is 1 (TRUE). (5 marks)

3). Determine the truth value of each of the following statements where U = {1, 2, 3} is the domain of discourse (a set of integer numbers):

(a) $\exists x\, \forall y\, (x^2 < y+1)$; (2 marks)

(b) $\forall x\, \forall y\, (x^2+y^2 < 12)$. (2 marks)

Explain the answers.

4). Without using any truth tables, prove or disprove that $\neg(p \wedge q) \vee (\neg p \wedge q) \equiv \neg p$.

(2 marks)

5). Let p denote "*He is rich*" and let q denote "*He is happy*." Write each statement in symbolic form using p and q. Note that "*He is poor*" and "*He is unhappy*" are equivalent to ¬p and ¬q, respectively.

(a) If he is rich, then he is unhappy. (2 marks)

(b) He is neither rich nor happy. (2 marks)

(c) It is necessary to be poor in order to be happy. (2 marks)

(d) To be poor is to be unhappy. (2 marks)

QUESTION III: Combinatorics (23 marks)

1). A student council consists of 15 students.

a). In how many ways can a committee of six be selected from the membership of the council?

(2 marks)

b). Two council members have the same major and are not permitted to serve together on a committee. How many ways can a committee of six be selected from the membership of the council? (2 marks)

c). Two council members always insist on serving on committees together. If they can not serve together, they will not serve at all. How many ways can a committee of six be selected from the council membership? (2 marks) d). Suppose the council contains eight men and seven women.

(i) How many committees of six contain three men and three women? (2 marks)

(ii) How many committees of six contain at least one woman? (2 marks)

e). Suppose the council consists of *three freshmen*, *four sophomores, three juniors*, and *five seniors*. How many committees of eight contain two representatives from each class?

(2 marks)

2). If one quarter of all subsets of size 5 of the set {1, 2, 3, …, m} contains the element 3, determine *m*. (4 marks)

3). How many 5-digit numbers can be formed from the integers 1, 2, ..., 9 if no digit can appear more than twice? (For instance, 41434 is not allowed.)

(4 marks)

4). Let A = {1, 2, ..., n + 1} and B = {1, 2, ..., n}. Find the number of functions f: A→B that are surjective. (3 marks)

QUESTION IV: Probability (28 marks)

1). Ten people (five men and five women) are standing in line. Assume that all possible ways in which they might line up are equally likely.

(a) What is the probability that they appear in line in alphabetical order by name? Please assume no two of them have the same English name. (3 marks)

(b) What is the probability that all the women precede all the men? (3 marks)

(c) What is the probability that between any two women there are no men? (3 marks)

(d) What is the probability that they alternate by gender in the line? (3 marks)

2). 50 butterflies are captured from a previously unexplored rainforest. 20 of these butterflies have stripes, 31 have spots and 14 have dots. 16 have stripes and spots, 9 have spots and dots and 5 have all three features. 12 of the 50 butterflies have none of these features.

(a) What is the probability that a butterfly randomly selected from the sample has stripes and dots? (3 marks)

(b) What is the probability that a butterfly randomly selected from the sample has at least two of these three features? (3 marks)

3). When a pair of balanced dice is rolled and the sum of the numbers showing face up is computed, the result can be any number from 2 to 12, inclusive.
What is the expected value of the sum?

(4 marks)

4). A man fires at a target n = 6 times and hits it k = 2 times,

(a) List the different ways that this can happen, (3 marks)

(b) How many ways are there? (3 marks)

END OF EXAM PAPER