PAPER CODE	EXAMINER	DEPARTMENT	TEL
CPT 107	K.L. Man	Discrete Mathematics and Statistics	1509

2024/25 SEMESTER 1 – Assessment I

BACHELOR DEGREE – Year 2

Discrete Mathematics and Statistics

DEADLINE: 01 November 2024 at 5 pm

LEARNING OUTCOMES

1. **Reason about simple datatypes using basic proof techniques.**
2. **Interpret set theory notation, perform operations on sets, and reason about sets.**
3. **Understand, manipulate and reason about unary relations, and binary relations.**

INSTRUCTIONS TO CANDIDATES

1. **The Assessment should be done individually.**
2. **Total marks available are 100, accounting for 10% of the overall module marks.**
3. **The number in the column on the right indicates the marks for each question.**
4. **Answer all questions.**
5. **Answers should be written in English and in the answer script provided by student/candidate.**
6. **Relevant and clear steps should be included in your answers.**
7. **Your solutions should be submitted electronically through the Learning Mall (LM) via the submission link.**
8. **The naming of Report (in pdf) is as follows: CPT107_StudentID_002.pdf (e.g CPT107_12345678_002.pdf).**
9. **Answers can also be handwritten and fully and clearly scanned or photographed for submission as one single PDF document via the LM.**

LATE SUBMISSIONS

Penalties will apply for any work submitted after the due date unless you have obtained a formal extension prior to the date for submission. The penalty applied will be 5% of the available marks for the assignment for each working day or part thereof that the assignment is late. The penalty will be capped at five working days (120 hours) from the assignment deadline. Work submitted after five working days will receive a grade of zero.

SPECIAL CONSIDERATION

Requests for an extension due to illness, misadventure, or other extenuating circumstances beyond your control will only be considered via a formal application for special consideration through e-Bridge.

PLAGIARISM AND ACADEMIC MISCONDUCT

It is assumed that you are thoroughly familiar with the policies of XJTLU regarding academic misconduct and plagiarism. Ignorance of the rules is not an acceptable defence against an allegation of academic misconduct. There are no excuses for engaging in plagiarism. Assignment answers will be checked for plagiarism. Impermissible similarities between student answers (current and former) can be detected by academic integrity software and by instructor, and will be referred to the School’s Examination Officer for investigation. Penalties will follow those of the University’s Academic Integrity Policy on e-Bridge and can ranged from capped marks to expulsions from the university.

USE OF GENERATIVE ARTIFICAL INTELLIGENCE (AI) IS NOT PERMITTED.

Notes:

- **To obtain full marks for each question, relevant and clear steps must be included in the answers.**
- **Partial marks may be awarded depending on the degree of completeness and clarity.**

QUESTION 1: Proof Techniques (20 marks)

(a) Use the method *proof by contradiction* to verify that there does not exist any integers x and y such that $5x + 15y = 1$.

(2 marks)

(b) Let r be a positive real number, and the Statement S be: "if r is not rational number, then $\sqrt[6]{r}$ is also not a rational number".

1. Prove or disprove the Statement S by *contradiction*.
2. If you think the converse of the Statement S is true, prove it. If not, give a counter-example.

(8 marks)

(c) For $n \in \mathbb{N}$, use proof by induction to show that $n^3 + 3 - n$ is divisible by 3.

(5 marks)

(d) For all natural numbers $n \geq 1$, use proof by induction to prove or disprove that:

$$2(1 + 2 + 4 + \ldots + 2^{n-1}) = 2^{n+1} - 2.$$

(5 marks)

QUESTION 2: Set Theory (32 marks)

(a) Let the universal set be $\mathbb{N}$, $\mathrm{P} = \{x \in \mathbb{N} \mid x \text{ is even and } 3 < x < 11\}$, $\mathrm{Q} = \{x \in \mathbb{N} \mid x \text{ is odd and } 2 < x < 10\}$, $\mathrm{R} = \{x \in \mathbb{N} \mid x \text{ is odd and } 3 < x < 9\}$ and $\mathrm{T} = \{x \in \mathbb{N} \mid x^2 = 5\}$. Find the elements of the following expressions:

1. $\mathrm{Q} \cap (\mathrm{P} \cup \mathrm{T})$
2. $\sim((\mathrm{Q} - \mathrm{T}) \cap \mathrm{P})$
3. $(\mathrm{R} \cup \mathrm{T}) \cap (\mathrm{Q} \,\Delta\, \mathrm{P})$
4. $\big(((\mathrm{T} \cup \mathrm{Q}) \cap (\mathrm{P} \,\Delta\, \mathrm{Q})\big) \cup (\mathrm{Q} - (\mathrm{P} - \mathrm{T}))) \cap \mathrm{R}$

(8 marks)

(b) Let X and Y be sets, then $X \subseteq Y$ if and only if $\sim X \subseteq \sim Y$. If you think that it is true, prove it. Otherwise, give a counterexample to show that it is false.

(6 marks)

(c) Let A, B, and C be sets. Prove or disprove that the following sets are equal:

1. A ∪ (B x C) = (A x B) ∪ (A x C)
2. A x (B − C) = (A x B) − (A x C)

(10 marks)

(d) Let X and Y be sets, and $X \subseteq Y$ if and only if $((Y - X) \cup X) = Y$. If you think that it is true, prove it. Otherwise, give a counterexample to show that it is false.

(8 marks)

QUESTION 3: Relations (48 marks)

(a) Let $B = \{n \in \mathbb{Z} \mid n \neq 0\}$ and R be the relation on the set B defined by R ={(a,b) | $a + b = 2c$}, where a,b,c $\in B$. Prove or disprove that R over B is an equivalence relation.

(8 marks)

(b) Let a set $S = \{1,2,3,4\}$. Using ordered pairs to show the transitive closure of the relation $\{(1,2),(2,3),(3,4),(1,3),(1,4),(2,4)\}$ on S.

(6 marks)

(c) Let R and S be two partial orders on a set Y, and let T be a relation on Y such that aTb (i.e. $a,b \in Y$) if and only if both aRb and aSb hold. Is T also a partial order on Y? Justify your answer with proofs and/or counterexamples.

(8 marks)

(d) Based on the partial order setting on the set A = {a, b, c, d, e} as {(a,a), (b,b), (c,c), (d,d), (e,e), (a,b), (a,c), (a,d), (a,e), (d,e)}, draw the corresponding Hasse diagram.

(4 marks)

(e) Let A be the points in the plane (A = R × R). We say two points are equivalent if they are equal distance from the origin. So (x, y) ~ (w, z) if $x^2+y^2 = w^2+z^2$. Prove or disprove that this relation (denoted by ~) is an equivalence relation on A. If it is an equivalence relation, then find its equivalent classes.

(12 marks)

(f) For all a, b $\in$ Z+, we define a◎b if a/b $\in$ Z+. Prove or disprove ◎ is a partial order and/or a total order on Z+.

(10 marks)

END OF ASSESSMENT I PAPER