Module: INT102 Assignment1

1. Assessment
The tasks contribute 10% to the overall assessment of INT102

2. Submission
Please submit assessment tasks via Learning Mall.

3. Deadline
3-April-2025, Thursday17:30.

Question 1 (15 marks)

Consider the following function: $f(n) = 7n + 3n^2 + 2n \log n + 2$

a. State the order of magnitude (in Big-O notation) of the function. (**5 marks**)

b. Prove that the function f(n) is of the order of magnitude as you stated above. (**10 marks**)

Question 2 (15 marks)

Given the algorithm as below:

Input: An array $A[0..n-1]$ of real numbers sorted in ascending order and a real number K.
$l \leftarrow 0;$
$r \leftarrow n - 1$
while $l \leq r$ **do**
    $m \leftarrow \lfloor (l + r)/2 \rfloor$
    if $K = A[m]$ **return** m
    else if $K < A[m]$ $r \leftarrow m - 1$
    else $l \leftarrow m + 1$
return -1

a. Which algorithm design technique is employed in the above algorithm? (**5 marks**)

b. What is the output of this algorithm for the input A[0..7] = [12, 23, 36 , 49, 51, 55, 99, 107] and K = 99? (**5 marks**)

c. What is the time complexity of the algorithm? (**5 marks**)

Question 3 (15 marks)

Given the Selection sort algorithm as below:

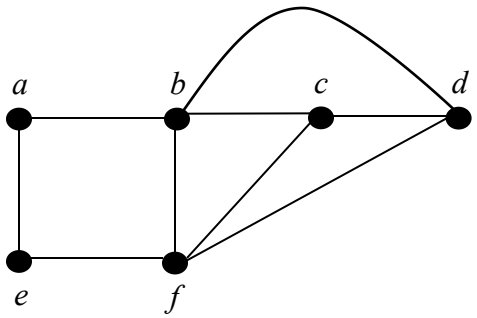
```
ALGORITHM SelectionSort(A[0..n − 1])
//Sorts a given array by selection sort
//Input: An array A[0..n − 1] of orderable elements
//Output: Array A[0..n − 1] sorted in ascending order
for i =0 to n − 2 do
    min = i
    for j =i + 1 to n − 1 do
        if A[j ]<A[min]  min = j
swap A[i] and A[min]
```

a. What is the number of key comparisons needed to sort the numbers A[0..5]=[6，1，2，3，4，5] in ascending order using the selection sort algorithm? **(6 marks)**

b. What is the number of swapping operations needed to sort the numbers A[0..5]= [6，1，2，3，4，5] in ascending order using the Bubble sort algorithm? **(9 marks)**

Question 4 (25 marks)

Consider the following graph G.



a. Give the adjacency matrix and adjacency list of the graph *G*. **(4 marks)**

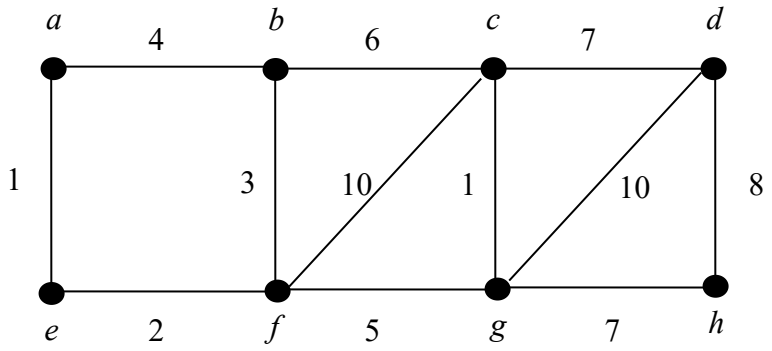
b. Give the incidence matrix and incidence list of the graph *G*. **(6 marks)**

c. Starting at the vertex a and resolving ties by the vertex alphabetical order traverse the graph by breadth-first-search (BFS) and construct the corresponding BFS tree. **(7 marks)**

d. Starting at the vertex a and resolving ties by the vertex alphabetical order traverse the graph by depth-first-search (DFS) and construct the corresponding DFS tree. **(8 marks)**

Question 5 (30 marks)

Consider the following graph G. The label of an edge is the cost of the edge.



a. Using *Prim's* algorithm, draw a *minimum spanning tree* (MST) of the graph. Also write down the change of the priority queue step by step and the order in which the vertices are selected. Is the MST drawn unique? (i.e., is it the one and only MST for the graph?) **(10 marks)**

b. Using *Kruskal's* algorithm, draw a *minimum spanning tree* (MST) of the graph G. Write down the order in which the edges are selected. Is the MST drawn unique? (i.e., is it the one and only MST for the graph?) **(10 marks)**

c. Referring to the same graph above, find the shortest paths from the vertex ***a*** to *all* other vertices in the graph G using *Dijkstra's* algorithm. Show the changes of the priority queue step by step and give the order in which edges are selected. **(10 marks)**

N.B. There may be more than one solution. You only need to give one of the solutions.