

all resistors have resistance =  $R \Omega$

using junction rule:  $\sum I_i = 0$

@  $V_1$  junction:

$$I_1 - I_2 - I_3 = 0$$

$$\frac{V_+ - V_1}{R} - \frac{V_1 - V_2}{R} - \frac{V_1 - V_3}{R} = 0$$

$$(V_+ - V_1) - (V_1 - V_2) - (V_1 - V_3) = 0$$

$$V_+ - 3V_1 + V_2 + V_3 = 0$$

$$3V_1 - V_2 - V_3 = V_+$$

@  $V_2$  junction:

$$I_1 + I_2 - I_3 - I_4 = 0$$

$$(V_+ - V_2) + (V_1 - V_2) - (V_2 - V_3) - (V_2 - V_4) = 0$$

$$0 = V_+ - 4V_2 + V_1 + V_3 + V_4$$

$$-V_1 + 4V_2 - V_3 - V_4 = 0$$

@  $V_{N-1}$  junction:

$$0 = I_1 + I_2 + (-I_3) - I_4$$

$$0 = (V_{N-3} - V_{N-1}) + (V_{N-2} - V_{N-1}) - (V_{N-1} - V_N) - (V_{N-1} - 0)$$

$$0 = V_{N-3} + V_{N-2} - V_{N-1} - V_{N-1} + V_N$$

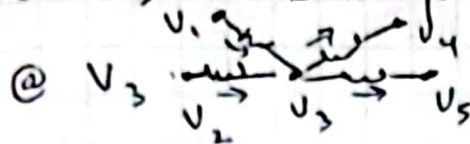
$$= -V_{N-3} - V_{N-2} + 2V_{N-1} - V_N$$

@  $V_N$  junction:

$$0 = (V_{N-2} - V_N) + (V_{N-1} - V_N) - (V_N - 0)$$

$$0 = -V_{N-2} - V_{N-1} + 3V_N$$

For  $V_3 - V_{N-2}$  junctions



2 currents in  
2 currents out  
so all will have

$$0 = I_{in1} + I_{in2} - I_{out1} - I_{out2}$$

all currents are equal to

$$I_i = \frac{\Delta V_i}{R} = \begin{cases} \frac{V_{i-2} - V_i}{R} \\ \frac{V_{i-1} - V_i}{R} \\ \frac{V_i - V_{i+1}}{R} \\ \frac{V_i - V_{i+2}}{R} \end{cases}$$

$i \neq 1, 2, N-1, N$

At  $i$ th junction  $0 = \sum I_i = \frac{1}{R} [(V_{i-2} - V_i) + (V_{i-1} - V_i) - \dots - (V_i - V_{i+1}) - (V_i - V_{i+2})]$

$$0 = V_{i-2} + V_{i-1} + V_{i+1} + V_{i+2} - 4V_i$$

$$= -V_{i-2} - V_{i-1} + 4V_i - V_{i+1} - V_{i+2}$$

Putting this into matrix form:  $\vec{A} \vec{V} = \vec{b}$

$\vec{b} = \vec{0}$

$$\vec{A} = \begin{pmatrix} -1 & -1 & 4 & -1 & -1 & \dots & 0 \\ -1 & 4 & -1 & -1 & -1 & \dots & 0 \\ 0 & -1 & -1 & 4 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & -1 & -1 & 4 & -1 \\ 0 & 0 & -1 & -1 & -1 & -1 & 4 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

$\vec{V} = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ \vdots \\ V_{N-1} \\ V_N \end{pmatrix}$

$\vec{b} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}$

- all blank elements are 0

$$\vec{A} = \begin{pmatrix} 3 & -1 & -1 & & & \\ & -1 & 4 & -1 & -1 & \\ & & -1 & -1 & 4 & -1 & -1 \\ & & & -1 & -1 & 4 & -1 & -1 \\ & & & \vdots & \vdots & & \ddots & \\ & & & -1 & -1 & 4 & -1 & -1 \\ & & & & -1 & -1 & 3 & -1 \\ & & & & & -1 & -1 & 3 \end{pmatrix}$$

all blank space  
are zero

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_{N-1} \\ v_N \end{pmatrix}$$

$$\vec{w} = \begin{pmatrix} w_1 \\ w_2 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$