PHYS6350 (Computational Physics)

Problem Set 1

Due:

Fri, Feb 3, 2023, end of the day

Instructions:

Send via Teams or email. Should include code (or, where applicable, note) which solves the problems and, where applicable, plots and/or annotations. The preferred languages are Python/Jupyter notebook/C++ while other choices may be accepted on an individual basis.

Problem 1.1

Write a program that finds both roots of the quadratic equation

$$ax^2 + bx + c = 0$$

while avoiding large round-off errors.

Calculate the roots to at least 10 decimal digits for the following choices of parameters:

1. $a = 1.6 \cdot 10^{-4}$, $b = 1.5 \cdot 10^{4}$, $c = 1.3 \cdot 10^{-4}$
2. $a = 1.6 \cdot 10^{-4}$, $b = -1.4 \cdot 10^{4}$, $c = 1.3 \cdot 10^{-4}$

Problem 1.2

The function $\cos(x)$ evaluated at $x = 0, 1, 2, 3$ reads

x	cos(x)
0	0.
1	0.5403023058681398
2	-0.4161468365471424
3	-0.9899924966004454

There exists a corresponding interpolating polynomial

$$p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

1. Calculate the values of coefficients a_i, $i = 0, 1, 2, 3$ of the interpolating polynomial p(x)
2. Compare $p(x)$ with $\cos(x)$ in a range $x = 0 \ldots 5$.

Problem 1.3

Consider the following equation

$$\frac{x}{2} = e^{1-x^2}$$

1. Find the root of this equation to at least 10 decimal digits accuracy using a method of your choice.
2. Find the root of this equation to at least 10 decimal digits accuracy using the relaxation method.
   Hint: Consider taking the logarithm of both sides of the equation before casting it in the $x = \varphi(x)$ form