Lecture 1: CS6250 Graphics & Visualization

- Introduction
- Coding Standards
- Geometric Background
- Transformations
- Image Formation
- Shape Representations

Relation between Areas

- Imaging
- Computer Graphics
- Visualization
- Computer Vision

Coding

All code for this class must be written in C++ and/or Tcl/Tk and run on prime. Coding standards are the same as for CS3610/5610 (see on-line document).

Any code used from any source ***MUST*** be properly credited.

VTK will provide a framework for graphics algorithms.

Geometry

Lines

Equations of a line:

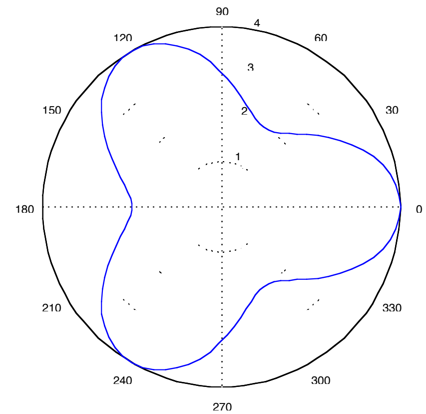
-
-
-
-

Curves

E.g. Circle:

Polar Coordinates

What does the curve $r = a + bcosn\theta$ look like?



3D Geometry

3D line equation:

Vector equations:

for point and slope, or

for two points:

Plane equation:

where p is the perpendicular distance to the origin.

Coordinate Systems

There are at least four coordinate systems commonly used in computer graphics.

- model
- world
- view
- display

Coordinate Transforms

How can we represent a point?

a.)

b.)

Which representation is better?

Why?

Homogenous Coordinate Transforms

Translation:

Scale:

Rotation

Reflections?

The Image Formation Process

Image Formation

- Shading is roughly independent of observer position
- Shading depends on:
  - Object reflectivity
  - Angle between source ray and surface normal
  - Source distance from object

Objectives

- Understand how images are formed
- Understand how they are sensed
- Understand mapping from world to image

Image Formation Overview

- Model Object Patch
- Model Projection
- Model Illumination
- Model Sensor
- Model Image

Notations

- Vectors $\vec{i}, \hat{i}$
- Hat vector is same as arrow, except it is a unit vector.
- Magnitude of a vector $i = |\mathbf{i}|$.

Surface Patch Model

1st order approximation: ??

Tangent Plane

Definition: set of all tangents to curves on the surface.

Parameterization:

- position
- orientation: normal to tangent plane.

2nd order approximation

principal curvatures

Surface Patch Diagram

Second Order Approximation

Normal to tangent plane

- Set of planes containing normal are called normal sections, they cut surface along curves
- The curvatures of these curves are called normal curvatures
- The maximum and minimum curvatures are called principal curvatures
- The maximum and minimum curvature directions are perpendicular.

Projective Geometry

Simplest sensor model is merely projection.

Assumption, light travels in straight lines, ignore diffraction.

We will consider projection through a point onto a plane.
We will need to represent the point and the plane.

Note that there are other camera geometries.

Pinhole Camera

The projection maps $\mathbf{p} = (x, y, z)^T$ in space to

$\mathbf{P} = (U, V, -W_p)^T$ in the image.

We must represent the center of the projection, a point $\mathbf{O_p}$

Image Representation

- represent the image plane

  normal to plane $\mathbf{W_p} = \mathbf{N_p}$

  let the optical axis be positive from the image plane through $\mathbf{O_p}$

  let $W_p = |\mathbf{W_p}|$

- image coordinates defined by $\mathbf{U_p}$, $\mathbf{V_p}$

  $\mathbf{U_p}$ defines the orientation of grid in image; its magnitude is the scale factor

  $\mathbf{V_p}$ is defined to make a right-handed triple:

$$\hat{\mathbf{W}}_\mathbf{p} = \hat{\mathbf{U}}_\mathbf{p} \times \hat{\mathbf{V}}_\mathbf{p}$$

  its magnitude is the scale factor.

E.G. $\mathbf{U_p}$ defines the orientation of scan lines and pixel spacing in television scan (almost true).

Space coordinates

- $\mathbf{z}$ is along normal to image plane $\mathbf{W_p}$
- vector $\mathbf{x}$ is normal to $\mathbf{z}$.
- vector y chosen to make a left handed triple:

$$\hat{\mathbf{x}} \times \hat{\mathbf{y}} = -\hat{\mathbf{z}}$$

Diagram!

Egocentric System

The image coordinate system induces a viewer centered system in space with axes $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$.

$$\hat{\mathbf{x}} = \hat{\mathbf{U}}_\mathbf{p}$$

$$\hat{\mathbf{y}} = \hat{\mathbf{V}}_\mathbf{p}$$

$$\hat{\mathbf{z}} = -\widehat{\mathbf{W}}_\mathbf{p}$$

Egocentric System Cont.

A space point $\mathbf{p_i}$ has space coordinates $(x_i,y_i,z_i)^T$ where:

$$x_i = \mathbf{p}_i \cdot \hat{\mathbf{x}}$$

$$y_i = \mathbf{p}_i \cdot \hat{\mathbf{y}}$$

$$z_i = \mathbf{p}_i \cdot \hat{\mathbf{z}} = -\,\mathbf{p}_i \cdot \widehat{\mathbf{W}}_\mathbf{p}$$

More Diagrams

Projection Equation

Image Plane is at $-W_p$. By similar triangles:

$$\frac{U}{-W_p} = \frac{x}{z}$$

$$\frac{V}{-W_p} = \frac{y}{z}$$

this gives:

$$U = \frac{-W_p}{z} x$$

$$V = \frac{-W_p}{z} y \qquad \text{simply scaled space}$$

$$W = -W_p \qquad \text{coordinates}$$

Inverted Coordinates

All image coordinates are inverted with respect to space coordinates. In addition, all points have the same z coordinate!

Projection Vector Form:

Map $\mathbf{p} = (x, y, z)^T$ to $\mathbf{P} = (U, V, -W_p)^T$.

$$\mathbf{P} = \frac{-W_p}{z}\, \mathbf{p}$$

$$z = -\,\mathbf{p} \cdot \widehat{\mathbf{W}}_{\mathbf{p}}$$

$$\mathbf{P} = \frac{W_p \mathbf{p}}{\mathbf{p} \cdot \widehat{\mathbf{W}}_{\mathbf{p}}}$$

Orthographic Projection

This projection is a parallel projection onto some plane. It is frequently used as a far field approximation. Distance to object large compared to object size, as in a telephoto lens. Projection is approximately constant for all object points.

$$(U, V, -W_p) = (x, y, z)$$

$$U = x; \quad V = y$$

It can be seen as the limiting case of the perspective projection.

Other Imaging Geometries

- Moving belt with linear sensor, or moving airplane with linear sensor.

  separate projection center for each line!
- Spherical projection. E.G. eye!

Magnification

Magnification is defined as the ratio of image dimension to object dimension.

$$\frac{U}{x} = \frac{V}{y} = \frac{W_p}{z}$$

magnification is thus just the ratio of image distance to object distance. i.e. magnification depends on distance.

E.G. given an object at 2m, and given a 50mm lens for a 35mm camera, magnification is roughly:

$$\frac{50\text{mm}}{2\text{m}} \approx 2.5 \times 10^{-2}$$

Brightness

How do we determine the brightness of a point in the image?

Three concepts:

- Intensity of source; model illumination
- Radiance; model reflectivity of surfaces
- Irradiance; model sensor system

Intensity of Source

To model general illumination, first model point source, and then integrate contributions.

The intensity of a light source is a measure of the source intensity as seen by the eye. Most sources have different intensities when viewed from different directions.

E.G. a lamp is brighter overhead than through the shade.

E.G. the sun. Sunspots make the illumination non-uniform!

Intensity of Source Diagram

Discussion

Point source has power **P** it radiates uniformly over a sphere. As the light spreads out, it neither increases nor decreases. Power is independent of radius.

Power through a patch of the surface is **P** times fraction of sphere the patch covers, independent of radius.

Surface area of a sphere is $4\pi r^2$, thus power per projected area on a sphere is:

$$\delta P = \frac{\mathbf{P}}{4\pi r^2}$$

Solid Angle

Consider first angle, a one-dimensional form:

$$angle = \frac{arc\ length}{radius}$$

angle is 2π times fraction of the circle subtended by a segment.

Extend to two-dimensional form solid angle Ω:

$$\Omega = \frac{projected\ area}{r^2}$$

Solid angle is 4π times the fraction of the sphere subtended by surface patch.

Illumination in General

- Point source at infinity
- Sun: Direct Illumination

  Sky: gives diffuse illumination ~ 10% of direct on clear day.

  The sky is an extended source, its spectrum is non-uniform.

  At sunset, the sky is red in west, blue in east, it can affect color judgement.

- Mutual Illumination (concavities in blocks, lying in grass).
- Under trees and clouds
- Indoors

  multiple sources

  extended sources

  mutual illumination

  variety of spectra

  spectra non-uniform in position

- Incandescent lamps

  roughly point source, comparable to sun

  energy in red
- Fluorescent tubes

  extended source

  broad spectrum like sun, but blue to green

Summary of Illumination

It is difficult to model illumination accurately.

- spectra of sources varies
- clouds, diffuse, mutual illumination are unknown and complex
- intensity non-uniform
- spectra non-uniform

Luckily, simplistic assumptions can be useful.

Irradiance

Irradiance is a measure of image brightness. It measures how much light falls on a unit area of surface or image. It is measured in W/m^2.

Radiance

Radiance is related to the energy flux emitted from a surface. It is the flux emitted per unit foreshortened surface area per unit solid angle. Measured in $W/m^2/\text{steradian}$.

Color

