Divide & Conquer

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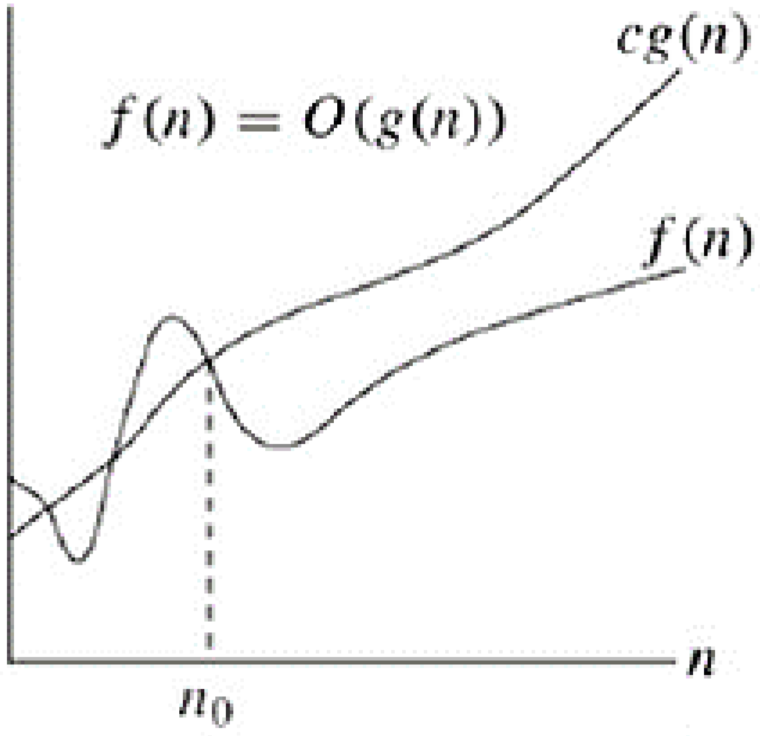
1. Introduction

Big-O notation

- $f(n)$ is $O(g(n))$,

  if there are positive constants c and n_0 such that $f(n) \leq cg(n)$ for $n \geq n_0$

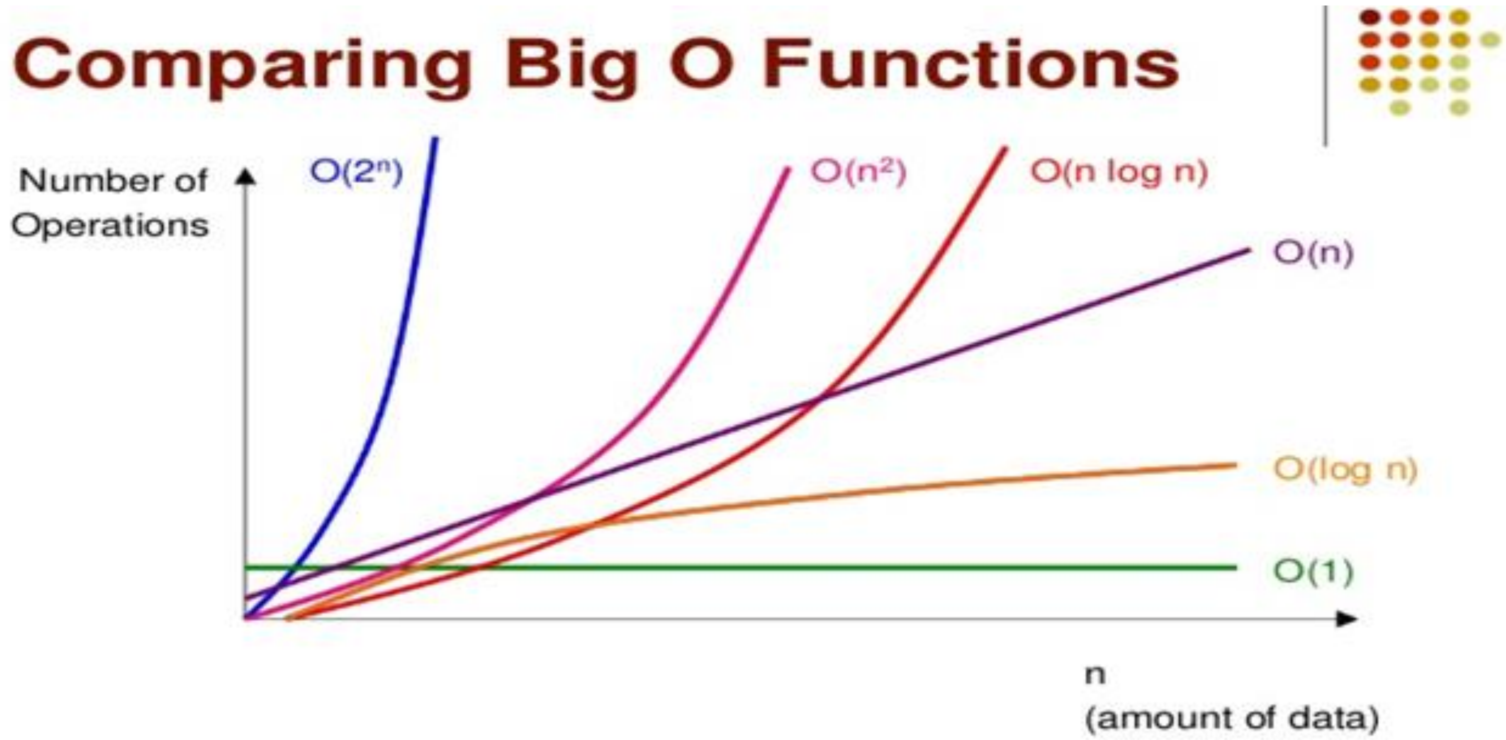


Big-O notation

$f(n)=4n+5$: $O(n)$

$f(n)=7n^2+40$: $O(n^2)$

Big-O notation

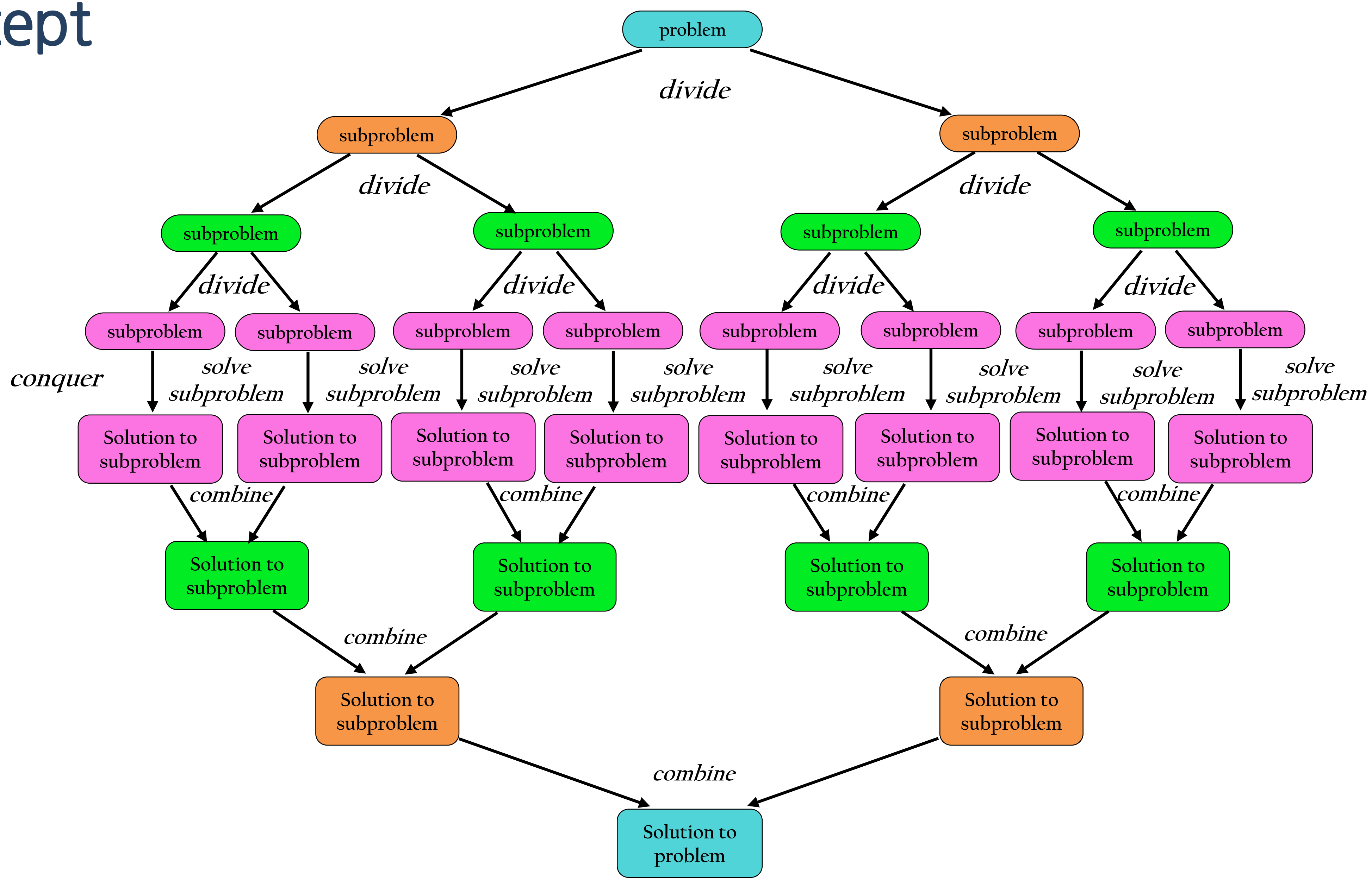


Performance :

O(1) > O(logn) > O(n) > O(nlogn) > O(n^2) > O(2^n)

2. Divide & Conquer

Concept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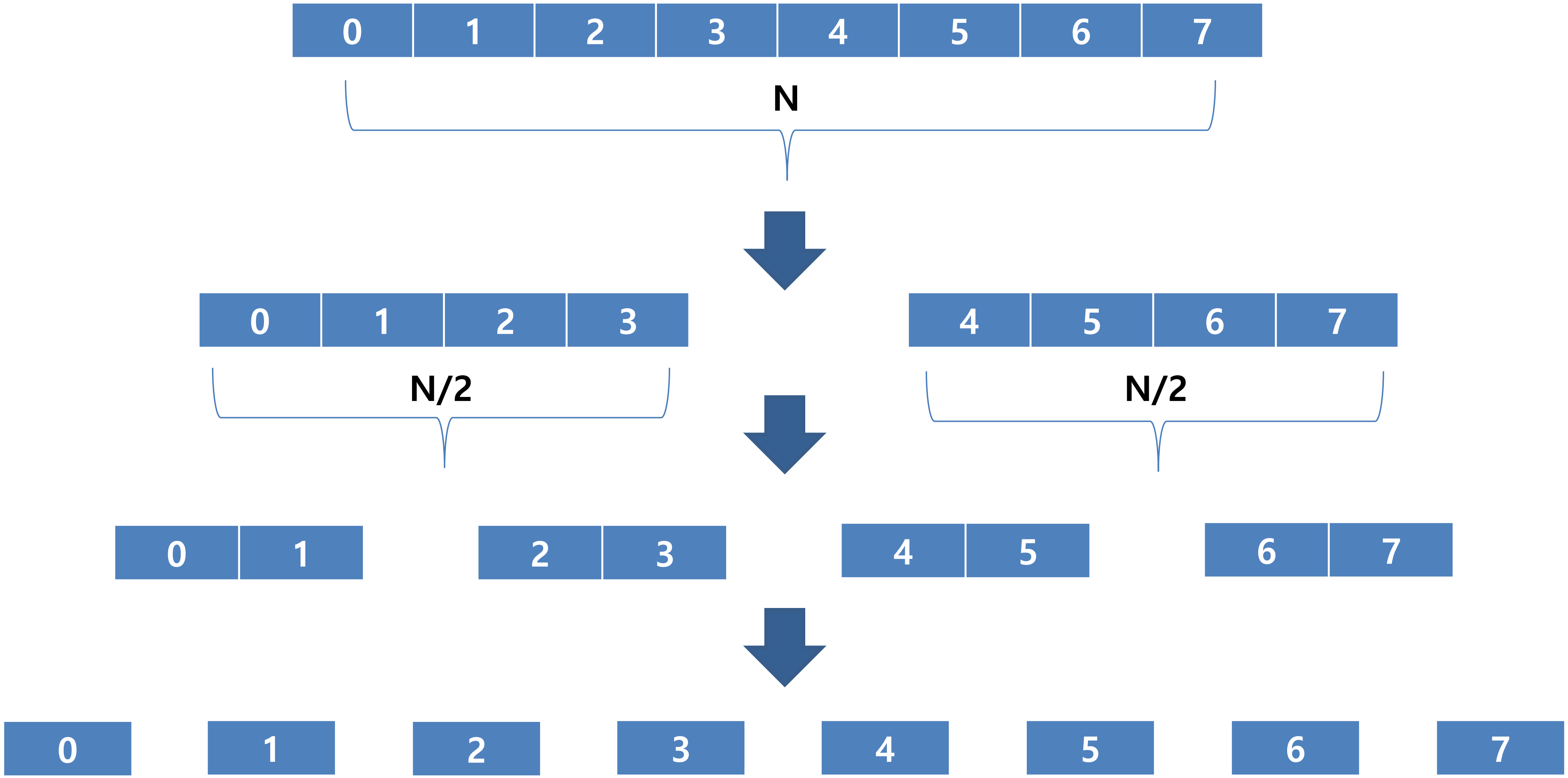
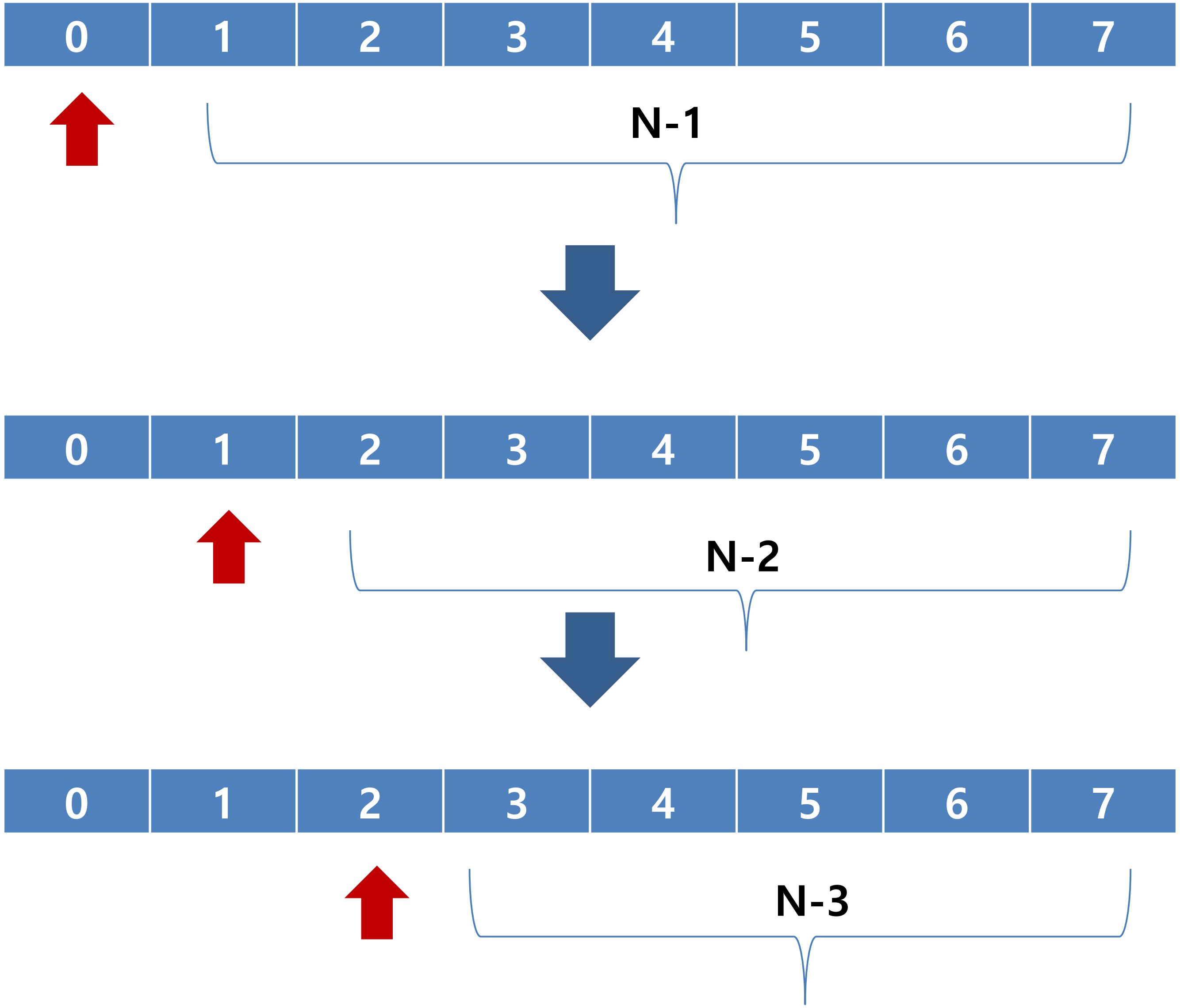
Concept



Concept



Master Theorem

Combining the three cases above gives us the following "master theorem".

Theorem 1 *The recurrence*

Number of problem

Size of problem

$$
\begin{aligned}
T(n) &= aT(n/b) + cn^k \\
T(1) &= c,
\end{aligned}
$$

Time of conquer step
for each sub-problem

where a, b, c, and k are all constants, solves to:

$$
\begin{aligned}
T(n) &\in \Theta(n^k) \text{ if } a < b^k \\
T(n) &\in \Theta(n^k \log n) \text{ if } a = b^k \\
T(n) &\in \Theta(n^{\log_b a}) \text{ if } a > b^k
\end{aligned}
$$

Master Theorem

$$T(n) = \begin{cases} c & \text{if } n = 1 \\ 2T(n/2) + cn & \text{if } n > 1 \end{cases}$$

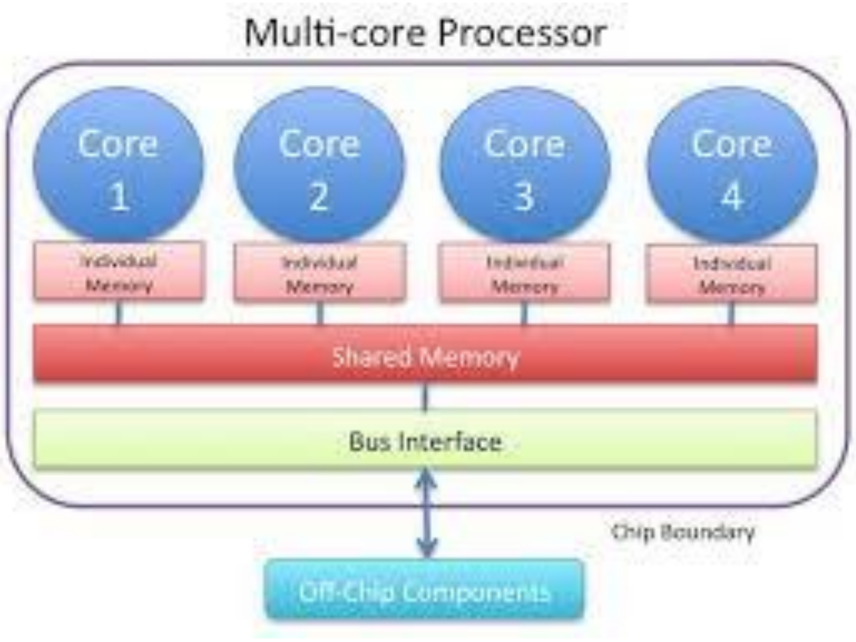
A=2 B=2 K=1

T(n)=2T(n/2)+n

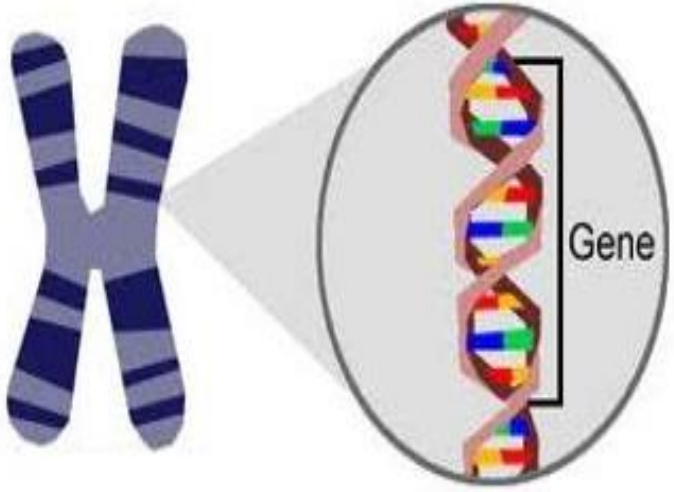
$$T(n) \in \Theta(n^k) \text{ if } a < b^k$$
$$\boxed{T(n) \in \Theta(n^k \log n) \text{ if } a = b^k}$$
$$T(n) \in \Theta(n^{\log_b a}) \text{ if } a > b^k$$

then $T(n) = \Theta(n^{\log_2 2} \log_2 n) = \Theta(n \log_2 n)$

Applications



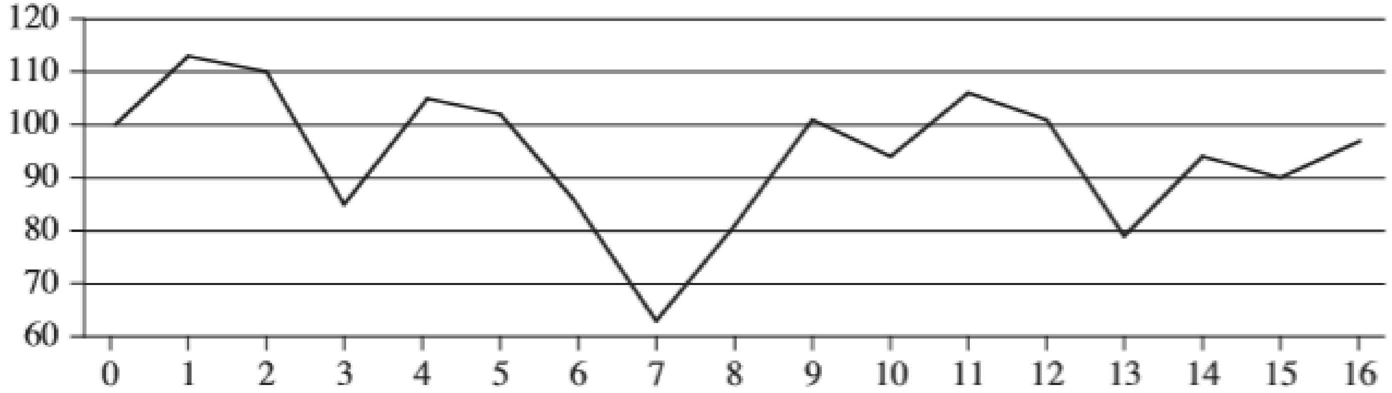
Merge Sort



Quick Sort

3. Maximum-subarray

Problem



Day	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Price	100	113	110	85	105	102	86	63	81	101	94	106	101	79	94	90	97
Change		13	−3	−25	20	−3	−16	−23	18	20	−7	12	−5	−22	15	−4	7

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	13	−3	−25	20	−3	−16	−23	18	20	−7	12	−5	−22	15	−4	7

maximum subarray

1. Brute-Force

1	-2	2	4	5	-6	3

Sum [illegible]

Max value -999

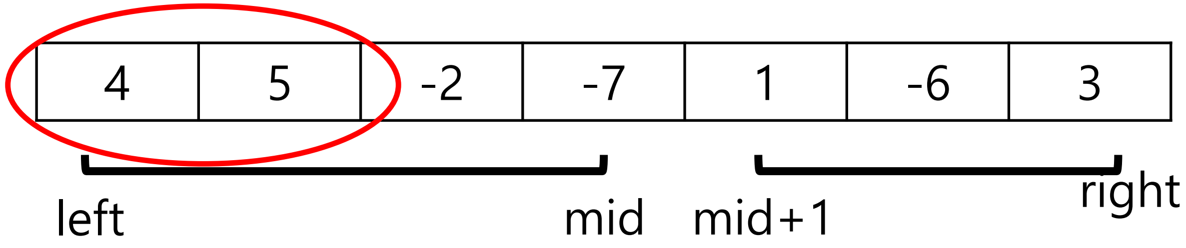
1. Brute-Force

```
for(int i=0; i<n; i++){

    int sum=0;

    for (int j = i; j < n; j++) {
            sum = sum + arr[j];
            ret = max(ret, sum);
        }

}
```

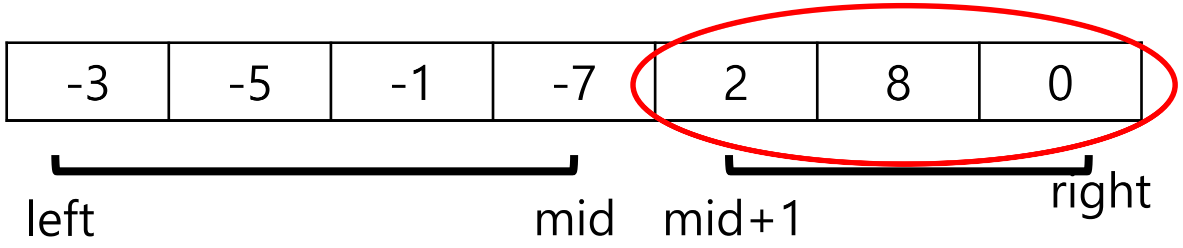
(n^2+n)/2 -> O(n^2)

2. Divide & Conquer

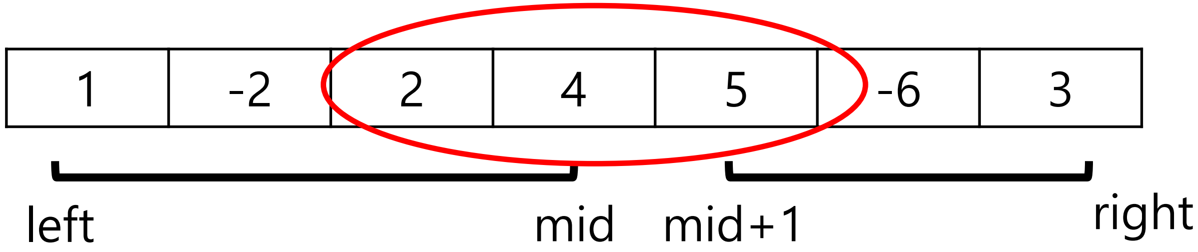
Case 1. Max value in the range of [left , mid]



Case 2. Max value in the range of [mid+1 , right]



Case 3. Max value overlap between the [left, mid] and [mid+1, right]



2. Divide & Conquer

1. If base case, return value
2. Find mid value
3. Divide the problem to sub-problem and compare case1 max value and case2 max value.
4. Find the case3 max value.
5. Return the highest value by comparing the maximum value of case 3 and single value(case1, case2)

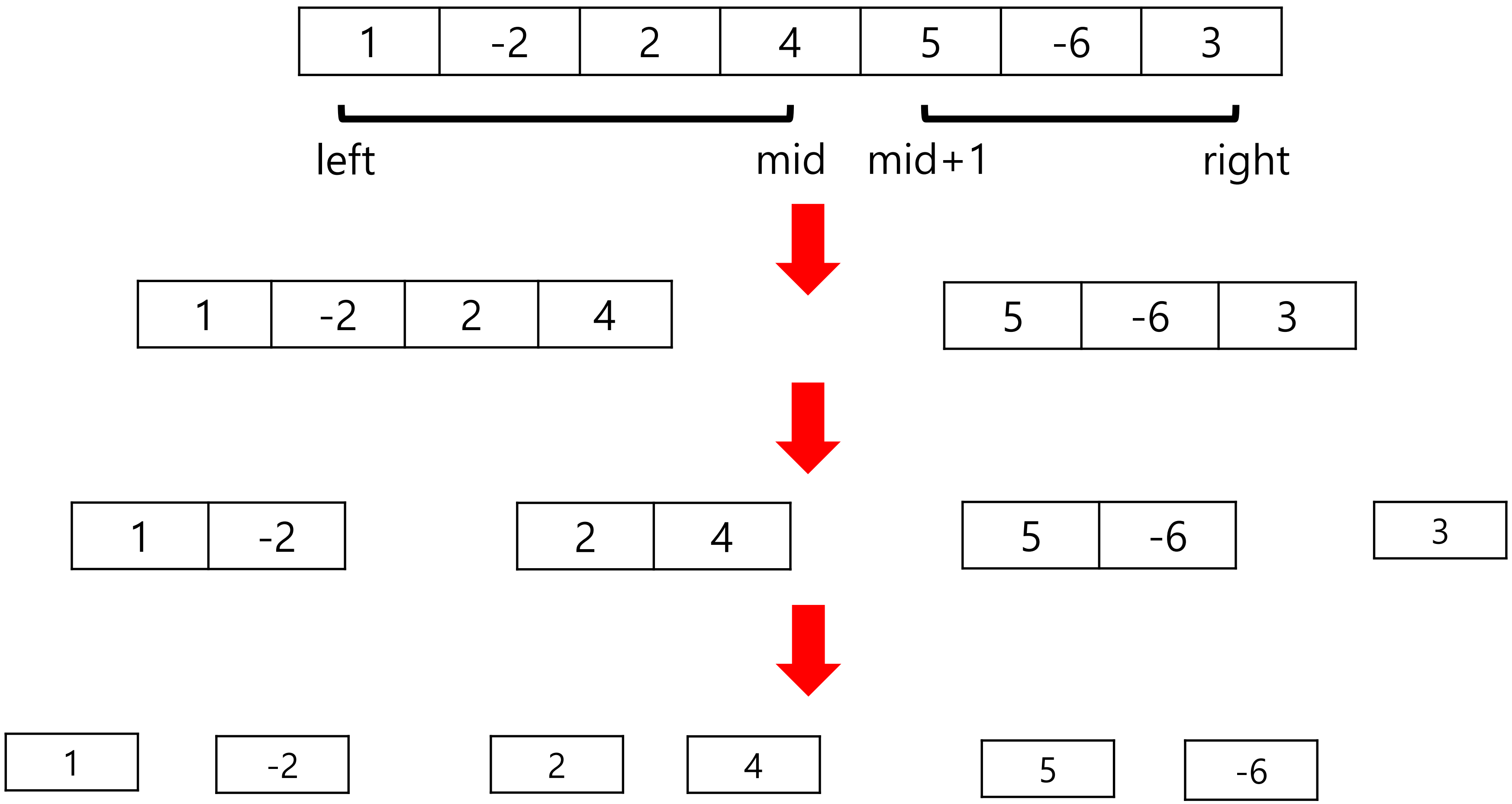
```
int recur(int left, int right) {

    //base case
    if (left == right) {
        return arr[left];
    }

    //recursive case
    int mid = (left + right) / 2;

    int single = max(recur(left, mid), recur(mid + 1, right));

    for (int i = mid; i >= left; i--) {
        sum = sum + arr[i];
        left_sum = max(left_sum, sum);
    }

    sum = 0;
    for (int i = mid + 1; i <= right; i++) {
        sum = sum + arr[i];
        right_sum = max(right_sum, sum);
    }

    return max(left_sum + right_sum, single);
}
```

Case 1

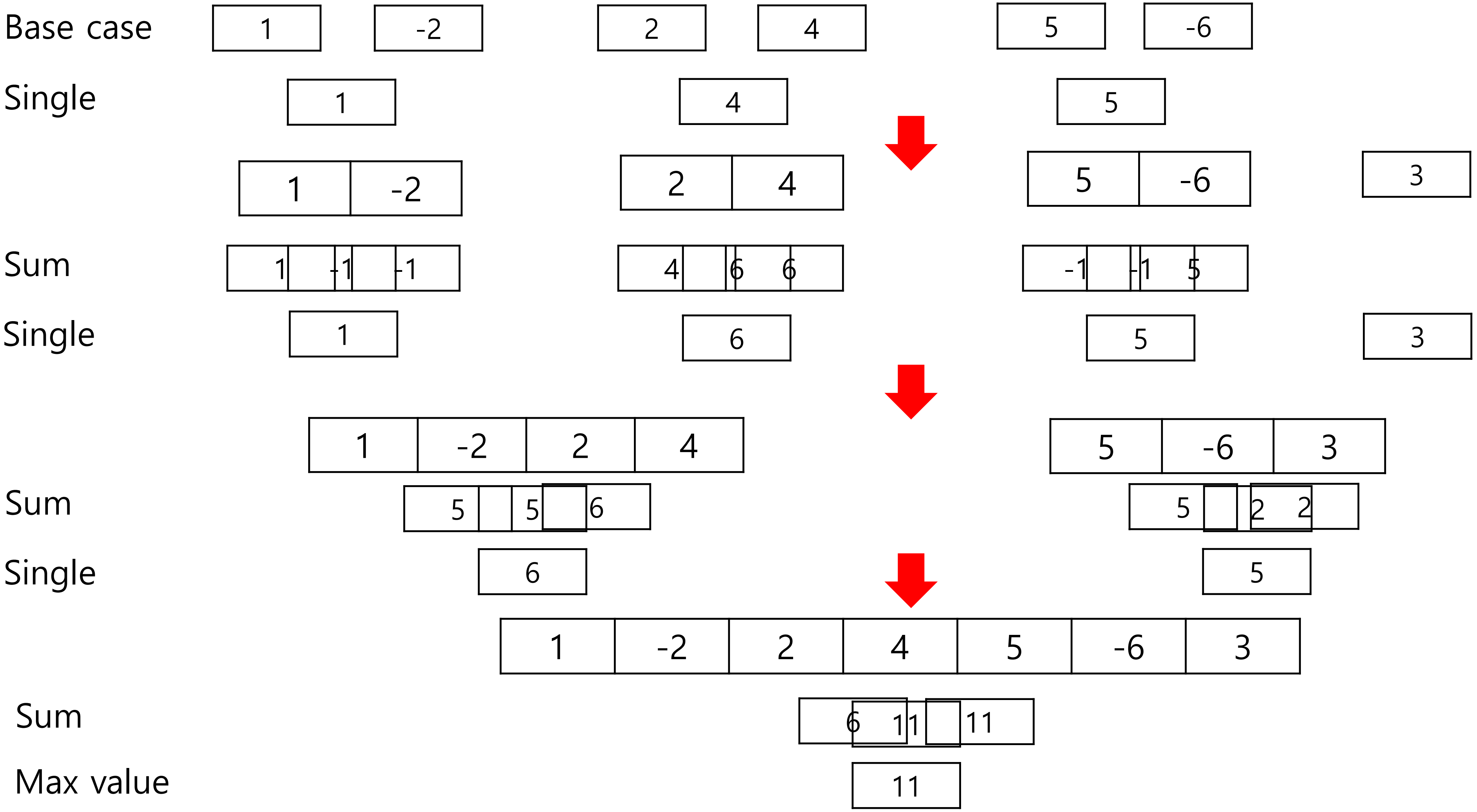
Case 2

Case 3

2. Divide



2. Conquer & Combine



2. Divide & Conquer

- $\Theta(1)$
- $2T(n/2)$
- $\Theta(n)$

(mid - left +1) + (right - (mid + 1) +1)
= right - left +1

```
int recur(int left, int right) {

    //base case
    if (left == right) {
        return arr[left];
    }

    //recursive case
    int mid = (left + right) / 2;

    int single = max(recur(left, mid), recur(mid + 1, right));

    for (int i = mid; i >= left; i--) {
        sum = sum + arr[i];
        left_sum = max(left_sum, sum);
    }

    sum = 0;
    for (int i = mid + 1; i <= right; i++) {
        sum = sum + arr[i];
        right_sum = max(right_sum, sum);
    }

    return max(left_sum + right_sum, single);
}
```

2. Divide & Conquer

$T(n) = 2T(n/2) + \Theta(n)$

1. If $f(n) = O(n^{\log_b a - \epsilon})$ for some constant $\epsilon > 0$, then $T(n) = \Theta(n^{\log_b a})$.
2. $\boxed{\text{If } f(n) = \Theta(n^{\log_b a}), \text{ then } T(n) = \Theta(n^{\log_b a} \lg n).}$
3. If $f(n) = \Omega(n^{\log_b a + \epsilon})$ for some constant $\epsilon > 0$, and if $af(n/b) \le cf(n)$ for some constant $c < 1$ and all sufficiently large n, then $T(n) = \Theta(f(n))$.

Using master theorem, a=2, b=2, $f(n) = \Theta(n)$

$\therefore T(n) = \Theta(n\log n)$

3. Dynamic programing

$$f(i) = \begin{cases} max(0, f(i-1)) + arr[i] & i > 0, \\ arr[i] & i == 0 \end{cases}$$

```
for(int i=0; i<n; i++){

    sum = max(sum,0) + arr[i];
    ret = max(ret, sum);

}
```

O(n) Time

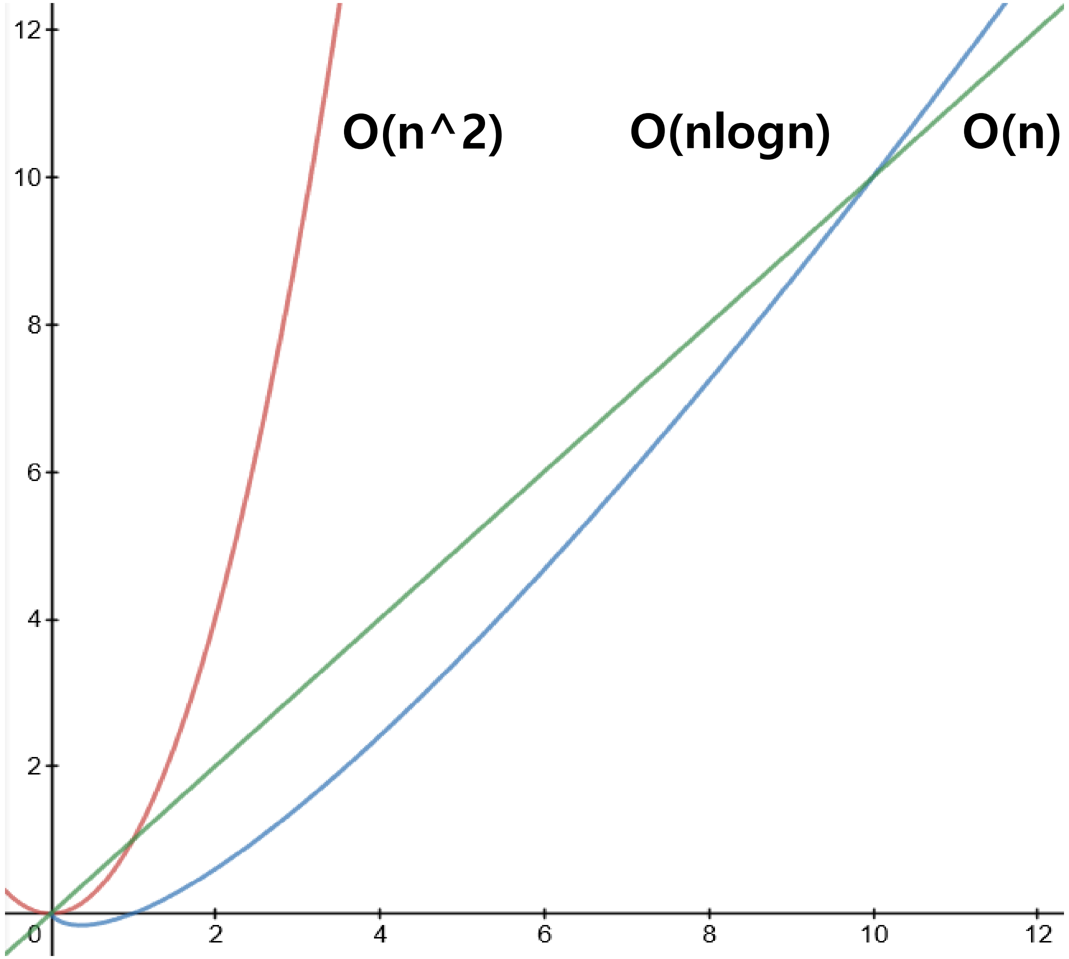
3. Dynamic programing

1	-2	2	4	5	-6	3

Sum -999

Max value -999

Analysis



N>10 이상일 경우

Performance : O(n^2) < O(nlogn) < O(n)

Analysis

1. Brute-Force => O(n^2)

```
n의 개수 : 10000
max value : 6569
걸린 시간 : 0.953000sec
```

2. Divide & Conquer => O(nlogn)

```
n의 개수 : 10000
max value : 6569
걸린 시간 : 0.004000sec
```

3. Dynamic Programing => O(n)

```
n의 개수 : 10000
max value : 6569
걸린 시간 : 0.001000sec
```

4. Conclusion

Conclusion

- **Advantages**

- Use Divide & Conquer, we can solve problems in less time than Brute-force.
- It can be available in many fields.
  (such as, Matrix multiplication, Parallelization, Bioinformatics)

- **Disadvantages**

- There is a possibility of stack overflow due to recursive.
- There is a possibility of better way such as dynamic programing.

Q & A

Thank you!