Harvesting Control Rules and Reference Points in Stochastic Age-Structured Fisheries Models

(MYFISH EU Project, *Anna Rindorf et al*)

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Motivation

- Fishery management is commonly based on the use of:
  - Harvesting Control Rules (HCR),
  - Reference points, and
  - The risk of the stock dropping below a limit point

Why?

It is important to avoid situations where the stock is at or below the limit reference point.

Accordingly, ***management should aim to target*** *a level of stock size that carries* ***a low risk*** *(allowing for scientific uncertainty) of the stock dropping below the limit reference point.*

This could mean having a target level of fishing mortality that provides stock sizes above B_{msy}.

(Beddington, Agnew, and Clark - Nature 2007)

State of art

- Stochastic feedback policies (i.e. constant escapement), **ignore the age-structured dinamics**
- Traditional age-structured Reference Points, **commonly ignore the stochastic component of the system (i.e.** B_{msy}**)**
- Management Strategy Evaluation (MSE) **use heuristics Harvest Control Rules** (i.e. based on **Biomass Model's**)

What Are We Interested In?

1. Define HCR for Stochastic (multispecies) age (-lenght and spatial) structured models, as

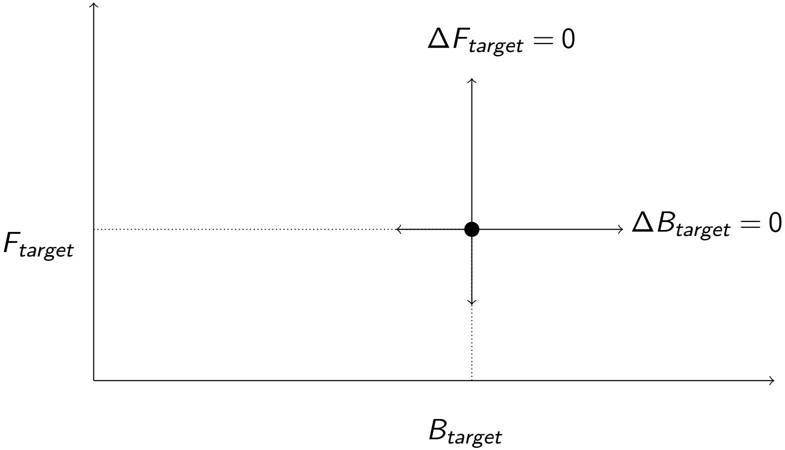
   "the optimal feedback policy" that minimizing the weighted sum of squares between the

   a) *stock assessment outputs and*
   b) *a given "biological reference point".*

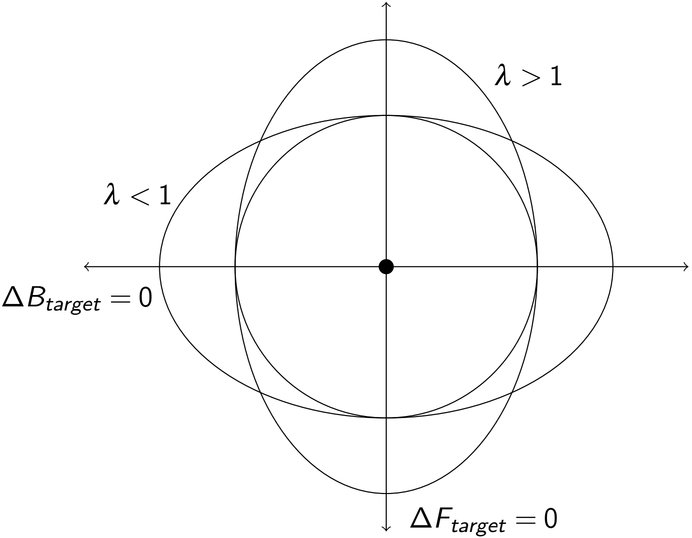
2. Given this HCR, explore the "dynamics" of the fishery

3. Change the "biological reference point" and start again ...

Guess a Target



Define a Distance



In short: Risk (v), Refence Points (B_{tar}) and HCR (λ)

- STEP 1: HCR(B_{tar}, λ) stabilize the fishery "around" B_{tar}

  .. and use the HCR to Generate the Stock dynamics Distribution

- STEP 2: Given $Pr(B \leq B_{lim}) \leq v$,

  Define the HCR(B_{tar}, λ) Feasible Set($v, B_{tar},$)

- STEP 3: Compute Expected Yield distribution *EYield*

$$\max_{B_{tar}} EYield(B_{tar}, \lambda) \;\; s.t. \;\; \lambda \in FeasibleSet(v, B_{tar})$$

A Simple 2 age Model: Assumptions

1. Consider a model with only two age classes: juveniles and adults
2. Each year, t, a stochastic exogenous number of juvenile fish are born
3. Additionally, assume that only a part of the juveniles survive to become adults next period
4. SSB is a (increasing) function of the number of Adults

A Simple 2 age Model

Model

		Time (t)			
age classes		t	$t+1$	$t+2$	$t+3$
Juveniles	N_1	e^{z_t}			
Adults	N_2		$e^{-pF_t-m}e^{z_t}$		

where z_t follows an AR(1) process

$$z_{t+1} = \rho z_t + \varepsilon_{t+1},$$

with zero mean, $E(\varepsilon_{t+1}) = 0$, and variance σ_z.

Change variables

Model in log

$$\log N_1 = z$$

$$\log N_2 = x$$

		Time (t)			
age classes		t	$t+1$	$t+2$	$t+3$
log Juveniles	z	z_t			
log Adults	x		$z_t - pF_t - m$		

HCR

The manager's objective is to minimize the distance between the fishing mortality, F_t, and the biomass, B_t to the target reference point, (B_{tar}, F_{tar}) subject to the stock dynamics:

$$\max_{F_t, B_{t+1}} E_0 \quad \sum_{t=0}^{\infty} -\beta^t \left\{ (F_t - F_{tar})^2 + \lambda (B_t - B_{tar})^2 \right\}$$

$$s.t. \quad \begin{cases} x_{t+1} = z_t - pF_t - m \\ z_{t+1} = \rho z_t + \varepsilon_{t+1}. \end{cases}$$

where λ weights the importance of biomass versus effort-oriented objectives

One more Change

$$\begin{aligned} x_{t+1} &= z_t - pF_t - m \\ x_{msy} &= z_{tar} - pF_{tar} - m \\ \Delta x = x_{t+1} - x_{msy} &= \Delta z_t - p\Delta F_t \end{aligned}$$

Dynamic are independent of natural mortality m **!**

Model in Δ log

		Time (t)			
age classes		t	$t+1$	$t+2$	$t+3$
Δ log Juveniles	Δz	Δz_t			
Δ log Adults	Δx		$\Delta z_t - p\Delta F_t$		

Biomass and Biodiversity

Assumption: Biomass equal to Biodiversity (*Shannon Index*)

$$B = \log N_2$$

REMARK: (Recruits are not in the B_t !)

The problem can be simplified

$$\max_{\Delta F_t, \Delta B_{t+1}} E_0 \qquad \sum_{t=0}^{\infty} -\beta^t \left\{ \Delta F_t^2 + \lambda \Delta B_t^2 \right\}$$

$$s.t. \qquad \begin{cases} \Delta B_{t+1} = \Delta z_t - p \Delta F_t \\ \\ \Delta z_{t+1} = \rho \Delta z_t + \varepsilon_{t+1}. \end{cases}$$

where $\Delta F = F_t - F_{tar}$, $\Delta z_t = z_t - z_{tar}$ and $\Delta B_t = B_t - B_{tar}$

STEP 1: Compute HCR

Solving for the HCR, ΔF_t, we have

$$\Delta F_t = \frac{p\beta\lambda}{1+p^2\beta\lambda}\Delta z_t,$$

Lemma

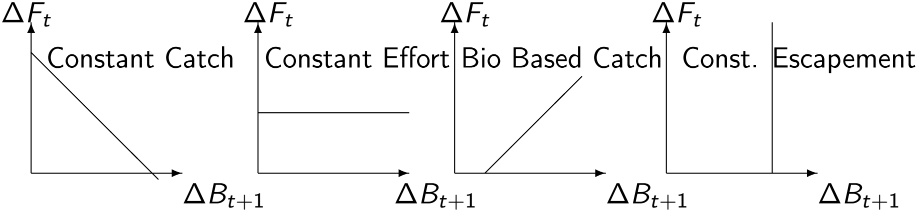
Good recruitments, imply a higher fishing mortality

Understanding the role of λ

- If $\lambda < 0$, the HCR generates a negative relationship between fishing mortality and biomass, similar to that of a constant catch rule
- If $\lambda = 0$, the HCR reproduces a constant fishing mortality rule.
- If $\lambda > 0$ the HCR reproduces a biomass-based rule.
- if $\lambda \to \infty$, the HCR reproduces a constant or fixed escapement rule

λ generates the basic types of control rules

$\lambda < 0$ $\lambda = 0$ $\lambda > 0$ $\lambda \to \infty$



Generate the Stock dynamics Distribution

$$\Delta B_{t+1} = \Delta z_t - p \underbrace{\Delta F_t}_{\frac{p\beta\lambda}{1+p^2\beta\lambda}\Delta z_t,} = \frac{1}{1+p^2\beta\lambda}\Delta z_t = \frac{1}{1+p^2\beta\lambda}(\rho\Delta z_{t-1} + \varepsilon_t)$$

Assuming that ε_t is a Gaussian process, then B_{t+1} also follows a Gaussian distribution. Thus, we can easily obtain the biomass moments: the expected value, μ_B, and its variance, σ_B, are given by

$$\mu_B = B_{tar}$$

and

$$\sigma_B = \sigma_z \sum_{k=0}^{\infty} \left(\frac{\rho}{1+p^2\beta\lambda} \right)^k = \frac{(1+p^2\beta\lambda)^2}{(1+p^2\beta\lambda)^2 - \rho^2}\sigma_z$$

STEP 2: HCR and risk

We can now relate the design of target reference points, in a stochastic environment, for given a predetermined risk level that we want to avoid. Thus, if we want to calculate the value of λ for which $Pr(B \leq B_{lim}) = v$, we can use the cumulative distribution

$$Pr(B \leq B_{lim}) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{B_{lim} - \mu_B}{\sigma_B\sqrt{2}}\right)\right] = v$$

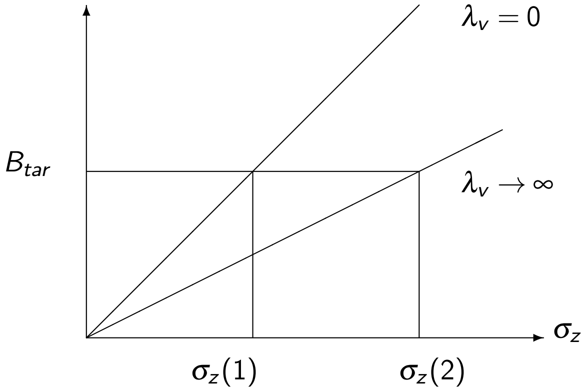
where erf is the Gaussian error function. Rearranging terms, we have

$$\lambda_v = \frac{\sqrt{\frac{\rho^2(B_{lim} - B_{tar})}{(B_{lim} - B_{tar}) - \sigma_z \text{erf}^{-1}(2v-1)\sqrt{2}}} - 1}{\rho^2\beta}$$

λ_v is the HCR for which v is the probability that the stock is below a given threshold B_{lim}.

STEP 3: Compute (Expected) Yield for each B_{tar} in the Feasible HCR Set

Feasible HCR Set for $Pr(B \leq B_{lim}) \leq v$



Numerical Simulations

- The model is applied to the Southern Hake, (MYFISH)
- In particular, we evaluate how much does a reference point lower than F_{max} decreases the Biomass volatility and increase Yiled !
- For these two target points, we then implement different HCR, thus characterizing the continuum of feasible HCR.
- The effect of each HCR is simulated 10000 times for each experiment and each simulation is run over 100 seasons.

Table: **Age structured model. Hake**

Age	N	weight	maturity	m_a	p_a
0	78856.7	0.00	0.0181	0.4	0.076
1	49006.5	0.05	0.1194	0.4	0.317
2	23915.4	0.33	0.5000	0.4	0.559
3	9164.5	0.90	0.8806	0.4	0.623
4	3293.3	1.71	0.9819	0.4	0.633
5	1171.9	2.70	0.9975	0.4	0.635
6	416.4	3.79	0.9997	0.4	0.635
7	148.0	4.93	1.0000	0.4	0.635
8	52.6	6.06	1.0000	0.4	0.635
9	18.7	7.14	1.0000	0.4	0.635
10	6.6	8.16	1.0000	0.4	0.635
11	2.4	9.09	1.0000	0.4	0.635
12	0.8	9.94	1.0000	0.4	0.635
13	0.3	10.70	1.0000	0.4	0.635

General LQ Model

The LQ problem can be written as:

$$\max_{\mathbf{y}_t} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \mathbf{x}_t^T \mathbf{R} \mathbf{x}_t + 2\mathbf{y}_t^T \mathbf{W} \mathbf{x}_t + \mathbf{y}_t^T \mathbf{Q} \mathbf{y}_t \right\}$$

$$\begin{bmatrix} 1 \\ z_{t+1} \\ x_{t+1,2} \\ \dots \\ x_{t+1,A} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ \rho & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ z_t \\ x_{t,2} \\ \dots \\ x_{t,A} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -p_1 \\ \dots \\ -p_{A-1} \end{bmatrix} F_t + \begin{bmatrix} 0 \\ \varepsilon_{t+1} \\ 0 \\ \dots \\ 0 \end{bmatrix}$$

where $\mathbf{y} = F_t - F_{tar}$, $\mathbf{x}^T = [1 \quad z_t \quad x_{t,2} - x_{msy,2} \quad \dots \quad x_{t,A} - x_{msy,A}]$ with $\mathbf{W}_{1\times A+1} = \mathbf{0}_{1\times A+1}$, $\mathbf{Q}_{1\times 1} = 1$ and

$$\mathbf{R}_{A+1\times A+1} = -\lambda \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & (\mu_1 e^{z_{tar}})^2 & \mu_1\mu_2 e^{z_{tar}} e^{x_{\mathbf{2},tar}} & \dots & \mu_1\mu_A e^{z_{tar}} e^{x_{A,tar}} \\ 0 & \mu_2\mu_1 e^{z_{tar}} e^{x_{\mathbf{2},tar}} & (\mu_2 e^{x_{\mathbf{2},tar}})^2 & \dots & \mu_2\mu_A e^{x_{\mathbf{2},tar}} e^{x_{A,tar}} \\ \dots. & \dots & \dots & \dots & \dots \\ 0 & \mu_A\mu_1 e^{z_{tar}} e^{x_{A,tar}} & \mu_A\mu_2 e^{x_{\mathbf{2},tar}} e^{x_{A,tar}} & \dots & (\mu_A e^{x_{A,tar}})^2 \end{bmatrix}$$

Results

R.P.	$Ftarget = Fmax$			$Ftarget = (2/3)Fmax$		
λ (HCR)	-0.1000	0.0000	0.2000	-0.1000	0.0000	0.2000
	Expected Values					
F	0.9959	1.0000	1.0072	0.6598	0.6667	0.6773
SSB	1.1166	1.1001	1.0759	1.5851	1.5401	1.4877
$Yield$	2.2638	2.2548	2.2362	2.3655	2.3733	2.3573
	Volatility					
std (F)	0.0284	0.0000	0.0496	0.0468	0.0000	0.0733
std (SSB)	0.3094	0.2769	0.2241	0.4683	0.3823	0.2629
std $(Yield)$	0.5854	0.5641	0.5262	0.5882	0.5853	0.5720
	Correlations					
corr(SSB, F)	-0.8734	0.0000	0.8719	-0.8652	0.0000	0.8571
corr$(Yield, SSB)$	0.9996	0.9991	0.9950	0.9958	0.9994	0.9839

Results

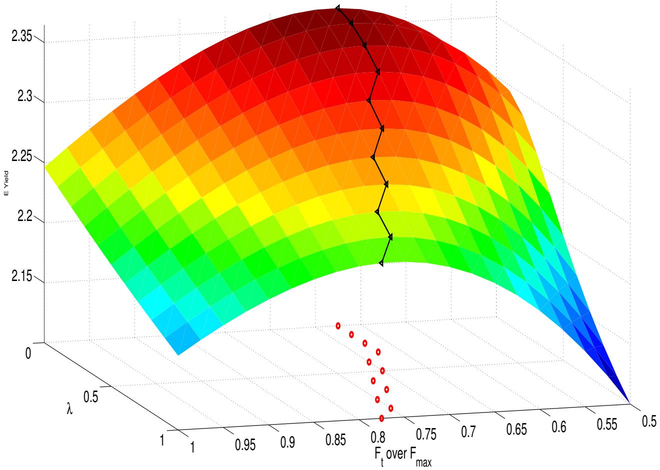


Figure: Expected Yield vs fishing mortality

Results

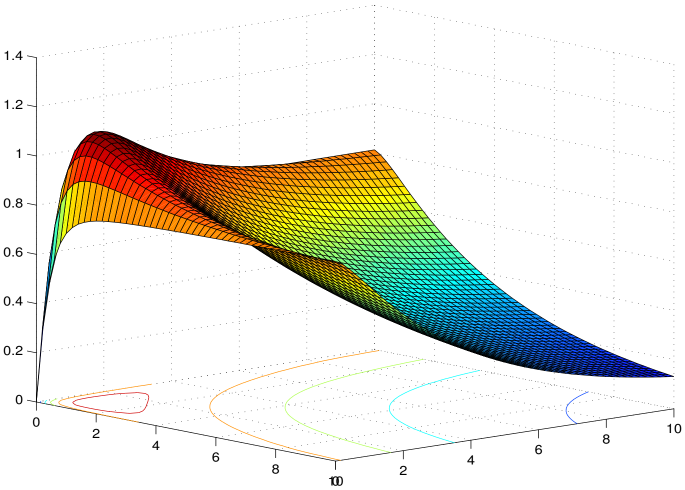


Figure: MSY is the optimal

Results

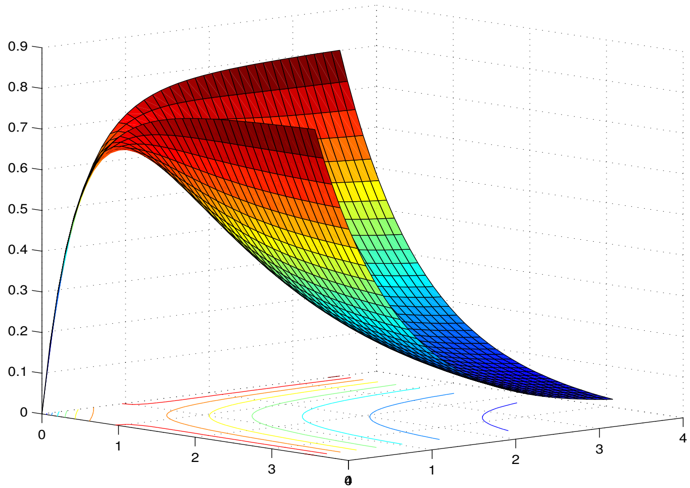


Figure: MSY is NOT the optimal

Implications ...

In more general models

Lemma

With more than two age classes, the HCR depends on the biodiversity of the spawning stock biomass.

$$HCR = \sum_{i=1}^{A} p_i \ln N_i^a$$

Lemma

For positive recruitment levels, even if the level of biomass is at the target level, its associated target fishing mortality might not be the optimal.

i.e. In Years with Good Recritments it is optimal to set $\Delta F_t > 0$

What are the implications of the omission of the biological structure?

$$HCR = \underbrace{\alpha}_{number} \times (SSB_t - K)$$

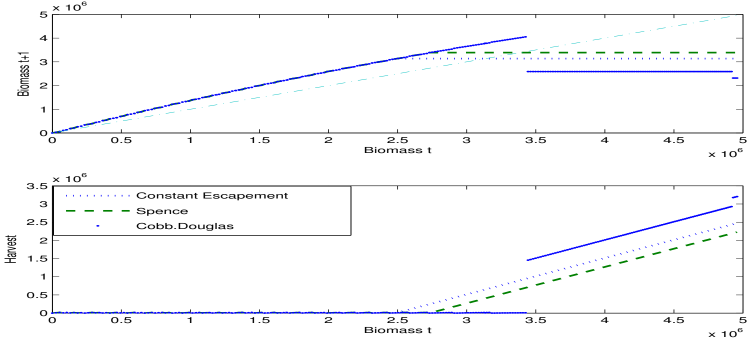


Figure: Da Rocha and Nøstbakken (ERE, 2014)

What are the implications of the omission of the biological structure?

$$HCR = \underbrace{\alpha}_{number} \times (SSB_t - K)$$

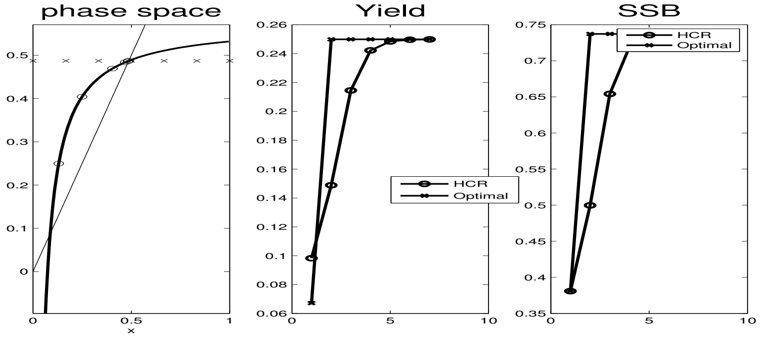


Figure: K is low !

What are the implications of the omission of the biological structure?

$$F_t = HCR(SSB_t) = \underbrace{\alpha}_{number} \times (SSB_t - K)$$

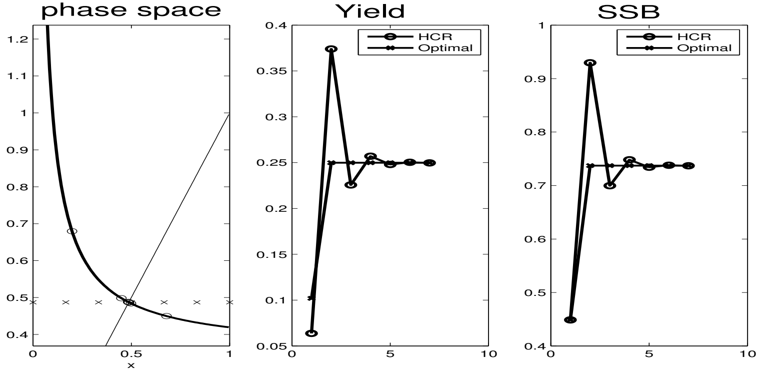


Figure: K is high !

Implications: Constant Effort HCR vs Biodiversity HCR

- Constant effort policies are quite common. Commonly HCR is as follows:

> *F should be F_{msy} when the stock is greater than B_{msy} and F should be reduced when the stock is lower than some trigger biomass lower than B_{msy}*

- We study the implications of constant effort rules, $\lambda = 0$, and risk.

Lemma

Let $F = F_{tar}$ for all possible states, i.e. $\lambda = 0$. (In general) Biomass volatility can be reduced by applying a HCR with a positive λ.

MYFISH: WW Case Study (Southern Hake Mixed Fishery)

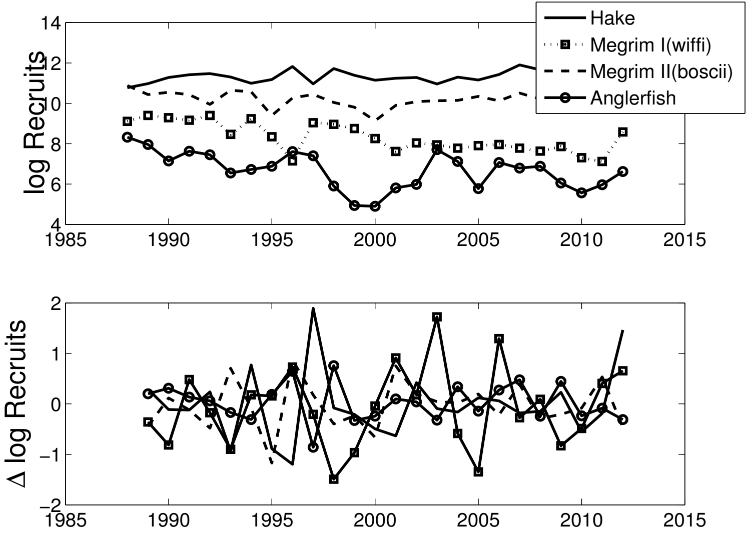


Figure: Recruitments of Hake, Megrim I (wiffi), Megrim II (boscii), Anglerfish

VAR Estimation of Δ log recruits

The VAR is an econometric model used to capture the linear inter-dependencies among multiple time series.

Run a var(1) on Δ log of recruits of: Hake, Megrim I (wiffi), Megrim II (boscii), Anglerfish

$$\begin{pmatrix} Hake_t \\ wiffi_t \\ boscii_t \\ Anglerfish_t \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} + \begin{pmatrix} a^1_{11} & a^1_{12} & a^1_{13} & a^1_{14} \\ a^1_{21} & a^1_{22} & a^1_{23} & a^1_{24} \\ a^1_{31} & a^1_{32} & a^1_{33} & a^1_{34} \\ a^1_{41} & a^1_{42} & a^1_{43} & a^1_{44} \end{pmatrix} \begin{pmatrix} Hake_{t-1} \\ wiffi_{t-1} \\ boscii_{t-1} \\ Angler_{t-1} \end{pmatrix} + \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix}$$

$$\begin{pmatrix} a^1_{11} & a^1_{12} & a^1_{13} & a^1_{14} \\ a^1_{21} & a^1_{22} & a^1_{23} & a^1_{24} \\ a^1_{31} & a^1_{32} & a^1_{33} & a^1_{34} \\ a^1_{41} & a^1_{42} & a^1_{43} & a^1_{44} \end{pmatrix} = \begin{pmatrix} -0.358 & 0.156 & -0.381 & 0.119 \\ 0.116 & -0.410 & 1.109 & 0.065 \\ 0.060 & -0.336 & -0.438 & 0.114 \\ -0.544 & -0.554 & 0.114 & -0.081 \end{pmatrix}$$

Estimation of AR of recruits

Run a AR(1) on data of recruits of: Hake, Megrim I (wiffi), Megrim II (boscii), Anglerfish

$$\begin{pmatrix} Hake_t \\ wiffi_t \\ boscii_t \\ Anglerfish_t \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} + \begin{pmatrix} a_{11}^1 & 0 & 0 & 0 \\ 0 & a_{22}^1 & 0 & 0 \\ 0 & 0 & a_{33}^1 & 0 \\ 0 & 0 & 0 & a_{44}^1 \end{pmatrix} \begin{pmatrix} Hake_{t-1} \\ wiffi_{t-1} \\ boscii_{t-1} \\ Angler_{t-1} \end{pmatrix} + \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix}$$

$$\begin{pmatrix} a_{11}^1 & 0 & 0 & 0 \\ 0 & a_{22}^1 & 0 & 0 \\ 0 & 0 & a_{33}^1 & 0 \\ 0 & 0 & 0 & a_{44}^1 \end{pmatrix} = \begin{pmatrix} -0,514 & 0 & 0 & 0 \\ 0 & -0,379 & 0 & 0 \\ 0 & 0 & -0,357 & 0 \\ 0 & 0 & 0 & -0,003 \end{pmatrix}$$

Moreover

Covariance matrix, Ω of recruits of: Hake, Megrim I (wiffi), Megrim II (boscii), Anglerfish

$$\Omega_{AR} = \begin{pmatrix} 0.10 & 0 & 0 & 0 \\ 0 & 0.41 & 0 & 0 \\ 0 & 0 & 0.19 & 0 \\ 0 & 0 & 0 & 0.63 \end{pmatrix}$$

$$\Omega_{VAR} = \begin{pmatrix} 0.06 & -0.003 & 0.008 & -0.007 \\ -0.003 & 0.144 & -0.008 & -0.068 \\ 0.008 & -0.008 & 0.13 & -0.002 \\ -0.008 & -0.068 & -0.002 & 0.54 \end{pmatrix}$$

MYFISH: WW Case Study

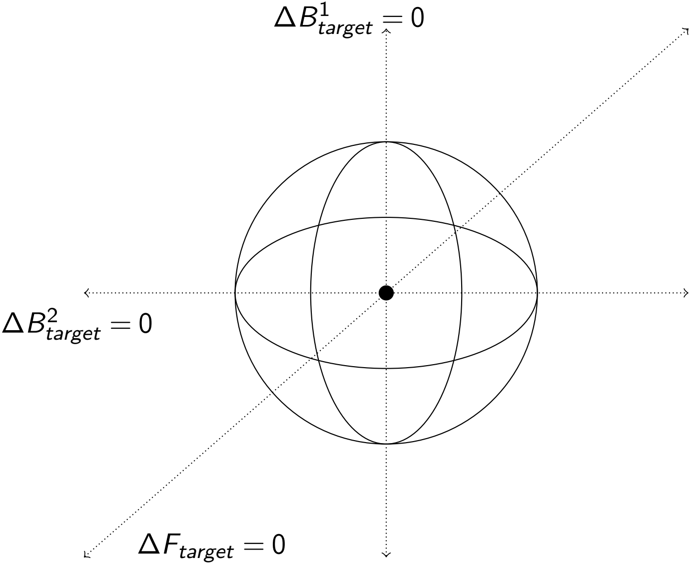
Let's consider the case of a **simple mixed** fishery : $F_t^i = \delta^i E_t$

$$\max_{E_t, B_{t+1}^1, B_{t+1}^2} E_0 \sum_{t=0}^{\infty} -\beta^t \left\{ (E_t - E_{tar})^2 + \lambda_1 (B_t^1 - B_{tar}^1)^2 + \lambda_2 (B_t^2 - B_{tar}^2)^2 \right\}$$

$$s.t. \begin{cases} B_{t+1}^1 = z_t^1 - p^1 \delta^1 E_t - m^1 \\ z_{t+1}^1 = \rho_{1,1} z_t^1 + \rho_{1,2} z_t^2 + \varepsilon_{t+1}^1 \\ B_{t+1}^2 = z_t^2 - p^2 \delta^2 E_t - m^2 \\ z_{t+1}^2 = \rho_{2,1} z_t^1 + \rho_{2,2} z_t^2 + \varepsilon_{t+1}^2 \end{cases}$$

where ε_{t+1}^i verifies $E\varepsilon_{t+1}^i = 0$ and $E\varepsilon_{t+1}^1 \varepsilon_{t+1}^2 = \Omega$

Define a Distance



$$
\begin{bmatrix} 1 \\ \\ z^1_{t+1} \\ x^1_{1,t+1} \\ x^1_{2,t+1} \\ \\ z^2_{t+1} \\ x^2_{1,t+1} \\ x^2_{2,t+1} \end{bmatrix}
=
\begin{bmatrix}
1 & \begin{vmatrix} 0 & 0 & 0 \end{vmatrix} & \begin{vmatrix} 0 & 0 & 0 \end{vmatrix} \\
\begin{vmatrix} \mathbf{c^2} \\ 0 \\ 0 \end{vmatrix} & \begin{vmatrix} \rho^{\mathbf{1,1}} & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} & \begin{vmatrix} \rho^{\mathbf{1,2}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \\
\begin{vmatrix} \mathbf{c^2} \\ 0 \\ 0 \end{vmatrix} & \begin{vmatrix} \rho^{\mathbf{2,1}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} & \begin{vmatrix} \rho^{\mathbf{2,2}} & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}
\end{bmatrix}
\begin{bmatrix} 1 \\ \\ z^1_{t} \\ x^1_{1,t} \\ x^1_{2,t} \\ \\ z^2_{t} \\ x^2_{1,t} \\ x^2_{2,t} \end{bmatrix}
$$

$$
+
\begin{bmatrix} 0 \\ \begin{vmatrix} 0 \\ \delta^{\mathbf{1}} p^{\mathbf{1}}_{\mathbf{1}} \\ \delta^{\mathbf{1}} p^{\mathbf{1}}_{\mathbf{2}} \end{vmatrix} \\ \begin{vmatrix} 0 \\ \delta^{\mathbf{2}} p^{\mathbf{2}}_{\mathbf{1}} \\ \delta^{\mathbf{2}} p^{\mathbf{2}}_{\mathbf{2}} \end{vmatrix} \end{bmatrix}
E_t +
\begin{bmatrix}
0 & \begin{vmatrix} 0 & 0 & 0 \end{vmatrix} & \begin{vmatrix} 0 & 0 & 0 \end{vmatrix} \\
\begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} & \underbrace{\begin{vmatrix} \varepsilon^{\mathbf{1}}\varepsilon^{\mathbf{1}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}}_{\Omega_{\mathbf{1}}} & \underbrace{\begin{vmatrix} \varepsilon^{\mathbf{1}}\varepsilon^{\mathbf{2}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}}_{\Omega_{\mathbf{1,2}}} \\
\begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} & \underbrace{\begin{vmatrix} \varepsilon^{\mathbf{1}}\varepsilon^{\mathbf{2}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}}_{\Omega_{\mathbf{1,2}}} & \underbrace{\begin{vmatrix} \varepsilon^{\mathbf{2}}\varepsilon^{\mathbf{2}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}}_{\Omega_{\mathbf{2}}}
\end{bmatrix}
$$

and in the mixed fishery model

$$
\mathbf{R}_{(1+2\times3)\times(1+2\times3)} = \left[\begin{array}{c|ccc|ccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & & & & 0 & 0 & 0 \\
0 & \multicolumn{3}{c|}{\lambda_{\mathbf{1}}\, \nabla^2 B^1\Big|_{B^1=B^1_{tar}}} & 0 & 0 & 0 \\
0 & & & & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & & & \\
0 & 0 & 0 & 0 & \multicolumn{3}{c}{\lambda_{\mathbf{2}}\, \nabla^2 B^2\Big|_{B^2=B^2_{tar}}} \\
0 & 0 & 0 & 0 & & &
\end{array}\right]
$$

where $\mathbf{W}_{1\times3+3} = \left[\begin{array}{ccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$ and $\mathbf{Q}_{1\times1} = [-1]$, and

$$
\nabla^2 B^i\Big|_{B^i_{tar}} = \left[\left[\begin{array}{ccc}
-\mu_z^2 e^{2z_{tar}} & -\mu_z\mu_{x_{\mathbf{1}}} e^{z_{tar}} e^{x_{\mathbf{1}},tar} & -\mu_z\mu_{x_{\mathbf{2}}} e^{z_{tar}} e^{x_{\mathbf{2}},tar} \\
-\mu_{x_{\mathbf{1}}}\mu_z e^{z_{tar}} e^{x_{\mathbf{1}},tar} & -\mu_{x_{\mathbf{1}}}^2 e^{2x_{\mathbf{1}},tar} & -\mu_{x_{\mathbf{1}}}\mu_{x_{\mathbf{2}}} e^{x_{\mathbf{1}},tar} e^{x_{\mathbf{2}},tar} \\
-\mu_{x_{\mathbf{2}}}\mu_z e^{z_{tar}} e^{x_{\mathbf{2}},tar} & -\mu_{x_{\mathbf{1}}}\mu_{x_{\mathbf{2}}} e^{x_{\mathbf{1}},tar} e^{x_{\mathbf{2}},tar} & -\mu_{x_{\mathbf{2}}}^2 e^{2x_{\mathbf{2}},tar}
\end{array}\right]\right|_{\vec{\mu}^i, z^i, \vec{x}^i}
$$

Impulse -Response

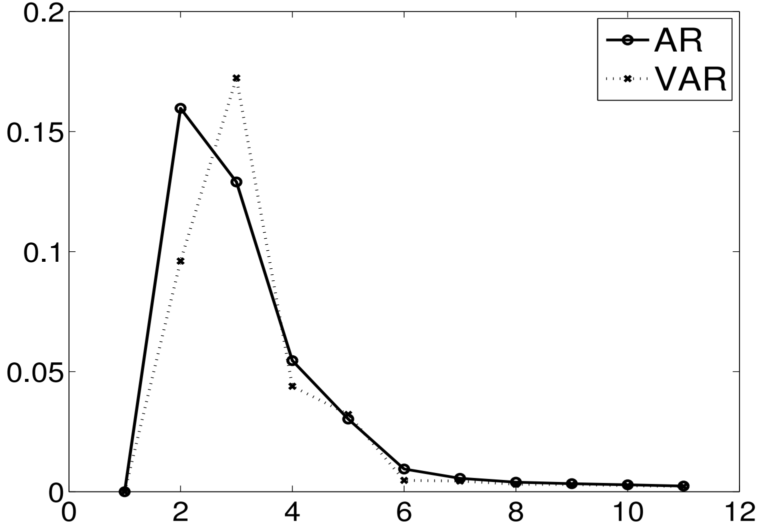


Figure: $\Delta F_t / F_{target}$ after a "Good year"

Minimizing Risk

1. Minimizing Risk

$$(\lambda_1, \lambda_2, \lambda_3, \lambda_4) \in argmin \sum Pr(SSB_i) < 2/3 \times SSS_i^{target}$$

2. Minimizing Risk and Model uncertainty

$$(\lambda_1, \lambda_2, \lambda_3, \lambda_4) \in argmin \left\{ \max_{j=AR,VAR} \sum Pr(SSB_i) \right\} < 2/3 \times SSS_i^{target}$$

3. **Robutness**: Model uncertainty represented by *dynamic misspecification*

$$\tilde{\varepsilon}_t = \varepsilon_t + \omega_t$$

Work in progress

- **YIELD**: *"Preferences for biodiversity"*

$$Yield_t = \sum_{j=1}^{N} \alpha_i {y_{i,t}}^{1/\sigma} \quad U(Yield_t) = \frac{{Yield_t}^{\sigma}}{1-\sigma}$$

$$\sigma = K \times \frac{\Delta \log Yield}{\Delta \sum_a \phi_a \log N_a}$$

- **Future prices uncertainty**: *Introduce "market shocks"*: compute VAR for prices and expand HCR

$$F_t = \underbrace{\sum_i \sum_a \lambda(a,i) \ln N_{a,i,t}}_{biodiversity} + \underbrace{\sum_i m(i) \ln pr_{i,t}}_{market \quad shocks}$$

Thank you for your attention

What We Do

- We write standard VPA models in continuous times
- We assume that **key** parameters are non constants
  1. discards
  2. selectivity patterns
- We compute **Dynamic Reference Points**

IDEA

Formally, the optimal management path is the solution of the following maximization problem

$$\max_{\left\{F_t, N^a_{t+2}\right\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t \left\{ \sum_{a=1}^{A} pr^a y^a(F_t) N^a_t - TC(F_t) \right\},$$

$$s.t. \begin{cases} N^{a+1}_{t+1} = e^{-z^a(F_t)} N^a_t \quad \forall t \quad \forall a = 1, A-1 \\ \\ N^1_{t+1} = \Psi\left(\sum_{a=1}^{A} \mu^a \omega^a N^a_t\right) \quad \forall t \\ \\ SSB_{pa} \leq \sum_{a=1}^{A} \mu^a \omega^a N^a_t \quad \forall t \end{cases}$$

where $y^a(F_t) = \omega^a \frac{F_t}{p^a F_t + m}\left(1 - e^{-p^a F_t - m}\right)$ and pr and TC represent the price and the total cost function which depends positively on fishery mortality, respectively.

IDEA

Let $n(a,t)$ be the number of fish of age a at time t.
Conservation law (Von Forester, 1959; McKendrick, 1926)

$$\frac{\partial n(a,t)}{\partial t} = -\frac{\partial n(a,t)}{\partial a} - [m(a) + (\underbrace{p(a,t) + d(a,t)}_{functions})F(t)]n(a,t).$$

The Stock Recruitment relationship a

$$n(0,t) = \Psi(\int_0^A \omega(a)\mu(a)n(a,t)da).$$

where, $\int_0^A \omega(a)\mu(a)n(a,t)da$ is the SSB. Finally we also assume that fish die at age A, i.e.

$$n(A,t) = 0.$$

IDEA

$$\max_{F(t)} \int_0^\infty \left[\left(\int_0^A pr(a)\omega(a)p(a,t)n(a,t)da \right) F(t) - C(F(t)) \right] e^{-rt}dt,$$

$$s.t. \begin{cases} \dfrac{\partial n(a,t)}{\partial t} = -\dfrac{\partial n(a,t)}{\partial a} - [m(a) + (\underbrace{p(a,t) + d(a,t)}_{functions})F(t)]n(a,t). \\ n(0,t) = \Psi(SSB(t)) \ \forall t, \\ SSB(t) = \displaystyle\int_0^\infty \mu(a)\omega(a)n(a,t)da. \ \forall t, \\ SSB(t) \geq B_{pa}. \ \forall t, \\ n(A,t) = 0 \ \forall t, \\ n(a,0) \quad \text{given}, \ \forall a, \end{cases}$$