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STABILITY

Vector Lyapunov Functions for Stability and Stabilization of Differential Repetitive Processes[1]

K. Galkowski[a], M. A. Emelianov[b], P. V. Pakshin[b], and E. Rogers[c]

[a] *Institute of Control and Computation Engineering, University of Zielona Góra, Zielona Góra, Poland*
[b] *Arzamas Polytechnic Institute of R.E. Alekseev Nizhny Novgorod State Technical University, Arzamas, Russia*
[c] *Department of Electronics and Computer Science, University of Southampton, Southampton, UK*
e-mail: pakshin@apingtu.edu.ru, pakshinpv@gmail.com

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Abstract—Differential repetitive processes arise in the analysis and design of iterative learning control algorithms. They belong to a class of mathematical models whose dynamic properties are defined by two independent variables, such as a time and a spatial coordinate, also known as 2D systems in the literature. Moreover, standard stability analysis methods cannot be applied to such processes. This paper develops a vector Lyapunov function-based approach to the exponential stability analysis of differential repetitive processes and applies the resulting conditions to develop linear matrix inequality based iterative learning control law design algorithms in the presence of model uncertainty.

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INTRODUCTION

In recent years, considerable effort has been directed to the development of a stability and control theory for nD systems. Starting from 1998, Workshops on Multidimensional (nD) Systems have been organized biannually. Unfortunately, there exists no generally accepted name for this class of systems in the Russian language literature, where the relatively few publications include the monographs [1, 2] and the papers [3–5]. A distinctive feature of nD systems is that their dynamic characteristics depend on more than one variable and such dependencies can be represented in different combinations (as functions of discrete, continuous or mixed discrete-continuous variables). The overwhelming majority of the currently available research considers 2D systems, which date back to the 1970s in control theory and circuit design. The most widespread models include the Roesser model [6], the Fornasini–Marchesini model [7] and the repetitive process model [8]. The Roesser model originates from image processing problems where the state vector is partitioned into two sub-vectors, termed horizontal and vertical, respectively. The Fornasini–Marchesini model (a doubly indexed dynamical system in the initial terminology of [7]) deals with a single state vector. A repetitive process differs from 2D systems described by the Roesser and Fornasini Marchesini models in the finite duration of one of the independent variables. An application area for 2D systems research and, in particular, repetitive processes can be traced to [9], where iterative learning control (ILC) was introduced to improve the accuracy of robots executing the same finite duration task over and over again with resetting to the initial location after each one is complete. The survey papers [10, 11], the monograph [12] and later publications demonstrate that ILC has many applications and research in this field continues to grow, both in terms of theory and applications.

As mentioned above, a generic application area for ILC is systems consisting of multiple repetitions of the same operation, e.g., a gantry robot moving payloads from one location point to another along a given path. The novelty of such control lies in using information from the previous execution, or pass, to design the control signal for the next one. Consequently, there is information propagation in two independent variables, from pass to pass and also over a finite time interval, i.e., the duration of the pass, termed the pass length. Hence, 2D systems and, in particular, repetitive process theory is applicable and ILC laws designed using linear repetitive process theory have been verified experimentally, e.g., in [13].

A very large volume of 2D control systems research (including the case of uncertain parameter systems) deals with linear stationary dynamics. Recent years have seen the emergence of the first results on 2D

[1] The article was translated by the authors.

nonlinear systems. For example, the stability of nonlinear Fornasini–Marchesini systems was analyzed in [14], whilst [15, 16] considered different types of stability for nonlinear discrete Roesser systems. Moreover, a series of applied problems dictate further development of the nonlinear theory to allow control law design. Among recent possible applications for repetitive process control theory in the nonlinear model setting are metal deposition processes [17].

This paper establishes exponential stability conditions for nonlinear differential repetitive processes using a nonstandard extension of the vector Lyapunov functions framework [18]. The case of possible failures in operation is also considered, where the failures are modeled as random switching, i.e., by a state-space model with jumps in the parameter values and/or structure governed by a Markov chain with a finite set of states. Such models are termed Markovian jump systems or systems with random structure [19]. Finally, the new theory is applied to ILC design for a linear system with model uncertainty and sensor failures.

1. EXPONENTIAL STABILITY OF REPETITIVE PROCESSES

Consider a nonlinear repetitive process with a pass length $T < \infty$ described over $0 \le t \le T$ by the state-space model

$$\begin{aligned} \dot{x}_{k+1}(t) &= f_1(x_{k+1}(t), y_k(t), t), \\ y_{k+1}(t) &= f_2(x_{k+1}(t), y_k(t), t), \quad t \in [0,T], \quad k = 0,1,\dots, \end{aligned} \tag{1.1}$$

where on pass k $x_k(t) \in \mathbb{R}^n$ denotes the state vector, $y_k(t) \in \mathbb{R}^m$ the pass profile vector, and f_1 and f_2 are nonlinear functions such that $f_1(0,0,t) = 0$ and $f_2(0,0,t) = 0$. In addition, the function f_1 satisfies the Lipschitz condition with constant L uniformly in t, i.e.,

$$|f_1(x',y',t) - f_1(x'',y'',t)| \le L(|x' - x''| + |y' - y''|), \quad x',x'' \in \mathbb{R}^n, \quad y',y'' \in \mathbb{R}^m,$$

where $|\cdot|$ denotes the Euclidean norm. The boundary conditions, i.e., the pass state initial vector sequence and the initial pass profile, are of the form

$$\begin{aligned} x_{k+1}(0) &= d_{k+1}, \quad k \ge 0, \\ y_0(t) &= f(t), \quad 0 \le t \le T, \\ y_0(t) &= 0, \quad t > T, \end{aligned} \tag{1.2}$$

where the entries in $d_{k+1} \in \mathbb{R}^n$ are known constants for each k and the entries in $f(t) \in \mathbb{R}^m$ are known functions of t, $0 \le t \le T$. Moreover, it is assumed that $f(t)$ and d_{k+1} satisfy the inequalities

$$|f(t)|^2 \le M_f, \quad |d_{k+1}|^2 \le \kappa_d z_d^k, \quad k = 0,1,\dots, \tag{1.3}$$

where M_f and $z_d < 1$ are positive real scalars and the latter determines the rate of convergence of the pass state initial vector sequence. In the systems theory developed for linear repetitive processes [8], stability along the pass (SAP) is the basic property in control law design and experimental verification. For nonlinear systems, the recent publications [20, 21] introduced the property of pass profile exponential stability (PPES). This was motivated by the fact that (a) in the special case of linear dynamics, PPES guarantees SAP and (b) in the nonlinear case, SAP cannot be applied as it is defined in terms of properties of a particular linear operator in a Banach space.

Introduce the pass profile vector norm as

$$\|y_k\| = \sqrt{\int_0^T |y_k(t)|^2 dt}, \tag{1.4}$$

and introduce the following definitions of stability.

D e f i n i t i o n 1. A nonlinear differential repetitive process described by (1.1) and (1.2) is said to be pass profile exponentially stable if there exist $\kappa > 0$ and $0 < z < 1$ such that

$$\|y_k\| \le \kappa z^k, \tag{1.5}$$

where κ depends on the pass length T and z depends on z_d from (1.3).

Definition 2. A nonlinear differential repetitive process described by (1.1) and (1.2) is said to be PPES if there exist constants $\kappa > 0$, $\lambda > 0$ and $0 < z < 1$ such that

$$|x_k(t)|^2 + |y_k(t)|^2 \leq \kappa \exp(-\lambda t) z^k. \tag{1.6}$$

Connections between the stability properties for nonlinear differential repetitive processes and SAP [8] are discussed later in this paper, see the remark associated with Theorem 1 below and Section 3. The last definition also has some similarities with the definition of exponential stability (ES) given in [22].

For repetitive processes described by (1.1) and (1.2), it is not possible to find an analog of the total derivative of the state vector along its paths without obtaining the explicit solution, in contrast to standard systems where the first derivatives of the state vector entries can be used. Therefore, the classical framework of Lyapunov functions is not applicable. An alternative is to use a vector Lyapunov function-based approach, where the traditional comparison system [18] is replaced by the divergence operator of a vector function along the paths of the process. To derive exponential stability conditions for systems described by (1.1) and (1.2), consider the candidate vector Lyapunov function

$$V(x_{k+1}(t), y_k(t)) = \begin{bmatrix} V_1(x_{k+1}(t)) \\ V_2(y_k(t)) \end{bmatrix}, \tag{1.7}$$

where $V_1(x) > 0,\ x \neq 0,\ V_2(y) > 0,\ y \neq 0,\ V_1(0) = 0, V_2(0) = 0$. The divergence operator of this function along the paths generated by (1.1) is defined as

$$\operatorname{div} V(x_{k+1}(t), y_k(t)) = \frac{dV_1(x_{k+1}(t))}{dt} + \Delta_k V_2(y_k(t)), \tag{1.8}$$

where $\Delta_k V_2(y_k(t)) = V_2(y_{k+1}(t)) - V_2(y_k(t))$ and the following result can be established.

Theorem 1. Assume that there exist a vector function of the form (1.7) and positive constants $c_1,\ldots,c_4$ such that

$$c_1|x|^2 \leq V_1(x) \leq c_2|x|^2, \tag{1.9}$$

$$c_1|y|^2 \leq V_2(y) \leq c_2|y|^2, \tag{1.10}$$

$$\operatorname{div} V(x, y) \leq -c_3(|x|^2 + |y|^2), \tag{1.11}$$

$$\left|\frac{\partial V_1(x)}{\partial x}\right| \leq c_4|x| \tag{1.12}$$

along the paths generated by (1.1) with the boundary conditions (1.2). Then a nonlinear differential repetitive process described by (1.1) and (1.2) is exponentially stable.

Proof. It follows from (1.9)–(1.11) that there exists a constant $\bar{c}_3 < c_3$ such that $\sqrt{z_d} < \zeta = 1 - \bar{c}_3/c_2 < 1$ and

$$\frac{dV_1(x_{k+1}(t))}{dt} + \lambda V_1(x_{k+1}(t)) + V_2(y_{k+1}(t)) - \zeta V_2(y_k(t)) \leq 0, \tag{1.13}$$

where $\lambda = \bar{c}_3 / c_2,\ \zeta = 1 - \bar{c}_3 / c_2 \in (0,1)$. Solving the differential inequality (1.13) with respect to $V_1(x_{k+1}(t))$ yields

$$V_1(x_{k+1}(t)) \leq V_1(x_{k+1}(0))\mathrm{e}^{-\lambda t} - \int_0^t \mathrm{e}^{-\lambda(t-s)}[V_2(y_{k+1}(s)) - \zeta V_2(y_k(s))]ds. \tag{1.14}$$

Introduce the variables

$$W_{k+1}(t) = V_1(x_{k+1}(0))\mathrm{e}^{-\lambda t} - V_1(x_{k+1}(t)),$$

$$H_k(t) = \int_0^t \mathrm{e}^{-\lambda(t-s)} V_2(y_k(s))ds$$

and rewrite (1.14) as

$$H_{k+1}(t) \le \zeta H_k(t) + W_{k+1}(t). \tag{1.15}$$

Solving inequality (1.15) with respect to the variable H_k gives

$$H_n(t) \le \zeta^n H_0(t) + \sum_{k=1}^{n} W_k(t)\zeta^{n-k}, \tag{1.16}$$

and hence

$$\sum_{k=1}^{n} V_1(x_k(t))\zeta^{n-k} + \int_0^t e^{-\lambda(t-s)} V_2(y_n(s))ds \le e^{-\lambda t}\sum_{k=1}^{n} V_1(x_k(0))\zeta^{n-k} + \zeta^n \int_0^t e^{-\lambda(t-s)} V_2(y_0(s))ds.$$

The last inequality is equivalent to

$$e^{\lambda t}\sum_{k=1}^{n} V_1(x_k(t))\zeta^{-k} + \zeta^{-n}\int_0^t e^{\lambda s} V_2(y_n(s))ds \le \zeta^{-n}\sum_{k=1}^{n} V_1(x_k(0))\zeta^{n-k} + e^{\lambda t}\int_0^t e^{-\lambda(t-s)} V_2(y_0(s))ds. \tag{1.17}$$

Evaluating the right-hand side of (1.17) and using (1.2) and (1.3) gives

$$\zeta^{-n}\sum_{k=1}^{n} V_1(x_k(0))\zeta^{n-k} + e^{\lambda t}\int_0^t e^{-\lambda(t-s)} V_2(y_0(s))ds \le \frac{c_2 M_f(e^{\lambda T} - 1)}{\lambda} + c_2\kappa_d$$
$$\times \sum_{k=1}^{\infty}\zeta^k = \frac{c_2 M_f(e^{\lambda T} - 1)}{\lambda} + \frac{c_2\kappa_d}{1-\zeta} = C.$$

Using this estimate and (1.9), (1.17) implies that

$$|x_n(t)|^2 \le \frac{C}{c_1}\zeta^n e^{-\lambda t}. \tag{1.18}$$

The Lipschitz condition and (1.12) give

$$\frac{dV_1(x_{k+1}(t))}{dt} = \frac{\partial V_1(x_{k+1}(t))}{\partial x_{k+1}(t)} f_1(x_{k+1}(t), y_k(t), t) \ge -\left|\frac{\partial V_1(x_{k+1}(t))}{\partial x_{k+1}(t)}\right| |f_1(x_{k+1}(t), y_k(t), t)|$$
$$\ge -c_4 L(|\,x_{k+1}(t)\,| + \varepsilon\,|\,y_k(t)\,|)(|\,x_{k+1}(t)\,| + |\,y_k(t)\,|) \ge -\alpha V_1(x_{k+1}(t) - \beta\varepsilon V_2(y_k(t)), \tag{1.19}$$

where $\alpha = c_4 L(\varepsilon + 1)^2/c_1\varepsilon, \beta = 4c_4 L/c_1$, and ε is an arbitrary positive number. It follows from (1.19) and (1.13) that

$$V_2(y_{k+1}(t)) - z_0 V_2(y_k(t)) \le \alpha V_1(x_{k+1}(t)), \tag{1.20}$$

where $z_0 = \zeta + \beta\varepsilon$. Choose ε small enough such that $0 < z_0 < 1$ and apply (1.18) to solve the difference inequality (1.20):

$$V_2(y_n(t)) \le z_0^n V_2(y_0(t)) + \frac{\alpha c_2}{c_1}\sum_{k=1}^{n} z_0^{n-k}\zeta^k e^{-\lambda t}.$$

Also given the assumption (1.3) on (1.2), it follows from the last inequality that for any $\zeta_0 > z_0$ the function $V_2(y_n(t))\zeta_0^{-n}e^{\lambda t}$ is bounded for $t \in [0,\infty]$, $n = 0,1,\ldots$ Moreover, given (10),

$$|y_n(t)|^2 \le \bar{C}\zeta_0^n e^{-\lambda t}, \tag{1.21}$$

where $\bar{C}$ is a positive constant and the proof is complete since (1.18) and (1.21) establish (1.6).

Remark 1. It is easy to see that the condition (1.21) guarantees pass profile exponential stability of a process described by (1.1) and (1.2).

2. STABILITY OF DIFFERENTIAL REPETITIVE PROCESSES WITH FAILURES

This section considers nonlinear differential repetitive processes in the presence of failures and, in particular, random jumping in the model parameters or/and structure. The failures are modeled by a Markov chain with a finite set of states, often termed Markovian jump systems or systems with random structure [19, 23].

There is a sizable literature on stability, stabilization, optimal and robust control of standard (or 1D) systems with random structure. The monograph [24] and the references therein are one starting point for a comprehensive treatment of research in this field up to the time of their publication. The papers [25, 26] have solved state-feedback stabilization and H_∞ control problems for 2D discrete Roesser systems with random structure.

In [4] a class of linear discrete repetitive processes with uncertain parameters was considered. Based on the linear-quadratic control theory, a parametric description of state- and output-feedback stabilizing control laws was developed, yielding computationally efficient algorithms for computing the stabilizing gain matrices using linear matrix inequalities (LMIs). The results in [4] were further developed in [27] to design ILC laws for systems with possible failures in a "leader–follower" network.

The differential nonlinear repetitive processes with possible failures considered in this section are described by the state-space model

$$\begin{gathered} \dot{x}_{k+1}(t) = \varphi_1(x_{k+1}(t), y_k(t), r(t)), \\ y_{k+1}(t) = \varphi_2(x_{k+1}(t), y_k(t), r(t)), \quad t \in [0,T], \quad k = 0,1,\ldots, \end{gathered} \tag{2.1}$$

where $r(t)$ $(t \geq 0)$ denotes a Markov chain with a finite set of states $\mathbb{N} = \{1,\ldots,\nu\}$ and a transition probability matrix admitting the representation [19]

$$\mathrm{P}(r(t+\tau) = j \mid r(t) = i) = \begin{cases} \pi_{ij}\tau + o(\tau), & j \neq i, \\ 1 + \pi_{ii}\tau + o(\tau), & j = i, \end{cases} \pi_{ij} > 0,\, i \neq j,\, \pi_{ii} = -\sum_{i \neq j}^{\nu} \pi_{ij}. \tag{2.2}$$

Also φ_1 and φ_2 are nonlinear functions such that, for all $r \in \mathbb{N}$, $\varphi_1(0,0,r) = 0, \varphi_2(0,0,r) = 0$, and φ_1 also satisfies the Lipschitz condition

$$|\varphi_1(x', y', r) - \varphi_1(x'', y'', r)| \leq L(|x' - x''| + |y' - y''|), \quad x', x'' \in \mathbb{R}^n, \quad y', y'' \in \mathbb{R}^m, \quad r \in \mathbb{N}.$$

The rest of the notation is the same as in (1.1), the boundary conditions again have the form (1.2) and are assumed to satisfy the conditions of (1.3).

Due to the stochastic dynamics, the pass profile vector norm is defined as

$$\| y_k \|_{\mathrm{E}} = \sqrt{\mathrm{E}\int_0^T |y_k(t)|^2 dt}, \tag{2.3}$$

where E denotes the expectation operator and the following are the stochastic counterparts of the stability properties for the processes considered in the previous section.

Definition 3. A nonlinear differential repetitive process described by (2.1), (2.2) and (1.2) is said to be pass profile exponentially stable in the mean square if there exist $\kappa > 0$ and $0 < z < 1$ such that

$$\|y_k\|_{\mathrm{E}} \leq \kappa z^k, \tag{2.4}$$

where κ depends on the pass length T and z depends on z_d from (1.3).

Definition 4. A nonlinear differential repetitive process described by (2.1), (2.2) and (1.2) is said to be exponentially stable in the mean square (ESMS) if there exist constants $\kappa > 0$, $\lambda > 0$ and $0 < z < 1$ such that

$$\mathrm{E}[|x_k(t)|^2 + |y_k(t)|^2] \leq \kappa \exp(-\lambda t) z^k. \tag{2.5}$$

To derive the conditions for ESMS of a process (2.1), the route is to extend the vector Lyapunov function-based approach of the previous section to the stochastic case. Consider, therefore, the candidate vector Lyapunov function

$$V(x_{k+1}(t), y_k(t), r(t)) = \begin{bmatrix} V_1(x_{k+1}(t), r(t)) \\ V_2(y_k(t), r(t)) \end{bmatrix}, \tag{2.6}$$

where for all $r \in \mathbb{N}: V_1(x,r) > 0,\ x \neq 0,\ V_2(y,r) > 0,\ y \neq 0,\ V_1(0,r) = 0,\ V_2(0,r) = 0$ and $V_1(x,r)$ is differentiable with respect to x.

Introduce the differential and difference operators $\mathcal{D}_1$ and $\mathcal{D}_2$ defined along the paths of (2.1) as

$$\mathcal{D}_1 V(\xi,\eta,i) = \lim_{\Delta t\to 0}\frac{1}{\Delta t}\mathrm{E}[V_1(x_{k+1}(t+\Delta t), r(t+\Delta t) - V_1(x_{k+1}(t), r(t))|x_{k+1}(t) = \xi, y_k(t) = \eta, r(t) = i],$$

$$\mathcal{D}_2 V(\xi,\eta,i) = \mathrm{E}[V_2(y_{k+1}(t), r(t)) - V_2(y_k(t), r(t))|x_{k+1}(t) = \xi, y_k(t) = \eta, r(t) = i].$$

Since $V_1(\xi,i)$ is differentiable with respect to ξ for each $i \in \mathbb{N}$, then (2.1) and (2.2) give the following explicit expression for the operator $\mathcal{D}_1$:

$$\mathcal{D}_1 V(\xi,\eta,i) = \frac{\partial V_1(\xi,i)}{\partial\xi}\varphi_1(\xi,\eta,i) + \sum_{j=1}^{\nu}\pi_{i,j}V_1(\xi,j). \tag{2.7}$$

The detailed development of (2.7) is similar to [19] and, in the spirit of [28], $\mathcal{D}_1$ could be termed the averaged derivative.

Define the operator $\mathcal{D}$ as the stochastic analog of the divergence operator:

$$\mathcal{D} V(\xi,\eta,i) = \mathcal{D}_1 V(\xi,\eta,i) + \mathcal{D}_2 V(\xi,\eta,i)). \tag{2.8}$$

And the following result can be established.

T h e o r e m 2. Assume that there exist a vector function of the form (2.6) and positive constants $c_1,\ldots,c_4$ such that

$$\begin{gathered} c_1|\xi|^2 \le V_1(\xi,i) \le c_2|\xi|^2,\\ c_1|\eta|^2 \le V_2(\eta,i) \le c_2|\xi|^2,\\ \mathcal{D} V(\xi,\eta,i) \le -c_3(|\xi|^2 + |\eta|^2),\\ \left|\frac{\partial V_1(\xi,i)}{\partial\xi}\right| \le c_4\,|\,\xi\,| \end{gathered} \tag{2.9}$$

along the paths of (2.1), $i \in \mathbb{N}$. Then a differential linear repetitive process described by (2.1) and (1.2) has the ESMS property.

P r o o f. The condition (2.9) guarantees the existence of a constant $\bar{c}_3 < c_3$ such that $\sqrt{z_d} < \zeta = 1 - \bar{c}_3/c_2 < 1$ and

$$\mathcal{D} V(\xi,\eta,i) \le -\bar{c}_3(|\xi|^2 + |\eta|^2). \tag{2.10}$$

Applying the expectation operator to (2.10), using (2.9) and well-known properties of the conditional expectation, gives

$$\mathrm{E}[\mathcal{D}_1 V(x_{k+1}(t), y_k(t), r(t))] + \mathrm{E}[V_1(x_{k+1}(t), r(t))] + \mathrm{E}[V_2(y_{k+1}(t), r(t))] - \zeta\,\mathrm{E}[V_2(y_k(t), r(t))] \le 0. \tag{2.11}$$

Due to regularity, which follows from the Lipschitz conditions (by analogy with Theorem 5.2 in [19] and Theorem 4.1 in [28]),

$$\mathrm{E}[\mathcal{D}_1 V(x_{k+1}(t), y_k(t), r(t))] = \frac{d}{dt}\mathrm{E}[V(x_{k+1}(t), y_k(t), r(t))].$$

And hence (2.11) can be written as

$$\frac{d}{dt}\mathrm{E}[V(x_{k+1}(t), y_k(t), r(t))] + \lambda\,\mathrm{E}[V_1(x_{k+1}(t), r(t))] + \mathrm{E}[V_2(y_{k+1}(t), r(t))] - \zeta\,\mathrm{E}[V_2(y_k(t), r(t))] \le 0, \tag{2.12}$$

where $\lambda = \bar{c}_3\,/\,c_2$, $\zeta = 1 - \bar{c}_3\,/\,c_2 \in (0,1)$. Solving inequality (2.12) with respect to $\mathrm{E}\,V_1(x_{k+1}(t))$ gives

$$\mathrm{E}[V_1(x_{k+1}(t), r(t))] \le \mathrm{E}[V_1(x_{k+1}(0), r(0))]\mathrm{e}^{-\lambda t} - \int_0^t \mathrm{e}^{-\lambda(t-s)}\mathrm{E}[V_2(y_{k+1}(s), r(s)) - \zeta\,V_2(y_k(s), r(s))]ds. \tag{2.13}$$

Also introduce the variables

$$W_{k+1}(t) = \mathrm{E}[V_1(x_{k+1}(0), r(0))\mathrm{e}^{-\lambda t} - V_1(x_{k+1}(t), r(t))],$$

$$H_k(t) = \int_0^t \mathrm{e}^{-\lambda(t-s)}\mathrm{E}[V_2(y_k(s), r(s))]ds$$

and rewrite (2.13) as

$$H_{k+1}(t) \le \zeta H_k(t) + W_{k+1}(t). \tag{2.14}$$

Solving (2.15) gives

$$H_n(t) \le \zeta^n H_0(t) + \sum_{k=1}^{n} W_k(t)\zeta^{n-k} \tag{2.15}$$

or

$$\sum_{k=1}^{n} \mathrm{E}[V_1(x_k(t), r(t))]\zeta^{n-k} + \int_0^t \mathrm{e}^{-\lambda(t-s)}\mathrm{E}[V_2(y_n(s), r(s))]ds \le \mathrm{e}^{-\lambda t}\sum_{k=1}^{n} \mathrm{E}[V_1(x_k(0), r(0))]\zeta^{n-k}$$
$$+ \zeta^n \int_0^t \mathrm{e}^{-\lambda(t-s)}\mathrm{E}[V_2(y_0(s), r(s))]ds.$$

This last inequality is equivalent to

$$\mathrm{e}^{\lambda t}\sum_{k=1}^{n} \mathrm{E}[V_1(x_k(t), r(t))]\zeta^{-k} + \zeta^{-n}\int_0^t \mathrm{e}^{\lambda s}\mathrm{E}[V_2(y_n(s), r(s))]ds$$
$$\le \zeta^{-n}\sum_{k=1}^{n} \mathrm{E}[V_1(x_k(0), r(0))]\zeta^{n-k} + \mathrm{e}^{\lambda t}\int_0^t \mathrm{e}^{-\lambda(t-s)}\mathrm{E}[V_2(y_0(s), r(s))]ds,$$

and the remainder of the proof follows identical steps to that of Theorem 1, with obvious modifications to the notation.

3. ITERATIVE LEARNING CONTROL DESIGN FOR LINEAR SYSTEMS WITH UNCERTAIN PARAMETERS

In this section, Theorem 1 is applied to ILC design under parameter uncertainty for linear systems described by the state-space model

$$\begin{aligned} \dot{x}(t) &= A(\delta)x(t) + B(\delta)u(t), \\ y(t) &= Cx(t), \end{aligned} \tag{3.1}$$

where $x \in \mathbb{R}^n$ denotes the state vector, $u \in \mathbb{R}^m$ the input vector, $y \in \mathbb{R}^p$ the output vector and $\delta \in \mathbb{R}^N$ is the vector of constant uncertain parameters. The uncertainty associated with the system dynamics is assumed to satisfy the affine model

$$A(\delta) = A + \sum_{j=1}^{N} \delta_j A_j, \quad B(\delta) = B + \sum_{j=1}^{N} \delta_j B_j, \tag{3.2}$$

where $A, B, A_j, B_j,\ j = 1,\cdots,N$, are constant matrices of compatible dimensions, $\delta_j,\ j = 1,\cdots,N$, denote the entries in the uncertainty vector δ that satisfy the constraints

$$\underline{\delta}_j \le \delta_j \le \overline{\delta}_j. \tag{3.3}$$

Define the uncertainty set

$$\Delta = \{\delta = [\delta_1 \ \ldots \ \delta_N]^{\mathrm{T}} : \underline{\delta}_j \le \delta_j \le \overline{\delta}_j\}$$

and the set

$$\Delta_v = \{\delta = [\delta_1 \ \dots \ \delta_N]^{\mathrm{T}} : \delta_j \in \{\underline{\delta}_j, \overline{\delta}_j\}\}, \tag{3.4}$$

also termed the vertex set of the constraints polyhedron in some of the literature. In contrast to Δ, it forms a finite collection of 2^N vectors of dimension N whose elements represent a certain combination of upper $\overline{\delta}_j$, $j = 1,\cdots,N$, and lower $\underline{\delta}_j$, $j = 1,\cdots,N$, bounds of the admissible uncertain parameter range.

In the remainder of this paper, it is assumed that $CB(\delta) \neq 0$, $\delta \in \Delta$. The case when $CB(\delta) = 0$ (at least, for one value $\delta \in \Delta$) requires additional research and is not considered in this paper.

Suppose that the process (3.1) evolves in the repetitive mode with a pass length T with resetting to the initial state after each pass is complete. Moreover, within the time interval $0 \leq t \leq T$, the output signal $y(t)$ is required to follow a reference signal $y_{ref}(t)$ with a given accuracy ε. An illustrative example is a gantry robot executing the same finite task over and over again, such as placing objects on a moving conveyor under synchronization. In such systems information from previous passes can be used to compute the control signal to be applied on the next pass. The ILC design problem is to construct algorithms to achieve the required accuracy, where the survey papers [10–11] and the monograph [12] are one starting point for the literature.

To formulate the ILC problem, let the integer k denote the pass (also termed trial in some literature) number and $u_k(t), x_k(t)$ and $y_k(t)$ denote the input, state and output vectors, respectively, on this pass. These vectors have the same dimensions as their counterparts in (3.1) and the dynamics of the uncontrolled process are described by

$$\begin{aligned} \dot{x}_k(t) &= A(\delta)x_k(t) + B(\delta)u_k(t), \\ y_k(t) &= Cx_k(t), \end{aligned} \tag{3.5}$$

with the boundary conditions

$$y_0(t) = 0, \quad 0 \leq t \leq T, \quad x_k(0) = x_0, \quad k = 0,1,\ldots \tag{3.6}$$

Suppose that the components of the reference signal $y_{\mathrm{ref}}(t)$ are differentiable on the interval $[0,T]$ and define the error on pass k as

$$e_k(t) = y_{\mathrm{ref}}(t) - y_k(t). \tag{3.7}$$

Then the aim of ILC is to construct a sequence of bounded inputs such that:

$$\lim_{k\to\infty}|e_k(t)| = 0, \quad \lim_{k\to\infty}|u_k(t) - u_\infty(t)| = 0. \tag{3.8}$$

A commonly used ILC law is to select the input on the current pass as that used on the previous pass plus a correction, i.e., the ILC law on pass $k+1$ is of the form

$$u_{k+1}(t) = u_k(t) + \Delta u_{k+1}(t), \tag{3.9}$$

where $\Delta u_{k+1}(t)$ is the correction term that can use information generated over the complete previous pass, in contrast to standard feedback laws. The use of such non-causal temporal information is the major advantage of ILC.

Introduce the auxiliary vector

$$\dot{\upsilon}_{k+1}(t) = x_{k+1}(t) - x_k(t). \tag{3.10}$$

Then, given (3.5),

$$\mathrm{e}_{k+1}(t) - \mathrm{e}_k(t) = -CA(\delta)\int_0^t (x_{k+1}(\tau) - x_k(\tau))d\tau - CB(\delta)\int_0^t (u_{k+1}(\tau) - u_k(\tau))d\tau. \tag{3.11}$$

Using (3.10) and (3.11), the ILC dynamics can be described by a linear differential repetitive process with uncertainty of the form

$$\begin{aligned}\dot{\upsilon}_{k+1}(t) &= A(\delta)\upsilon_{k+1}(t) + B(\delta)\int_0^t \Delta u_{k+1}(\tau)d\tau,\\ e_{k+1}(t) &= -CA(\delta)\upsilon_{k+1}(t) + e_k(t) - CB(\delta)\int_0^t \Delta u_{k+1}(\tau)d\tau.\end{aligned} \tag{3.12}$$

Choose the correction term in (3.9) as

$$\Delta u_{k+1}(t) = F_1\dot{\upsilon}_{k+1}(t) + F_2\dot{e}_k(t). \tag{3.13}$$

Then if (3.13) guarantees exponential stability of (3.12), Theorem 1 gives that the ILC law (3.9) is convergent in the sense of (3.8). Substituting (3.13) into (3.12) yields the following equations describing the ILC dynamics

$$\begin{aligned}\dot{\upsilon}_{k+1} &= [A(\delta) + B(\delta)F_1]\upsilon_{k+1}(t) + B(\delta)F_2 e_k(t),\\ e_{k+1}(t) &= [-CA(\delta) - CB(\delta)F_1]\upsilon_{k+1}(t) + [I - CB(\delta)F_2]e_k,\end{aligned} \tag{3.14}$$

and the problem is to construct the stabilizing control law matrices F_1 and F_2 such that (3.14) is exponentially stable. To apply Theorem 1, choose the entries in the candidate vector Lyapunov function (1.7) as the quadratic forms $V_1(\upsilon_{k+1}(t)) = \upsilon_{k+1}^{\mathrm{T}}(t)P_1\upsilon_{k+1}(t)$ and $V_2(e_k(t)) = e_k^{\mathrm{T}}(t)P_2 e_k(t)$, where P_1 and P_2 are symmetrical positive definite matrices (denoted by $\succ 0$) of compatible dimensions. Calculating the divergence operator of the function (1.7) and applying Theorem 1 gives the following sufficient conditions for exponential stability of (3.14):

$$\begin{gathered}P_1 \succ 0,\ P_2 \succ 0,\\ A_c^{\mathrm{T}}(\delta)P^{1,0} + P^{1,0}A_c(\delta) + A_c^{\mathrm{T}}(\delta)P^{0,1}A_c(\delta) - P^{0,1} \prec 0,\ \delta \in \Delta,\end{gathered} \tag{3.15}$$

where

$$A_c(\delta) = \begin{bmatrix} A(\delta) + B(\delta)F_1 & B(\delta)F_2 \\ -CA(\delta) - CB(\delta)F_1 & I - CB(\delta)F_2 \end{bmatrix}, \quad P^{1,0} = \mathrm{diag}[P_1\ 0], \quad P^{0,1} = \mathrm{diag}[0\ P_2].$$

If δ is fixed, these conditions coincide with those for stability along the pass of differential linear repetitive processes [8]. Using Schur's complement formula, the conditions of (3.15) can be rewritten as the following bilinear inequalities:

$$\begin{bmatrix} A_{c1}^{\mathrm{T}}(\delta)P + PA_{c1}(\delta) - P^{0,1} & A_{c2}^{\mathrm{T}}(\delta)P \\ PA_{c2}(\delta) & -P \end{bmatrix} \prec 0,\ P \succ 0,\ \delta \in \Delta,$$

where

$$A_{c2}(\delta) = \begin{bmatrix} 0 & 0 \\ -CA(\delta) - CB(\delta)F_1 & I - CB(\delta)F_2 \end{bmatrix}, \quad A_{c1}(\delta) = \begin{bmatrix} A(\delta) + B(\delta)F_1 & B(\delta)F_2 \\ 0 & 0 \end{bmatrix}, \quad P = \mathrm{diag}[P_1\ P_2].$$

Introduce the new variables $X_1 = P_1^{-1}$, $X_2 = P_2^{-1}$, $Y_1 = F_1X_1$ and $Y_2 = F_2X_2$. Then applying Schur's complement formula and routine calculations gives the following coupled set of LMIs with respect to these variables:

$$\begin{bmatrix} D_{11}(\delta) & D_{12}(\delta) & 0 & D_{14}(\delta) \\ D_{12}(\delta)^{\mathrm{T}} & -X_2 & 0 & D_{24}(\delta) \\ 0 & 0 & -X_1 & 0 \\ D_{14}(\delta)^{\mathrm{T}} & D_{24}(\delta)^{\mathrm{T}} & 0 & -X_2 \end{bmatrix} \prec 0,\ X_1 \succ 0,\ X_2 \succ 0,\ \delta \in \Delta, \tag{3.16}$$

where $D_{11}(\delta) = A(\delta)X_1 + B(\delta)Y_1 + (A(\delta)X_1 + B(\delta)Y_1)^{\mathrm{T}}$, $D_{12}(\delta) = B(\delta)Y_2$, $D_{14}(\delta) = [-CA(\delta)X_1 - CB(\delta)Y_1]^{\mathrm{T}}$, $D_{24}(\delta) = (X_2 - CB(\delta)Y_2)^{\mathrm{T}}$.

Since the parameter uncertainty has an affine model, these inequalities hold for all $\delta \in \Delta$ if and only if they do so on a finite set $\delta \in \Delta_v$ [29], and the following result has been established.

T h e o r e m 3. Suppose that the LMIs (3.16) with $\delta \in \Delta_v$ are feasible with respect to X_1, X_2, Y_1, Y_2 and set $F_1 = Y_1 X_1^{-1}$, $F_2 = Y_2 X_2^{-1}$. Then the ILC law defined by (3.9) and (3.13) for systems described by (3.1) satisfies (3.8).

It is of interest to remove possibly undesirable effects resulting from the presence of derivatives in the ILC law in the above design. One possible way is to introduce the auxiliary variable

$$\vartheta_{k+1}(t) = x_{k+1}(t) - x_k(t) \tag{3.17}$$

and assume that the correction $\Delta u_{k+1}(t)$ in (3.9) involves no derivatives:

$$\Delta u_{k+1}(t) = K_1 \vartheta_{k+1}(t) + K_2 e_k(t). \tag{3.18}$$

Then using (3.9), (3.17) and (3.18), the controlled dynamics can be rewritten as

$$\begin{gathered} \dot{\vartheta}_{k+1} = [A(\delta) + B(\delta)K_1]\vartheta_{k+1}(t) + B(\delta)K_2 e_k(t), \\ e_{k+1}(t) = -C\vartheta_{k+1}(t) + e_k, \end{gathered} \tag{3.19}$$

where

$$A_c(\delta) = \begin{bmatrix} A(\delta) + B(\delta)K_1 & B(\delta)K_2 \\ -C & I \end{bmatrix}.$$

However, this system has no form of repetitive process stability [8] and hence (3.8) cannot hold. This problem area is one for possible future research.

4. ITERATIVE LEARNING CONTROL DESIGN FOR SYSTEMS WITH UNCERTAIN PARAMETERS AND SENSOR FAILURES

Consider the system (3.1) under possible sensor failures modeled by

$$\begin{gathered} \dot{x}(t) = A(\delta)x(t) + B(\delta)u(t), \\ y(t) = C(r(t))x(t), \end{gathered} \tag{4.1}$$

where $r(t)$ denotes a Markov chain with a finite set of states $\mathbb{N} = \{1, \ldots, \nu\}$ corresponding to the number of possible failures with transition probabilities given by (2.2).

With the ILC law (3.9) applied, the controlled dynamics are described by the state-space model

$$\begin{gathered} \dot{x}_k(t) = A(\delta)x_k(t) + B(\delta)u_k(t), \\ y_k(t) = C(r(t))x_k(t), \end{gathered} \tag{4.2}$$

where notation is that of the previous section. Due to the stochastic nature of (4.1), the following modified definition of ILC convergence is used.

D e f i n i t i o n 5. A system described by (4.1) is said to be ILC convergent if for all $0 \le t \le T$:

$$\mathrm{E}[\|e_k(t)\|^2] = \mathrm{E}[\|y_{\mathrm{ref}}(t) - y_k(t)\|^2] \to 0, \quad k \to \infty \tag{4.3}$$

and

$$\mathrm{E}[\|u_k(t) - u_\infty(t)\|^2] \to 0, \quad k \to \infty. \tag{4.4}$$

Using (3.10) and (3.11), the ILC dynamics can be written as a differential repetitive process with the random structure

$$\begin{gathered} \dot{\upsilon}_{k+1}(t) = A(\delta)\upsilon_{k+1}(t) + B(\delta)\Delta u_{k+1}(t), \\ e_{k+1}(t) = -C(r(t))A(\delta)\upsilon_{k+1}(t) + e_k(t) - C(r(t))B(\delta)\Delta u_{k+1}(t). \end{gathered} \tag{4.5}$$

Choose the ILC law correction term as

$$\Delta u_{k+1}(t) = F_1(i)\dot{\upsilon}_{k+1}(t) + F_2(i)\dot{e}_k(t), \text{ if } r(t) = i. \tag{4.6}$$

Then, if (4.6) guarantees the ESMS property of (4.5), Theorem 2 implies that the ILC law (3.9) is convergent.

To construct the stabilizing control law matrices $F_1(i)$ and $F_2(i)$, $i \in \mathbb{N}$, the stability conditions of Theorem 2 can be applied. Choose the entries in the candidate vector Lyapunov function as (2.6) with $V_1(\upsilon_{k+1}(t), r(t)) = \upsilon_{k+1}^{\mathrm{T}}(t)P_1(r(t))\upsilon_{k+1}(t)$, $V_2(e_k(t), r(t)) = e_k^{\mathrm{T}}(t)P_2(r(t))e_k(t)$, where $P_1(i) > 0$, $P_2(i) > 0 i \in \mathbb{N}$. Calculating the stochastic divergence operator of the function (2.6) along the paths of the system described by (4.5) gives the following sufficient conditions for ESMS:

$$P(i) = \mathrm{diag}[P_1(i)\ P_2(i)] \succ 0,$$

$$A_{c1}^{\mathrm{T}}(\delta, i)P(i) + P(i)A_{c1}(\delta, i) + \sum_{j=1}^{\nu} \pi_{ij} I^{1,0} P(j) - I^{0,1}P(i) + A_{c2}^{\mathrm{T}}(\delta, i)P(i)A_{c2}(\delta, i) \prec 0, \quad i \in \mathbb{N}, \quad \delta \in \Delta, \quad (4.7)$$

where $I^{1,0} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, I^{0,1} = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}$.

By setting $X(i) = P^{-1}(i)$, $Y(i) = F_1(i)X_1(i)$, $Y_2(i) = F_2(i)X_2(i)$ and using Schur's complement formula, a series of routine calculations yield the following coupled set of LMIs with respect to these variables:

$$\begin{bmatrix} S_{11}(\delta, i) & S_{12}(\delta, i) & S_{13}(i) \\ S_{12}^{\mathrm{T}}(\delta, i) & -X(i) & 0 \\ S_{13}^{\mathrm{T}}(i) & 0 & S_{33}(i) \end{bmatrix} \succ 0,\ X(i) \succ 0,\ \delta \in \Delta,\ i \in \mathbb{N}, \quad (4.8)$$

where

$$S_{11}(\delta, i) = \begin{bmatrix} A_{c11}(\delta, i) & B(\delta)Y_1(i) \\ (B(\delta)Y_1(i))^{\mathrm{T}} & -X_2(i) \end{bmatrix}, \quad S_{12}(\delta, i) = \begin{bmatrix} 0 & 0 \\ A_{c12}(\delta, i) & A_{c22}(\delta, i) \end{bmatrix}^{\mathrm{T}},$$

$$A_{c11}(\delta, i) = A(\delta)X(i) + B(\delta)Y_1(i) + (A(\delta)X(i) + B(\delta)Y_1(i))^{\mathrm{T}} + \pi_{ii}X_1(i),$$

$$A_{c12}(\delta, i) = -C(i)A(\delta)X_1(i) - C(i)B(\delta)Y_1(i),\ A_{c22}(\delta, i) = X_2(i) - C(i)B(\delta)Y_2(i),$$

$$S_{13}(i) = [\pi_{i1}^{\frac{1}{2}} X(i)I^{1,0} \ldots \pi_{i\,i-1}^{\frac{1}{2}} X(i)I^{1,0} \pi_{i\,i+1}^{\frac{1}{2}} X(i)I^{1,0} \ldots \pi_{i\nu}^{\frac{1}{2}} X(i)I^{1,0}],$$

$$S_{33}(i) = \mathrm{diag}[-X(1) \ldots - X(i-1) - X(i+1) \ldots - X(\nu)].$$

Since the parameter uncertainty has an affine model, these inequalities hold for all $\delta \in \Delta$ if and only if they do so on a finite set $\delta \in \Delta_v$ [29]. Hence, the following result has been established.

T h e o r e m 4. Suppose that the LMIs (4.8) with $\delta \in \Delta_v$ are feasible with respect to $X_1(i), X_2(i)$, $Y_1(i), Y_2(i)$, $i \in \mathbb{N}$ and set $F_1(i) = Y_1(i)X_1^{-1}(i)$, $F_2(i) = Y_2(i)X_2^{-1}(i)$, $i \in \mathbb{N}$. Then the ILC law defined by (3.9), (4.6) for system described by (4.1) is convergent in the sense of Definition 5.

5. CONCLUSIONS

This paper has developed new results on stability of nonlinear differential repetitive processes using a non-standard vector Lyapunov function-based approach. To demonstrate the practical potential of the derived theoretical results, their application in ILC design under parameter uncertainty and sensor failures has been investigated and the results obtained are a starting point for further research. In addition, these results can be treated as a starting point for further research on the stabilization of nonlinear repetitive processes based on vector Lyapunov functions, such as the development of an analog to the inversion theorem established for another definition of stability of Roesser systems [16].

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