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DATA PROCESSING AND IDENTIFICATION

# An Algebraic Criterion for Detecting the Fact and Time a Fault Occurs in Control Systems of Dynamic Plants

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**Abstract**—This paper proposes a new algebraic criterion for fault detection in control systems of dynamic plants; this criterion uses only the measurements of input and output signals and is based on the solvability condition for the problem of identifying the mathematical model of a dynamic plant. To estimate the proposed criterion, it is compared with a criterion based on analyzing prediction errors in solving a fault detection problem for a stabilizer actuator.

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## INTRODUCTION

The fault detection methods presently available can be classified into two groups [1–7]. The methods of the first group employ, directly or indirectly, mathematical models of plants, while the methods of the second group use only the measurements of input and output signals.

The model-based fault detection methods are the most popular and regarded as the classical ones [8–17]. These methods employ three different approaches for fault detection. The first approach is based on determining the parameters (invariants) of plant models, the second is based on solving the modeling (prediction) problem, and the third is based on analytical redundancy.

Using the model-based methods involves a number of difficulties due to the nonlinearity of such models, inaccuracy in determining their parameters, the impossibility of finding a unique solution, etc. The parameter errors in plant models inevitably increase the threshold values of the fault detection criteria, thus increasing the time of fault detection and decreasing the accuracy of determining the time the fault occurred. In some cases (see below), the accurate determination of the time that the fault occurred proves to be impossible in principle.

In contrast to the model-based methods, the methods based only on the measurements of input and output signals need no a priori information about plant parameters. These methods use the specific qualitative or quantitative characteristics of a plant [18, 19].

The fault detection methods that use qualitative characteristics analyze the behavior of processes or employ expert systems. In turn, the methods that use quantitative characteristics are based on the pattern recognition theory and can be subdivided into statistical and nonstatistical methods. The statistical methods include principal component methods, partial least square methods, and methods based on classification algorithms. The nonstatistical methods include only methods based on artificial neural networks.

The presently available methods that use measurement data only either require preliminary training/tuning for a particular plant (which makes these methods plant-specific) or are based on statistical algorithms. By definition, all statistical algorithms need a rather large sample of measurement data to reveal the statistical properties of the variables being analyzed, which inevitably increases the time of fault detection. In practice, to detect faults, the statistical algorithms may need time exceeding the critical response time of a control system during which a plant may enter the nonrestorable state [20].

This paper proposes a new quantitative fault detection criterion that uses only the measurements of input and output signals in a control system. This criterion needs no a priori information about the plant parameters, involves no statistical computations, and does not require solving the prediction problem; instead, it is based on the algebraic condition of solvability for the problem of identifying the mathematical model of a dynamic plant.

## 1. FORMULATION OF THE PROBLEM

Let the model of a nonfaulted dynamic plant with a closed control system be represented in the state space as

$$x_{i+1} = Ax_i + B(u_i + u_o), \tag{1.1}$$

$$u_i = Kx_i + Gv_i, \tag{1.2}$$

where $A$, $B$, $K$, and $G$ are the matrices of eigen dynamics, control efficiency, controller, and precompensator, respectively; $x$ is the state vector of length $n_x$; $u$ is the output signal of the control system that, if no faults occurred, coincides with the control deflection vector of length $n_v$; $u_o$ is the vector of the trim deflections corresponding to the equilibrium state of the plant; $i = \overline{0, l-1}$ is the discrete time before the occurrence of faults; and $l$ is the instant a fault occurs.

Let us write model (1.1) as

$$x_{i+1} = Ax_i + Bu_i + m_o, \tag{1.3}$$

where $m_o = Bu_o$ is the vector of the constant coefficients that depend on the trim deflections of the controls. When faults occur in the control system, the model of the plant is rewritten as

$$x_{j+1}^f = Ax_j^f + Bu_j^f + m_o, \tag{1.4}$$

$$u_j = Kx_j^f + Gv_j, \tag{1.5}$$

where $j = l, l+1, \ldots$ is the discrete time after a fault occurs and $x^f$ is the state vector of the faulted plant whose control deflection is described by the expression [21, 22]

$$u_j^f = Fu_j + (I - F)u_o^f, \tag{1.6}$$

where $F$ is the matrix of faults (loss of efficiency) of the control system

$$F = \mathrm{diag}\left[f(1) \cdots f(k) \cdots f(n_u)\right] \tag{1.7}$$

and $u_o^f$ is the vector of control jamming in the case of faults

$$u_o^f = \left[u_o^f(1) \cdots u_o^f(k) \cdots u_o^f(n_u)\right]^{\mathrm{T}}. \tag{1.8}$$

Let us substitute (1.6) into (1.4) and write the model of the plant with the faulted control system as

$$x_{j+1}^f = Ax_j^f + B_f u_j + m_o^f, \tag{1.9}$$

where $B_f = BF$ is the matrix of control efficiency for the faulted plant and $m_o^f = B(I - F)u_o^f + m_o$ is the constant vector characterizing the combined control deflection in the case of faults. Thus, it is required, based only on the measurements of control signals and states, to detect faults in the control system of the dynamic plant.

## 2. SOLUTION OF THE PROBLEM

Assume that the plant is observed over a certain period of time. Then, the models of the plant in the nonfaulted (1.3) and faulted (1.9) states are written in matrix form as

$$X_{i+1} = AX_i + BU_i + m_o e, \quad X_{j+1}^f = AX_j^f + B_f U_j + m_o^f e,$$

where $X_i = [x_i \ldots x_{i+h}]$, $X_j^f = [x_j^f \ldots x_{j+h^f}^f]$, $U_i = [u_i \ldots u_{i+h}]$, $U_j = [u_j \ldots u_{j+h^f}]$, and $e = [1 \cdots 1]$, where $h$ and $h^f$ characterize the number of the observation steps for the nonfaulted and faulted plants, respectively. For both these cases, the problems of identifying the model parameters of the plant are described by the linear right-hand matrix equations in the unknown $A, B, m_o, B_f, m_o^f$:

$$\begin{bmatrix} A & B & m_o \end{bmatrix} \begin{bmatrix} X_i \\ U_i \\ e \end{bmatrix} = X_{i+1}, \quad \begin{bmatrix} A & B_f & m_o^f \end{bmatrix} \begin{bmatrix} X_j^f \\ U_j \\ e \end{bmatrix} = X_{j+1}^f. \tag{2.1}$$

To solve matrix equations (2.1), we use the results obtained in [23], where it was shown that a linear matrix equation of the form

$$XC = D$$

with known matrices $C$ and $D$ is solved for $X$ if and only if the solvability condition

$$D\bar{C}^R = 0 \tag{2.2}$$

is met; in this case, the whole set of solutions is defined by the formula

$$X = \begin{bmatrix} D\tilde{C}^R & \Theta \end{bmatrix} \begin{bmatrix} \tilde{C}^L \\ \bar{C}^L \end{bmatrix} = D\tilde{C} + \Theta\bar{C}^L, \tag{2.3}$$

where $\Theta$ is arbitrary matrix; $\bar{C}^L$ and $\bar{C}^R$ are the left-hand and right-hand zero divisors of the maximum rank, respectively (i.e., the matrices for which the conditions $\bar{C}^L C = 0$ and $C\bar{C}^R = 0$ hold); $\tilde{C}^L$ and $\tilde{C}^R$ are the left-hand and right-hand divisors of unity, respectively ($\tilde{C}^L C\tilde{C}^R = I$); and $\tilde{C} = \tilde{C}^R\tilde{C}^L$ is the generalized inverse matrix defined by the canonical decomposition

$$C = \begin{bmatrix} \tilde{C}^L \\ \bar{C}^L \end{bmatrix}^{-1} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{C}^R & \bar{C}^R \end{bmatrix}^{-1}. \tag{2.4}$$

In the general case, canonical decomposition (2.4) has a nonunique form and actually formalizes direct and inverse equivalent transformations of matrices [24]. The canonical decomposition allows obtaining the set of all solutions of the minimum rank in an analytical form.

Then, according to (2.2), the solvability conditions for the problems of identifying models (2.1) (i.e., the conditions that there is at least one solution) correspond to the following expressions that are equivalent up to the designation of variables:

$$X_{i+1} \overline{\begin{bmatrix} X_i \\ U_i \\ e \end{bmatrix}}^R = 0, \quad X_{j+1}^f \overline{\begin{bmatrix} X_j^f \\ U_j \\ e \end{bmatrix}}^R = 0. \tag{2.5}$$

When solvability conditions (2.5) are met, according to (2.3), all identified models can be written as the sets

$$\begin{bmatrix} A & B & m_o \end{bmatrix} = X_{i+1} \widetilde{\begin{bmatrix} X_i \\ U_i \\ e \end{bmatrix}} + \Psi \overline{\begin{bmatrix} X_i \\ U_i \\ e \end{bmatrix}}^L, \quad \begin{bmatrix} A & B_f & m_o^f \end{bmatrix} = X_{j+1}^f \widetilde{\begin{bmatrix} X_j^f \\ U_j \\ e \end{bmatrix}} + \Upsilon \overline{\begin{bmatrix} X_j^f \\ U_j \\ e \end{bmatrix}}^L, \tag{2.6}$$

where $\Psi$ and $\Upsilon$ are arbitrary matrices.

According to (2.6), the identifiability conditions (i.e., the conditions of obtaining a unique solution) correspond to the following equalities for the left-hand zero divisors:

$$\overline{\begin{bmatrix} X_i \\ U_i \\ e \end{bmatrix}}^L = 0, \quad \overline{\begin{bmatrix} X_j^f \\ U_j \\ e \end{bmatrix}}^L = 0. \tag{2.7}$$

To determine the conditions of their existence, expressions (1.2) and (1.5) are substituted into (2.7):

$$\begin{bmatrix} X_i \\ U_i \\ e \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ K & G & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_i \\ V_i \\ e \end{bmatrix}, \quad \begin{bmatrix} X_j^f \\ U_j \\ e \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ K & G & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_j^f \\ V_j \\ e \end{bmatrix},$$

where $V_i = \begin{bmatrix} v_i & \cdots & v_{i+h} \end{bmatrix}$ and $V_j = [v_j \ \cdots \ v_{j+h^f}]$. Then, identifiability conditions (2.7) are written as the zero divisors of the products of the block matrices

$$\overline{\begin{bmatrix} I & 0 & 0 \\ K & G & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_i \\ V_i \\ e \end{bmatrix}}^L = 0, \quad \overline{\begin{bmatrix} I & 0 & 0 \\ K & G & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_j^f \\ V_j \\ e \end{bmatrix}}^L = 0,$$

which, with the left-hand zero divisor of the precompensator matrix ($\bar{G}^L G = 0$), can always be written as

$$\begin{bmatrix} 0 & \bar{G}^L & 0 \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ K & G & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_i \\ V_i \\ e \end{bmatrix} = 0, \quad \begin{bmatrix} 0 & \bar{G}^L & 0 \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ K & G & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_j^f \\ V_j \\ e \end{bmatrix} = 0.$$

The precompensator matrix $G$ transforms the input signals $v$ into the signals $u$ received directly by the controls. In practical problems of controlling advanced dynamic plants, the number of input signals (columns in the matrix $G$) is generally smaller than the number of the controls (rows in the matrix $G$); therefore, the precompensator matrix contains dependent rows that determine the existence of its zero divisor. In [25], it was shown that, in this case, the problem of finding a unique solution when identifying closed plants is relevant for any input signals, regardless of the availability of the information about the parameters of a control system. This shows that, in practice, parametric identification methods are used either taking into account a priori information about the plant model or by feeding test signals straight to the controls of the plant [26, 27].

Let us analyze in more detail the solvability conditions without solving the identification problem directly. Expressions (2.5) suggest that the problem of identifying the linear model of the plant is solvable both before and after the occurrence of faults. At the very instant of the occurrence of the fault, however, the behavior of the plant cannot be described by the single linear model

$$\begin{bmatrix} X_{i+1} & X_{j+1}^f \end{bmatrix} = A\begin{bmatrix} X_i & X_j^f \end{bmatrix} + B\begin{bmatrix} U_i & U_j \end{bmatrix} + m_o\begin{bmatrix} e & e \end{bmatrix} + \Delta B_f \begin{bmatrix} 0 & U_j \end{bmatrix} + \Delta m_o^f \begin{bmatrix} 0 & e \end{bmatrix},$$

where $\Delta B_f = B(F - I)$ and $\Delta m_o^f = B(I - F)u_o^f$. Hence, in this case, the identification problem does not have an exact solution and the solvability condition does not hold:

$$\begin{bmatrix} X_{i+1} & X_{j+1}^f \end{bmatrix} \overline{\begin{bmatrix} X_i & X_j^f \\ U_i & U_j \\ e & e \end{bmatrix}}^R = \begin{bmatrix} \Delta B_f & \Delta m_o^f \end{bmatrix} \begin{bmatrix} 0 & U_j \\ 0 & e \end{bmatrix} \overline{\begin{bmatrix} X_i & X_j^f \\ U_i & U_j \\ e & e \end{bmatrix}}^R \neq 0. \tag{2.8}$$

This allows us to use the norm of condition (2.8), which characterizes the accuracy of solving the identification problem [21]

$$\varepsilon = \left\| \begin{bmatrix} X_{i+1} & X_{j+1}^f \end{bmatrix} \overline{\begin{bmatrix} X_i & X_j^f \\ U_i & U_j \\ e & e \end{bmatrix}}^R \right\|_2 \tag{2.9}$$

when finding the orthogonal zero divisor of the matrix of the input-output data

$$\left( \overline{\begin{bmatrix} X_i & X_j^f \\ U_i & U_j \\ e & e \end{bmatrix}}^R \right)^{\mathrm{T}} \overline{\begin{bmatrix} X_i & X_j^f \\ U_i & U_j \\ e & e \end{bmatrix}}^R = I,$$

as a simple and valid criterion of the occurrence of a fault in the control system of a dynamic plant (1.1).

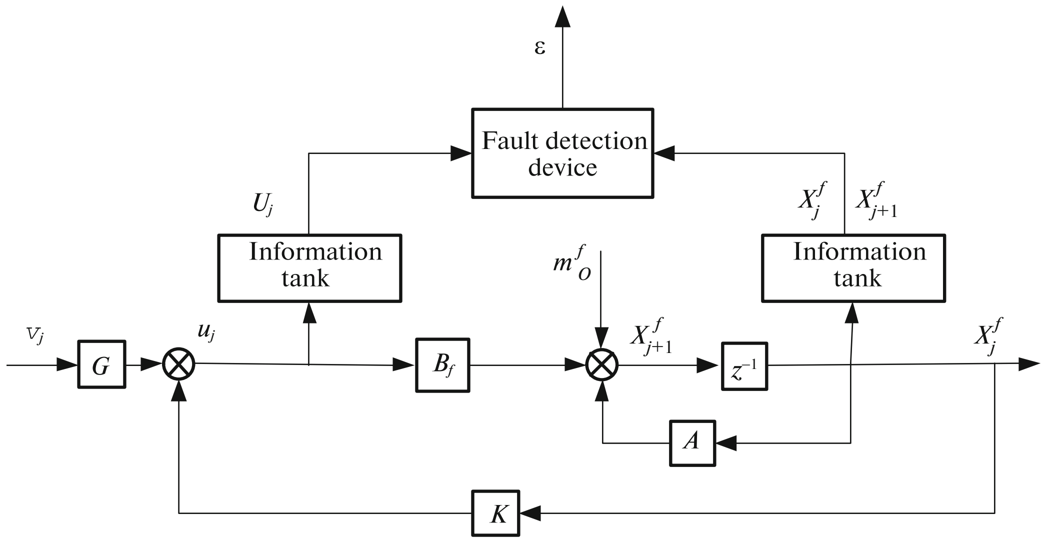


**Fig. 1.** Flow chart of fault detection.

Figure 1 shows the flow chart of the fault detection algorithm corresponding to the proposed method. To form the matrices of the input-output data according to the desired width of the identification window, the information tanks should be used that provide the data for the fault detection device implementing the algorithm for the calculation norm (2.9).

For a nonfaulted plant, the value of norm (2.9) is zero (or a small number due to calculation errors) and exceeds it when faults occur. The instant when the norm deviates from zero coincides with the instant that the fault occurs in the control system. Thus, the efficiency and accuracy of detecting the time the fault occurs are determined by the sampling frequency and coincide with the time interval between two consecutive measurements.

In this case, the fault detection depends only on the width of the identification window. The maximum required number of measurements is determined by summing the lengths of the state and control vectors. In the case of a low information value of the control signals or given a degenerate model of a plant, the number of measurements can be reduced. The minimum required number of measurements is determined by the width of the identification window for which the data matrix contains dependent columns.

## 3. METHODOLOGICAL EXAMPLE

Let us demonstrate the efficiency of the proposed method on the example of fault detection in the control system of a first-order dynamic plant represented in symbol form

$$x_{i+1} = ax_i + bu_i,;$$

the dynamics of this plant simulate the zero initial states and a constant unity control action.

Let a fault of form (1.6) occur at the third step, resulting in the loss of control efficiency ($f < 1$) without jamming ($u_o^f = 0$) for $b_f = bf = b + \Delta b$, where $\Delta b = b(1 - f)$. Thus, the matrix of input-output data for six steps of simulation is written as

$$\begin{bmatrix} X \\ U \\ e \end{bmatrix}^T = \begin{bmatrix} 0 & 1 & 1 \\ b & 1 & 1 \\ b(a+1) & 1 & 1 \\ b(a^2 + a + 1) & 1 & 1 \\ b + \Delta b + ab(a^2 + a + 1) & 1 & 1 \\ (b + \Delta b)(1 + a) + a^2 b(a^2 + a + 1) & 1 & 1 \\ (ba^3 + b + \Delta b)(a^2 + a + 1) & 1 & 1 \end{bmatrix}.$$

Next, set $h = h^f = 2$, determine the right-hand zero divisor of the data matrix for the first step of the fault detection algorithm

$$\overline{\begin{bmatrix} X_0 \\ U_0 \\ e \end{bmatrix}}^R = \overline{\begin{bmatrix} 0 & b & b(a+1) \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}}^R = \begin{bmatrix} a \\ -(a+1) \\ 1 \end{bmatrix},$$

and check if solvability condition (2.5) holds:

$$X_1 \overline{\begin{bmatrix} X_0 \\ U_0 \\ e \end{bmatrix}}^R = \begin{bmatrix} b & b(a+1) & b(a^2+a+1) \end{bmatrix} \begin{bmatrix} a \\ -(a+1) \\ 1 \end{bmatrix} = 0.$$

The fact that this condition equals zero indicates that the plant is nonfaulted; therefore, the input-output data can be described by a first-order model.

Then, determine the right-hand zero divisor of the data matrix for the second step of the algorithm

$$\overline{\begin{bmatrix} X_1 \\ U_1 \\ e \end{bmatrix}}^R = \overline{\begin{bmatrix} b & b(a+1) & b(a^2+a+1) \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}}^R = \begin{bmatrix} a \\ -(a+1) \\ 1 \end{bmatrix},$$

which is completely similar to the zero divisor at the previous step, and substitute this expression into condition (2.5):

$$X_2 \overline{\begin{bmatrix} X_1 \\ U_1 \\ e \end{bmatrix}}^R = \begin{bmatrix} b(a+1) & b(a^2+a+1) & b+\Delta b+ab(a^2+a+1) \end{bmatrix} \begin{bmatrix} a \\ -(a+1) \\ 1 \end{bmatrix} = \Delta b.$$

Note that the solvability condition no longer holds, which implies that the input-output data cannot be described by the first-order linear model and that the fault occurs in the control system at the third step of simulation. For the given width of the identification window, the solvability condition should also not hold at the next step, which can be proved by determining the right-hand zero divisor of the data matrix

$$\overline{\begin{bmatrix} X_2 \\ U_2 \\ e \end{bmatrix}}^R = \overline{\begin{bmatrix} b(a+1) & b(a^2+a+1) & b+\Delta b+ab(a^2+a+1) \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}}^R = \begin{bmatrix} ba^3+\Delta b \\ -\Delta b(ba^3+\Delta b) \\ a^2 b \end{bmatrix}$$

and by substituting it into expression (2.5):

$$X_3 \overline{\begin{bmatrix} X_2 \\ U_2 \\ e \end{bmatrix}}^R = -\Delta b(ba^3+\Delta b).$$

At the subsequent steps, the right-hand zero divisor remains constant and the solvability condition

$$\overline{\begin{bmatrix} X_3 \\ U_3 \\ e \end{bmatrix}}^R = \begin{bmatrix} a \\ -(a+1) \\ 1 \end{bmatrix}, \quad X_4 \overline{\begin{bmatrix} X_3 \\ U_3 \\ e \end{bmatrix}}^R = 0$$

holds, which indicates that there are no further variations in the parameters of the plant after the occurrence of the fault.

## 4. FAULT DETECTION IN A FLIGHT CONTROL SYSTEM

Let us demonstrate the validity of the proposed criterion on the example of fault detection in the right stabilizer actuator of a highly-maneuverable aircraft. For one of the flight modes, the matrices of the eigen dynamics, control efficiency, controller, and precompensator of model (1.1) and (1.2) take the following values, respectively:

$$A = \begin{bmatrix} -5.44 & -0.96 & 0 & 0 & -14.6 & 0 & 0 \\ 0.144 & -0.331 & 0 & 0 & -7.2 & 0 & 0 \\ 0 & 0 & -1.74 & 10.13 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2.02 & 0 & 0 & 0 \\ -0.0102 & 1 & 0 & 0 & -0.43 & 0.036 & 0 \\ 1 & 0.0102 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} -27.0685 & 27.0685 & -17.0754 & 17.0754 & -1.2262 & -1.2262 \\ 0.785 & -0.785 & 0.7105 & -0.7105 & -2.6587 & -2.6587 \\ -9.75 & -9.75 & -0.05 & -0.05 & 0 & 0 \\ -0.178 & -0.178 & -0.092 & -0.092 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0458 & -0.0458 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$K = \begin{bmatrix} -0.05 & 0 & -0.3 & -0.5 & 0 & 0 & 0 \\ 0.05 & 0 & -0.3 & -0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.8 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.8 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} 0.38 & 0.049 & 0 \\ 0.38 & -0.049 & 0 \\ 0 & 1.75 & 0 \\ 0 & -1.75 & 0 \\ 0 & 0 & 0.03 \\ 0 & 0 & 0.03 \end{bmatrix}.$$

In this case, the vectors of states, input signals, and control signals are written as

$$x = [\omega_x \ \omega_y \ \omega_z \ \alpha \ \beta \ \gamma \ \vartheta]^{\mathrm{T}}, \quad v = [v_\vartheta \, v_\gamma \, v_\psi]^{\mathrm{T}},$$

$$u = [\varphi_{\mathrm{r}} \ \varphi_{\mathrm{l}} \ \delta_{\mathrm{f.r}} \ \delta_{\mathrm{f.l}} \ \delta_{\mathrm{r.r}} \ \delta_{\mathrm{r.l}} \ \delta_{\mathrm{hc}}]^{\mathrm{T}},$$

where $\omega_x, \omega_y, \omega_z$ are the angular rates of roll, yaw, and pitch (deg/s), respectively; $\alpha, \vartheta, \gamma, \beta$ are the angles of attack, pitch, roll, and slip (deg), respectively; $v_\vartheta$, $v_\gamma$, and $v_\psi$ are the input control signals from the pilot for pitch (deg), roll (deg), and yaw (mm), respectively; $\varphi_{\text{пр}}, \varphi_{\text{л}}, \delta_{\text{ф.пр}}, \delta_{\text{ф.л}}, \delta_{\text{н.пр}}, \delta_{\text{н.л}}$, and$\delta_{\text{п.г.о}}$ are the mistrim angles for the right and left stabilizers, flaperons, rudders, and horizontal canard (deg), respectively.

Let us simulate the flight of the aircraft with the pitch control ($v_\vartheta = 5°, v_\gamma = 0, v_\psi = 0$) during 10 s by using the first-order Euler method with the integration step of 0.1 s. A fault in the form of a right stabilizer actuator jamming in the neutral position occurs in the fifth second of the flight. In this case, the fault parameters (1.7) and (1.8) are $F = \mathrm{diag}[0 \ 1 \ \cdots \ 1]$ and $u_o^f = [0 \ 0 \ \cdots \ 0]^{\mathrm{T}}$.

Below, we demonstrate the fulfillment of the solvability conditions for the problems of identifying the model of the aircraft in the nonfaulted and faulted states (2.5). Tables 1 and 2 show the values of the aerodynamic parameters and control deflections on the time interval of 1 s immediately before and after the

**Table 1.** Input-output data before a fault occurs

| Variables | Values of the variables | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $t$ | 4.0 | 4.1 | 4.2 | 4.3 | 4.4 | 4.5 | 4.6 | 4.7 | 4.8 | 4.9 |
| $i$ | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 |
| $x_i$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | −5 | −5 | −5 | −5 | −5 | −5 | −5 | −5 | −5 | −5 |
| | −2.34 | −2.34 | −2.34 | −2.34 | −2.34 | −2.34 | −2.34 | −2.34 | −2.34 | −2.34 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | −19.27 | −19.77 | −20.27 | −20.77 | −21.27 | −21.77 | −22.27 | −22.77 | −23.27 | −23.77 |
| $u_i$ | −0.77 | −0.77 | −0.77 | −0.77 | −0.77 | −0.77 | −0.77 | −0.77 | −0.77 | −0.77 |
| | −0.77 | −0.77 | −0.77 | −0.77 | −0.77 | −0.77 | −0.77 | −0.77 | −0.77 | −0.77 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

**Table 2.** Input-output data after a fault occurs

| Variables | Values of the variables | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $t$ | 5 | 5.1 | 5.2 | 5.3 | 5.4 | 5.5 | 5.6 | 5.7 | 5.8 | 5.9 |
| $j$ | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 |
| $x_j$ | 0 | −2.08 | −3.39 | −4.36 | −5.14 | −5.82 | −6.44 | −7.02 | −7.55 | −8.06 |
| | 0 | 0.06 | 0.07 | 0.07 | 0.05 | 0.04 | 0.02 | 0.01 | 0.01 | 0.01 |
| | −5 | −5.75 | −6.26 | −6.64 | −6.95 | −7.22 | −7.46 | −7.69 | −7.91 | −8.12 |
| | −2.34 | −2.35 | −2.44 | −2.55 | −2.68 | −2.81 | −2.94 | −3.07 | −3.19 | −3.3 |
| | 0 | 0 | 0.01 | 0.02 | 0.02 | 0.03 | 0.03 | 0.03 | 0.03 | 0.02 |
| | 0 | 0 | -0.21 | -0.55 | -0.98 | -1.5 | -2.08 | -2.72 | -3.43 | -4.18 |
| | −24.27 | −24.77 | −25.34 | −25.97 | −26.63 | −27.32 | −28.05 | −28.79 | −29.56 | −30.35 |
| $u_j$ | −0.77 | −1.1 | −1.36 | −1.58 | −1.78 | −1.96 | −2.13 | −2.29 | −2.44 | −2.59 |
| | −0.77 | −0.9 | −1.03 | −1.15 | −1.27 | −1.38 | −1.49 | −1.59 | −1.69 | −1.78 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | 0.05 | 0.06 | 0.05 | 0.04 | 0.03 | 0.02 | 0.01 | 0.01 | 0.01 |
| | 0 | 0.05 | 0.06 | 0.05 | 0.04 | 0.03 | 0.02 | 0.01 | 0.01 | 0.01 |

fault. Let us calculate the orthogonal zero divisors of the data matrix for $h = h^f = 8$ ($i = 41:49$ and $j = 51:59$)

$$\overline{\begin{bmatrix} X_i \\ U_i \\ e \end{bmatrix}}^R = \begin{bmatrix} 0.46 & 0.11 & 0.15 & 0.06 & -0.06 & -0.02 \\ -0.76 & 0.23 & -0.19 & 0.12 & 0.21 & 0.13 \\ -0.08 & -0.62 & 0.09 & -0.52 & -0.28 & 0.16 \\ 0.17 & 0 & -0.27 & -0.01 & 0.23 & -0.77 \\ 0.17 & -0.08 & -0.1 & 0.7 & -0.38 & 0.27 \\ 0.29 & 0.16 & 0.16 & -0.15 & 0.66 & 0.46 \\ -0.19 & 0.35 & 0.72 & -0.07 & -0.28 & -0.24 \\ 0.08 & 0.36 & -0.56 & -0.37 & -0.34 & 0.12 \\ -0.14 & -0.52 & 0.01 & 0.23 & 0.24 & -0.12 \end{bmatrix}, \quad \overline{\begin{bmatrix} X_j^f \\ U_j \\ e \end{bmatrix}}^R = \begin{bmatrix} 0 \\ 0.02 \\ -0.11 \\ 0.31 \\ -0.56 \\ 0.62 \\ -0.42 \\ 0.16 \\ -0.03 \end{bmatrix},$$

and multiply the state matrices for the next instant at the right by these divisors:

$$X_{i+1}\overline{\begin{bmatrix} X_i \\ U_i \\ e \end{bmatrix}}^R = 10^{-14}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.92 & 0.1 & -0.25 & 0.11 & 0.17 & 0.18 \\ -0.45 & 0.11 & -0.13 & 0.04 & 0.06 & 0.08 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -4.12 & 0.85 & -0.66 & 0.23 & 0.51 & 0.57 \end{bmatrix}, \quad X_{j+1}^f\overline{\begin{bmatrix} X_j^f \\ U_j \\ e \end{bmatrix}}^R = 10^{-14}\begin{bmatrix} 0.42 \\ 0.01 \\ 0.42 \\ 0.11 \\ 0 \\ 0.06 \\ 1.39 \end{bmatrix}.$$

The fulfillment of solvability conditions (2.5) indicates that the input-output data can be accurately described by the linear model of form (1.3) both before and after the occurrence of the fault.

Let us now analyze the time interval that includes the instant the fault occurred ($i = 42:49$ and $j = 50$). The calculated right-hand zero divisor of the data matrix

$$\overline{\begin{bmatrix} X_i & X_j^f \\ U_i & U_j \\ e & e \end{bmatrix}}^R = \begin{bmatrix} 0.08 & 0.41 & 0 & 0.28 & -0.03 & -0.02 \\ -0.15 & -0.55 & -0.05 & -0.39 & 0.04 & 0.5 \\ -0.19 & -0.02 & 0.2 & -0.34 & 0.22 & -0.73 \\ 0.62 & -0.34 & 0.07 & 0.44 & -0.2 & -0.09 \\ -0.5 & 0.22 & -0.37 & 0.14 & -0.55 & 0.05 \\ 0.21 & 0.35 & -0.43 & -0.06 & 0.63 & 0.23 \\ 0.15 & 0.37 & 0.65 & -0.31 & -0.21 & 0.3 \\ -0.44 & -0.26 & 0.28 & 0.51 & 0.33 & 0.03 \\ 0.22 & -0.18 & -0.36 & -0.28 & -0.24 & -0.26 \end{bmatrix}$$

is substituted into condition (2.8)

$$\begin{bmatrix} X_{i+1} & X_{j+1}^f \end{bmatrix}\overline{\begin{bmatrix} X_i & X_j^f \\ U_i & U_j \\ e & e \end{bmatrix}}^R = \begin{bmatrix} -0.47 & 0.37 & 0.74 & 0.59 & 0.49 & 0.55 \\ 0.01 & -0.01 & -0.02 & -0.02 & -0.01 & -0.02 \\ -0.17 & 0.13 & 0.27 & 0.21 & 0.18 & 0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \neq 0,$$

and norm (2.9) is calculated ($\varepsilon = 1.43$). It can be seen that the input-output data can no longer be described by the linear model of form (1.3), which indicates the occurrence of the fault in the control system.

Figure 2 shows the proposed fault detection criterion (2.9) for the entire time interval of the flight simulation, including the instant a fault occurs.

It can be seen that the fault is detected in the shortest possible time, corresponding to the integration step (1 s). In this case, before and after the occurrence of the fault, the value of criterion (2.9) is zero, while the fault itself is characterized by a spike the width of which corresponds to that of the identification window. Such a drastic change in the behavior of the plot makes it possible to accurately determine the time the fault occurred on both the time scales.

Figure 3 shows the norms of the prediction errors $\varepsilon_{pr}$ that are the differences between the real values of the state vector of the aircraft and the predicted values of the state vector of the nonfaulted aircraft model connected in parallel to the plant.

Analyzing the graph of the prediction errors (see Fig. 3a) on the time interval of 10 s allows detecting the fact of the occurrence of a fault. However, the exact time the fault occurred can be determined only by scaling-up the graph (see Fig. 3b where the instant the stabilizer jammed is characterized by a visible rise of the curve in the fifth second). In this case, the fault is also detected as promptly as possible, which suggests the similar efficiency of the criteria being compared.

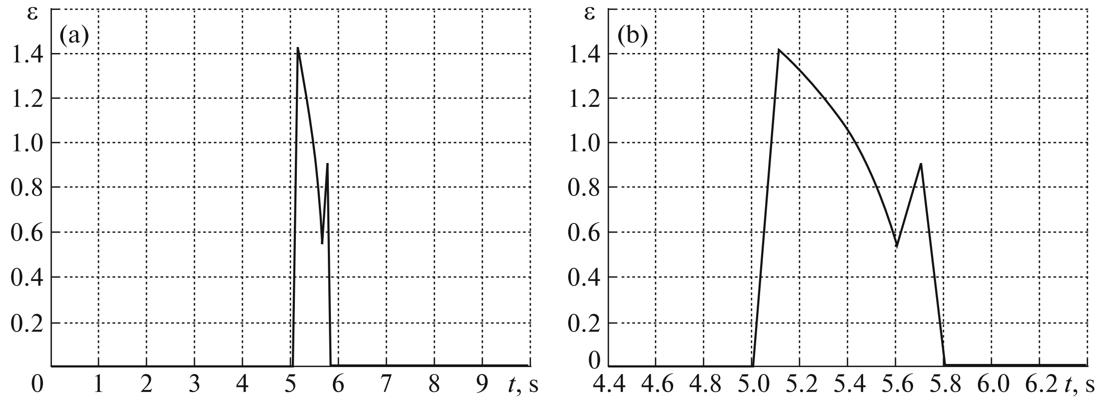


**Fig. 2.** Algebraic fault detection criterion: (a) the time interval of simulation and (b) the instant a fault occurs.

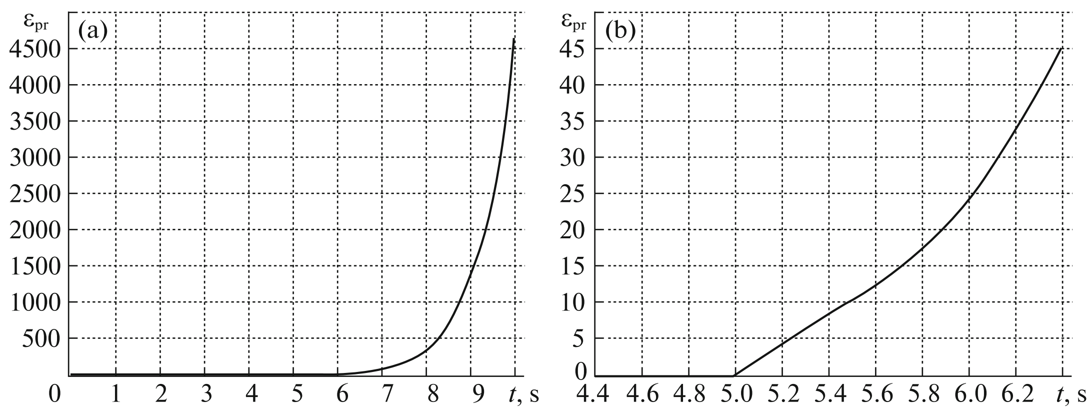


**Fig. 3.** Norms of prediction errors: (a) the time interval of simulation and (b) the instant a fault occurs.

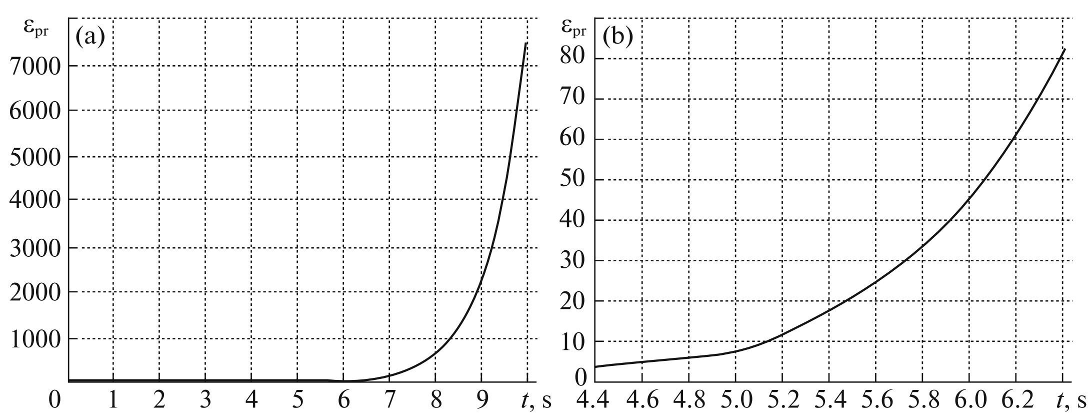


**Fig. 4.** Prediction error norms in the case of the errors in the model parameters: (a) the time interval of simulation and (b) the instant a fault occurs.

However, the situation changes considerably when the aircraft model contains even slight errors. For instance, Fig. 4 shows the norms of the prediction errors for different time scales with the disturbance (0.2%) of the model parameters according to the expression

$$\left[\hat{A}\ \ \hat{B}\right] = \left[A\ \ B\right] + w\left[A\ \ B\right],$$

where $\hat{A}$ and $\hat{B}$ are the matrices of the parameters for the model containing errors and $w = 0.002$ is the weight coefficient.

Although the norms of prediction errors increase considerably, the time the fault occurred can no longer be estimated reliably on almost any time scale, even when analyzing their statistical and/or dynamic properties. In this case, the prediction errors due to the disturbance of the model parameters become comparable to the errors caused by the fault in the control system without a rise in the graph at the instant the fault occurred.

In contrast, criterion (2.9) remains valid due to the fact that it does not depend on the model parameters at all; therefore, it does not depend on the errors in determining the parameters. The proposed criterion is based only on the information about the observed signals and involves no other auxiliary variables. Hence, the graphs shown in Fig. 2 will correspond to the stabilizer fault for any errors in the aircraft model.

## CONCLUSIONS

As a result of the investigation, a new algebraic criterion is developed for fault detection in control systems of dynamic plants that uses only the measurements of the input and output signals. The main featuresof the proposed criterion is that it does not depend on the parameters of the plant model, which ensures its validity in the absence of a priori information, and that it does not require identifying the model parameters of a plant and predicting its dynamics. This criterion allows detecting the facts of both occurrence and elimination of faults. The use of algebraic expressions makes it possible to increase the speed and improve the reliability of detecting the time a fault occurred and tracing the problem by executing analytic transformations.

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