A SEARCH FOR THE STANDARD MODEL HIGGS DECAYING TO TWO MUONS AT THE CMS EXPERIMENT

By

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A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL
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Major: Physics

In 2012 two collaborations at the Large Hadron Collider announced the discovery of a new boson at 125 GeV with properties similar to the Standard Model Higgs. In order to determine whether the boson is actually the Standard Model Higgs, all of the different decay modes need to be investigated. Insofar, any deviations from the Standard Model in one of these channels would imply new physics.

This dissertation presents the search for the Standard Model Higgs Boson decaying to $\mu^+\mu^-$. The search uses the 35.9 ± 0.9 fb^{-1} of $\sqrt{\mathrm{s}} = 13$ TeV proton-proton collision data recorded by the CMS detector in 2016. The observed and expected upper limits on the rate at a 95 % confidence level are presented for Higgs masses in the range 120 to 130 GeV. The expected and observed upper limits at a mass of 125 GeV are x.xx and $1.98^{+0.81}_{-0.57}$ $\times$ SM respectively. These results provide the best results to date on the Higgs coupling to second generation fermions. No deviations from the Standard Model are observed.

CHAPTER 1
INTRODUCTION AND OPENING REMARKS

We don't make the Chapter titles in All Caps Automatically because it is easier for you to type your Chapter Titles in uppercase than for those that need to have mixed case in their titles to find the correct command in the ufthesis.cls file and change it there. [*] We don't recommend that you change much of anything in the class file unless you're absolutely sure of what your are doing.[1]

1.1 The Section Command Text Should Be in Title Case

Title case is where all principal words are capitalized except prepositions, articles, and conjunctions. Green (2008)

1.1.1 Subsection Commands Are Also in Title Case

The difference, of course, are the second level headings are left-aligned

1.1.1.1 Subsubsections are in sentence case

The third level subheadings are left-aligned but in sentence case. Only the first letter and any proper nouns are capitalized. Strickler et al. (1998)

1.1.1.2 If you divide a section, you must divide it into two, or more, parts

Paragraph headings. There is no official fourth level heading. Do not use the Paragraph heading feature in LaTeX, simply apply the bold characteristic to the first few words of a paragraph followed by a colon or period.

1.1.2 I Need Another Second Level Heading in This Section

Aliquam mi nisi, tristique at rhoncus quis, consectetur non mi. Phasellus blandit quam ligula, a viverra lacus commodo at. In iaculis nisl vel pretium sollicitudin. In efficitur massa vel elit sollicitudin, vel auctor sapien cursus. Proin feugiat sapien a mi tempus;

---

[*] an un-numbered footnote - this is how you tell the readers that this chapter was previously published and then cite the Journal where it was published

[1] and now we're back to normal footnote marking

$\mathrm{X} - \mathrm{X}' = \mathrm{D} + \mathrm{D}'$

in consequat augue cursus. Nulla sed sagittis purus. Nunc eu consequat orci, eu laoreet enim. Ut euismod tincidunt sem, eget lacinia dui luctus eu. Aliquam mi augue, faucibus id semper vitae, porta ac ligula. Morbi sed ultrices odio. Mauris id luctus ex. Nulla ac libero dictum, interdum turpis lacinia, scelerisque leo. Praesent varius orci ac eros varius pharetra.

1.2 Image Handling in XeLaTeX

One of the biggest reasons for switching from the dvipdfm/dvipdfmx methods of compiling is the improved image handling capabilities. EPS, Bit-mapped, PDF, JPG, and PNG formats work well with the xelatex process.

1.2.1 The Traditional EPS Format

EPS format is the traditional format for LaTeX, but EPS files can be very large and many programs can't create or view these images. There are many programs that are used to interpret data and output the results as an EPS format image. It has been my experience that there are bounding box problems with these figures. On many occasions we have opened the image in Adobe Photoshop and, without making any changes, saved the document as a Photoshop EPS file, re-compiled the document, and the image worked correctly, so if you are having problems with an EPS image not showing in your document correctly, try this fix first.

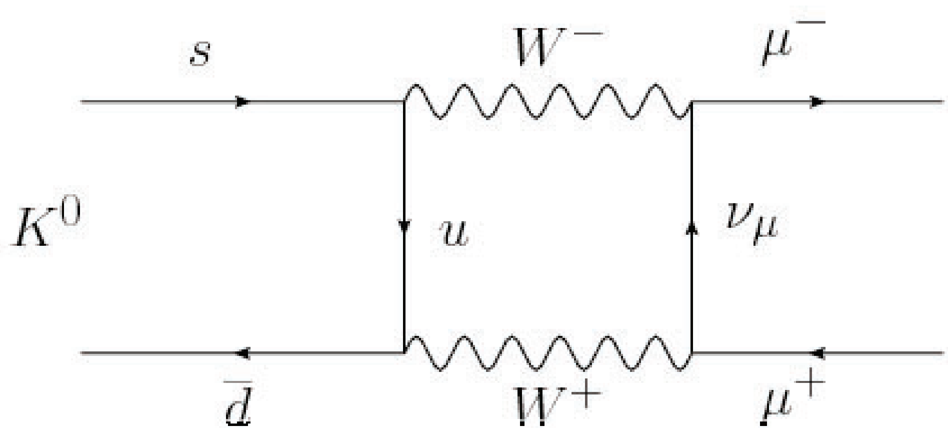


Figure 1-1. EPS format diagram. Note: no filetype is designated by adding an extension. The file type is determined and the correct procedure is automatically chosen by xelatex.

Quisque malesuada a leo eget ullamcorper. Curabitur ut aliquam quam. Nam quis quam id mauris aliquam blandit porttitor sit amet quam. Donec ut erat eleifend turpis finibus pulvinar.

1.2.2 Bitmapped Images Work As Well

Bitmapped images are a standard file type on PCs, but these files are also usually very large so compressed images may be a better alternative.

Figure 1-2. BMP format drawing. Note: no filetype is designated by adding an extension. The file type is determined and the correct procedure is automatically chosen by xelatex.

Morbi hendrerit risus nec quam posuere viverra. Donec quis tellus faucibus, molestie arcu sed, congue urna. Duis eget neque ac libero pulvinar porta eget et magna. Donec a magna eu eros suscipit cursus ac vitae nisl. Vivamus ligula purus, congue sed tortor blandit, ultrices egestas nisl.

1.2.3 Not to Mention PDF

It is often very handy to be able to include a pdf file as an image. By using XeLaTeX this is usually just matter of setting the size, or scale properties correctly.

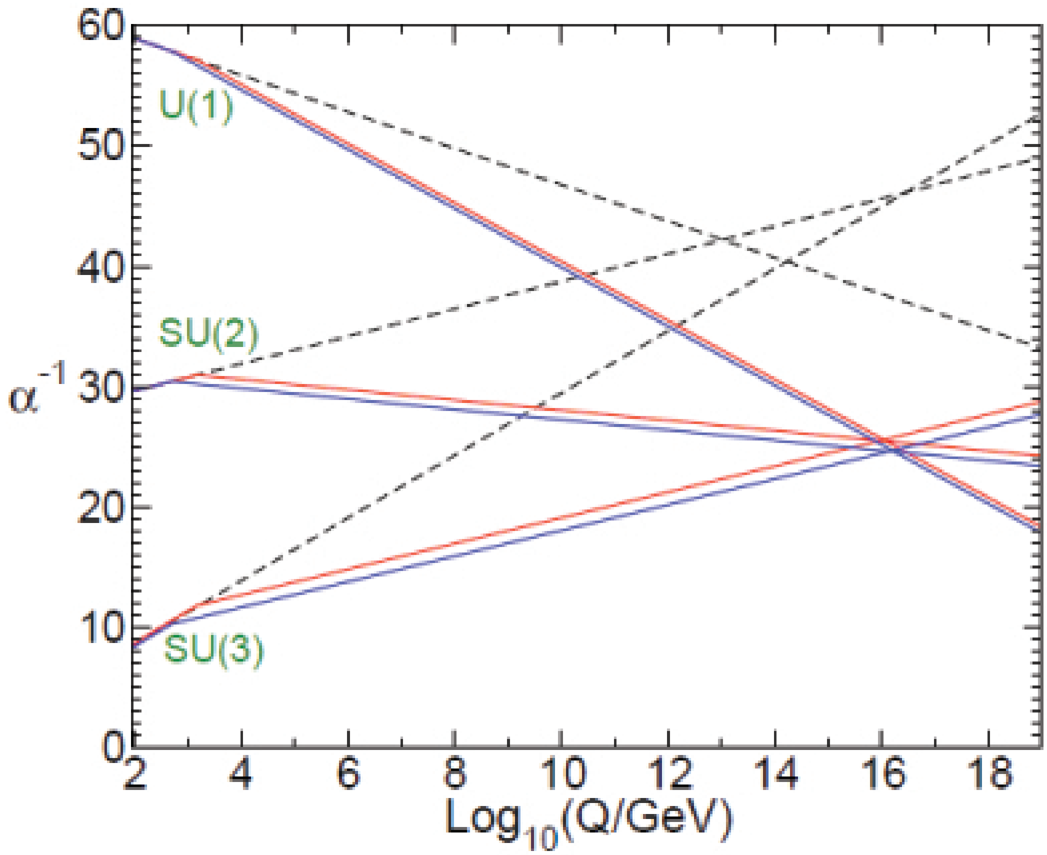


Figure 1-3. PDF format graph. Note: no filetype is designated by adding an extension. The file type is determined and the correct procedure is automatically chosen by xelatex.

Nulla mattis augue lacus. Nam non lectus dolor. Cras ac quam vel justo elementum vestibulum. Integer vulputate pulvinar lacus sit amet pulvinar.

1.2.4 JPG Is Absolutely Necessary

For photographs, JPG is the most common format. This format is a fraction of the size of Bit-mapped images and can deliver very good quality at a much smaller overhead. Vestibulum eu lectus vel orci dictum vehicula. Proin id maximus dolor. Integer augue ante, pulvinar ac erat vitae, porttitor ullamcorper libero. L'engle (2012)

Figure 1-4. JPG format image. Note: no filetype is designated by adding an extension. The file type is determined and the correct procedure is automatically chosen by xelatex.

Nunc blandit scelerisque velit, ac facilisis dui finibus et. Sed facilisis tortor vel commodo luctus. Donec est felis, malesuada id nibh in, accumsan malesuada lectus. Sed lobortis volutpat felis, vitae aliquet augue congue id. Fusce ut odio tincidunt, condimentum nulla vel, pharetra arcu.

1.2.5 PNGs Will Help Make Files Smaller

PNG files are even smaller than JPGs and are very good when text and images are combined.

Aenean condimentum libero sed mi porta, tempus ullamcorper lectus venenatis. Aliquam in diam dolor. Maecenas tempus consectetur sem et pulvinar. Aenean aliquam at metus ut hendrerit. Vivamus molestie ac neque eu luctus. Nam convallis maximus quam non lobortis. Fusce sit amet lorem et massa convallis aliquet at sit amet nulla. Suspendisse nec ex elit. Aenean gravida, sapien vitae congue commodo, urna turpis ornare libero, at cursus risus libero in erat. **?**

Figure 1-5. PNG format map. Note: no filetype is designated by adding an extension. The file type is determined and the correct procedure is automatically chosen by xelatex.

1.3 GIF, TIF, and Others

Other file formats have not been successful, with or without file extensions. The tests have not been exhaustive so if you have a different type, give it a try. GIF, and TIF both do NOT work at this time. The next image demonstrates how to use multiple images as a single figure. Notice, there is a single caption for ALL figures and that caption starts with a discription of the ENTIRE figure before breaking off into the subfigure descriptions.

Aliquam mi nisi, tristique at rhoncus quis, consectetur non mi. Phasellus blandit quam ligula, a viverra lacus commodo at. In iaculis nisl vel pretium sollicitudin. In efficitur massa vel elit sollicitudin, vel auctor sapien cursus. Proin feugiat sapien a mi tempus, in consequat augue cursus. Nulla sed sagittis purus. Nunc eu consequat orci, eu laoreet enim. Ut euismod tincidunt sem, eget lacinia dui luctus eu. Aliquam mi augue, faucibus id semper vitae, porta ac ligula. Morbi sed ultrices odio. Mauris id luctus ex. Nulla ac libero dictum, interdum turpis lacinia, scelerisque leo. Praesent varius orci ac eros varius pharetra.

Nunc blandit scelerisque velit, ac facilisis dui finibus et. Sed facilisis tortor vel commodo luctus. Donec est felis, malesuada id nibh in, accumsan malesuada lectus.

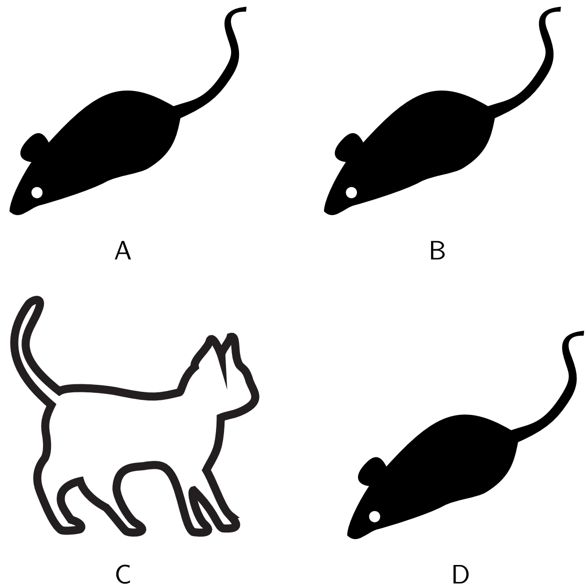


Figure 1-6. Tom and Jerries. This caption demonstrates how the sub-captions are left out of the List of Figures, but included in the figure itself. A) Tom the first; B) Tom the second; C) Jerry; D) Tom the third.

- WinEDT: This text editor is recommended for use editing TeX-files as it has many useful built in macros and is easy to use
- This program can be found and downloaded here: http://www.winedt.com/
- The GIMP (GNU Image Manipulation Program)
  - A freeware graphics editing program for picture editing and file conversions
  - Comparable to Adobe Photoshop
  - Can be downloaded here: http://www.gimp.org/
- A good reference of LaTeX2ϵ commands
  - This should be included on the ETD website here: http://etd.helpdesk.ufl.edu/tex.php

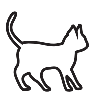
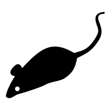
Sed lobortis volutpat felis, vitae aliquet augue congue id. Fusce ut odio tincidunt, condimentum nulla vel, pharetra arcu. In ultricies libero diam, nec rutrum magna vehicula nec. Praesent dictum eros sit amet turpis ultricies, eleifend condimentum dui imperdiet. Donec congue urna ante, id rutrum mi commodo a. Vivamus id tincidunt nunc. Morbi id lacus ut augue ultricies convallis. Duis a lectus quis ante pretium scelerisque nec nec nisi. In id porta justo, at euismod diam. Suspendisse vel tempus arcu. Praesent vel cursus nisi, ac rhoncus odio.

CHAPTER 2
LITERATURE REVIEW

2.1 Dolor Sit Amet

Many of the problems in theses and dissertations involve tables. The UF Graduate Counsel is very specific in the Table Requirements.There should be no vertical lines in tables and only three horizontal lines. No bold text, etc., see the web site for the complete list of requirements. One simple improvement can be incorporated by using tabularx instead of the tabular environment. This allows a table to be stretched the full text width easily, which avoids the centered or left aligned issue. Garfinkle et al. (1991) Table 2-2 is an examble of the tabularx code. Consectetur adipiscing elit. Fusce eget tempus lectus, non porttitor tellus. Aliquam molestie sed urna quis convallis. Aenean nibh eros, aliquam non eros in, tempus lacinia justo. In magna sapien, blandit a faucibus ac, scelerisque nec purus.

Table 2-1. A sample Table using tabularx

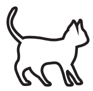
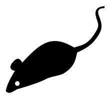
First	Second	Third
12	45	26
17	32	93
text	51	can be there too.
	28	Figures too - a cat.
	000	and a mouse!

Praesent fermentum felis nec massa interdum, vel dapibus mi luctus. Cras id fringilla mauris. Ut molestie eros mi, ut hendrerit nulla tempor et. Pellentesque tortor quam, mattis a scelerisque nec, euismod et odio. Mauris rhoncus metus sit amet risus mattis, eu mattis sem interdum.

2.1.1 Platea Dictumst

Donec convallis scelerisque ante, in sollicitudin orci laoreet eu. Nam arcu magna, semper vel lorem eu, venenatis ultrices est. Nam aliquet ut erat ac scelerisque. Maecenas ut molestie

Table 2-2. A sample Table using standard tablular

First	Second	Third
12	45	26
17	32	93
text	51	can be there too.
	28	Figures too - a cat.
	000	and a mouse!

mi. Phasellus ipsum magna, sollicitudin eu ipsum quis, imperdiet cursus turpis. Etiam pretium enim a fermentum accumsan. Morbi vel vehicula enim.

2.1.2 Long (and/or Wide) Tables

Another problem in LaTeX is the inability to handle long tables. While there are some packages that address this problem none of them quite fit the Editorial Office guidelines. The caption is not repeated but we do need "Table x-y. Continued" on each subsequent page and a repeat of the column headings on each page as well. The following table is the best example of the correct format I can produce. The disadvantage of this method is that much of it is manually set up and changes in the text will cause changes in the table. (**?**) For best results avoid the use of footnotemark and footnotetext commands inside of tables and try to keep your footnotes outside of floats whenever possible.

2.2 Ex id ullamcorper commodo

Augue sapien mattis leo, nec accumsan turpis quam at neque. Ut pellentesque velit sed placerat cursus. Integer congue urna non massa dictum, a pellentesque arcu accumsan. Nulla posuere, elit accumsan eleifend elementum, ipsum massa tristique metus, in ornare neque nisl sed odio. Nullam eget elementum nisi. Duis a consectetur erat, sit amet malesuada sapien. Aliquam nec sapien et leo sagittis porttitor at ut lacus. Vivamus vulputate elit vitae libero condimentum dictum. Nulla facilisi. Quisque non nibh et massa ullamcorper iaculis.

Integer laoreet bibendum arcu non pulvinar. Curabitur ac magna nibh. Phasellus sed nisi semper, molestie neque at, tempus lacus. Aenean vitae lacinia est. Phasellus aliquam lacus sit

Table 2-3. Feasible triples for highly variable Grid, MLMMH.

Time (s)	Triple chosen	Other feasible triples
0	(1, 11, 13725)	(1, 12, 10980), (1, 13, 8235), (2, 2, 0), (3, 1, 0)
2745	(1, 12, 10980)	(1, 13, 8235), (2, 2, 0), (2, 3, 0), (3, 1, 0)
5490	(1, 12, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
8235	(1, 12, 16470)	(1, 13, 13725), (2, 2, 2745), (2, 3, 0), (3, 1, 0)
10980	(1, 12, 16470)	(1, 13, 13725), (2, 2, 2745), (2, 3, 0), (3, 1, 0)
13725	(1, 12, 16470)	(1, 13, 13725), (2, 2, 2745), (2, 3, 0), (3, 1, 0)
16470	(1, 13, 16470)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
19215	(1, 12, 16470)	(1, 13, 13725), (2, 2, 2745), (2, 3, 0), (3, 1, 0)
21960	(1, 12, 16470)	(1, 13, 13725), (2, 2, 2745), (2, 3, 0), (3, 1, 0)
24705	(1, 12, 16470)	(1, 13, 13725), (2, 2, 2745), (2, 3, 0), (3, 1, 0)
27450	(1, 12, 16470)	(1, 13, 13725), (2, 2, 2745), (2, 3, 0), (3, 1, 0)
30195	(2, 2, 2745)	(2, 3, 0), (3, 1, 0)
32940	(1, 13, 16470)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
35685	(1, 13, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
38430	(1, 13, 10980)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
41175	(1, 12, 13725)	(1, 13, 10980), (2, 2, 2745), (2, 3, 0), (3, 1, 0)
43920	(1, 13, 10980)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
46665	(2, 2, 2745)	(2, 3, 0), (3, 1, 0)
49410	(2, 2, 2745)	(2, 3, 0), (3, 1, 0)
52155	(1, 12, 16470)	(1, 13, 13725), (2, 2, 2745), (2, 3, 0), (3, 1, 0)
54900	(1, 13, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
57645	(1, 13, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
60390	(1, 12, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
63135	(1, 13, 16470)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
65880	(1, 13, 16470)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
68625	(2, 2, 2745)	(2, 3, 0), (3, 1, 0)
71370	(1, 13, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
74115	(1, 12, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
76860	(1, 13, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
79605	(1, 13, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
82350	(1, 12, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
85095	(1, 12, 13725)	(1, 13, 10980), (2, 2, 2745), (2, 3, 0), (3, 1, 0)
87840	(1, 13, 16470)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
90585	(1, 13, 16470)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
93330	(1, 13, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
96075	(1, 13, 16470)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
98820	(1, 13, 16470)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
101565	(1, 13, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
104310	(1, 13, 16470)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
107055	(1, 13, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
109800	(1, 13, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
112545	(1, 12, 16470)	(1, 13, 13725), (2, 2, 2745), (2, 3, 0), (3, 1, 0)

Table 2-3. Continued

Time (s)	Triple chosen	Other feasible triples
115290	(1, 13, 16470)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
118035	(1, 13, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
120780	(1, 13, 16470)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
123525	(1, 13, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
126270	(1, 12, 16470)	(1, 13, 13725), (2, 2, 2745), (2, 3, 0), (3, 1, 0)
129015	(2, 2, 2745)	(2, 3, 0), (3, 1, 0)
131760	(2, 2, 2745)	(2, 3, 0), (3, 1, 0)
134505	(1, 13, 16470)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
137250	(1, 13, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
139995	(2, 2, 2745)	(2, 3, 0), (3, 1, 0)
142740	(2, 2, 2745)	(2, 3, 0), (3, 1, 0)
145485	(1, 12, 16470)	(1, 13, 13725), (2, 2, 2745), (2, 3, 0), (3, 1, 0)
148230	(2, 2, 2745)	(2, 3, 0), (3, 1, 0)
150975	(1, 13, 16470)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
153720	(1, 12, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
156465	(1, 13, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
159210	(1, 13, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
161955	(1, 13, 16470)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)
164700	(1, 13, 13725)	(2, 2, 2745), (2, 3, 0), (3, 1, 0)

amet placerat molestie. Sed sit amet bibendum lectus, ac ornare ligula. Curabitur porttitor interdum tortor a dignissim. Quisque a placerat nibh. Phasellus lobortis imperdiet augue, non congue est bibendum eu. Vivamus tincidunt quam eu fringilla laoreet.

Maecenas efficitur dolor et ipsum convallis, ut fringilla neque luctus. Donec ac nisl quis leo gravida accumsan sit amet sed tellus. Quisque placerat hendrerit augue sit amet aliquet. Vestibulum laoreet consequat nunc, et egestas nisl auctor et. Duis scelerisque vulputate placerat. Proin tempus ligula ac tempor eleifend. Nullam est odio, commodo quis nisl eu, feugiat efficitur purus.

Duis egestas in mauris vel efficitur. Sed a faucibus sem, non euismod enim. Maecenas nec nulla justo. Suspendisse ut orci ac mi aliquet tincidunt ac eget quam. Quisque ac mi sagittis, dapibus dui a, facilisis neque. Aenean euismod orci sem, non imperdiet ipsum pulvinar ac. Proin eu vestibulum magna, eu ullamcorper nulla. Etiam enim felis, dignissim eget commodo ac, faucibus nec justo. Nulla condimentum velit imperdiet ligula aliquam semper. Nulla facilisi. Ut in lobortis metus, at dictum ipsum. Suspendisse facilisis nec eros eget mollis. Vestibulum eget dolor ac mauris lobortis gravida. Suspendisse consectetur orci in risus pharetra, sed eleifend nisl lacinia. Mauris augue nibh, commodo sed sem at, congue molestie massa. Suspendisse sodales aliquet tellus, a tristique nunc aliquam id.

CHAPTER 3
MATERIALS ANS METHODS

3.1 Consectetur Adipiscing Elit

Fusce eget tempus lectus, non porttitor tellus. Aliquam molestie sed urna quis convallis. Aenean nibh eros, aliquam non eros in, tempus lacinia justo. In magna sapien, blandit a faucibus ac, scelerisque nec purus. Praesent fermentum felis nec massa interdum, vel dapibus mi luctus. Cras id fringilla mauris. Ut molestie eros mi, ut hendrerit nulla tempor et. Pellentesque tortor quam, mattis a scelerisque nec, euismod et odio. Mauris rhoncus metus sit amet risus mattis, eu mattis sem interdum.

3.1.1 This Is an Isolated Heading

Either promote this to a section heading, add another subsection heading, or delete this heading.

3.2 Augue sapien mattis leo

Nec accumsan turpis quam at neque. Ut pellentesque velit sed placerat cursus. Integer congue urna non massa dictum, a pellentesque arcu accumsan. Nulla posuere, elit accumsan eleifend elementum, ipsum massa tristique metus, in ornare neque nisl sed odio. Nullam eget elementum nisi. Duis a consectetur erat, sit amet malesuada sapien. Aliquam nec sapien et leo sagittis porttitor at ut lacus. Vivamus vulputate elit vitae libero condimentum dictum. Nulla facilisi. Quisque non nibh et massa ullamcorper iaculis.

CHAPTER 4
RESULTS

4.1 Fusce Eget Tempus Lectus,

Algorithm 4.1. *Euclids algorithm*

```
procedure Euclid(a, b)                          ▷ The g.c.d. of a and b
    r ← a mod b
    while r ≠ 0 do                              ▷ We have the answer if r is 0
        a ← b
        b ← r
        r ← a mod b
    end while
    return b                                    ▷ The gcd is b
end procedure
```

Proposition 4.1. *The Upsilon Function*

(1) If $\beta > 0$ and $\alpha \neq 0$, then for all $\mathrm{n} \geq -1$,

$$\mathrm{I_n}(\mathrm{c};\alpha;\beta;\delta) = -\frac{\mathrm{e}^{\alpha\mathrm{c}}}{\alpha}\sum_{\mathrm{i}=0}^{\mathrm{n}}(\frac{\beta}{\alpha})^{\mathrm{n-i}}\mathrm{Hh_i}(\beta\mathrm{c}-\delta)$$

$$+(\frac{\beta}{\alpha})^{\mathrm{n}+1}\frac{\sqrt{2\pi}}{\beta}\mathrm{e}^{\frac{\alpha\delta}{\beta}+\frac{\alpha^2}{2\beta^2}}\phi(-\beta\mathrm{c}+\delta+\frac{\alpha}{\beta})$$

(2) If $\beta < 0$ and $\alpha < 0$, then for all $\mathrm{x} \geq -1$

$$\mathrm{I_n}(\mathrm{c};\alpha;\beta;\delta) = -\frac{\mathrm{e}^{\alpha\mathrm{c}}}{\alpha}\sum_{\mathrm{i}=0}^{\mathrm{n}}(\frac{\beta}{\alpha})^{\mathrm{n-i}}\mathrm{Hh_i}(\beta\mathrm{c}-\delta)$$

$$-(\frac{\beta}{\alpha})^{\mathrm{n}+1}\frac{\sqrt{2\pi}}{\beta}\mathrm{e}^{\frac{\alpha\delta}{\beta}+\frac{\alpha^2}{2\beta^2}}\phi(\beta\mathrm{c}-\delta-\frac{\alpha}{\beta})$$

Proof. Case 1.

$\beta > 0$ and $\alpha \neq 0$. Since, for any constant α and $\mathrm{n} \geq 0$, $\mathrm{e}^{\alpha\mathrm{x}}\mathrm{Hh_n}(\beta\mathrm{x}-\delta) \to 0$ as $\mathrm{x} \to \infty$ thanks to (B4), integration by parts leads to

$$\mathrm{I_n} = -\frac{1}{\alpha}\mathrm{Hh}(\beta\mathrm{c}-\delta)\mathrm{e}^{\alpha\mathrm{c}} + \frac{\beta}{\alpha}\int_{\mathrm{c}}^{\infty}\mathrm{e}^{\alpha\mathrm{x}}\mathrm{Hh_{n-1}}(\beta\mathrm{c}-\delta)\mathrm{dx}$$

In other words, we have a recursion, for $\mathrm{n} \geq 0$, $\mathrm{I_n} = -(\mathrm{e}^{\alpha\mathrm{c}}\alpha)\mathrm{Hh_n}(\beta\mathrm{c} - \delta) + (\frac{\beta}{\alpha})\mathrm{I_{n-1}}$ with

$$\mathrm{I_{-1}} = \sqrt{2\pi}\int_{\mathrm{c}} \infty \mathrm{e}^{\alpha\mathrm{x}}\varphi(-\beta\mathrm{x} + \delta)\mathrm{dx}$$

$$= \frac{\sqrt{2\pi}}{\beta}\mathrm{e}^{\frac{\alpha\delta}{\beta} + \frac{\alpha^2}{2\beta^2}}\phi(-\beta\mathrm{c} + \delta + \frac{\alpha}{\beta})$$

Solving it yields, for $\mathrm{n} \geq -1$,

$$\mathrm{I_n} = -\frac{\mathrm{e}^{\alpha\mathrm{c}}}{\alpha}\sum_{\mathrm{i}=0}^{\mathrm{n}}(\frac{\beta}{\alpha})^{\mathrm{i}}\mathrm{Hh_{n-i}}(\beta\mathrm{c} + \delta) + (\frac{\beta}{\alpha})^{\mathrm{n}+1}\mathrm{I_{-1}}$$

$$= -\frac{\mathrm{e}^{\alpha\mathrm{c}}}{\alpha}\sum_{\mathrm{i}=0}^{\mathrm{n}}(\frac{\beta}{\alpha})^{\mathrm{n-i}}\mathrm{Hh_i}(\beta\mathrm{c} + \delta)$$

$$+(\frac{\beta}{\alpha})^{\mathrm{n}+1}\frac{\sqrt{2\pi}}{\beta}\mathrm{e}^{\frac{\alpha\delta}{\beta} + \frac{\alpha^2}{2\beta^2}}\phi(-\beta\mathrm{c} + \delta + \frac{\alpha}{\beta})$$

where the sum over an empty set is defined to be zero. □

Proof. Case2. $\beta < 0$ and $\alpha < 0$. In this case, we must also have, for $\mathrm{n} \geq 0$ and any constant $\alpha < 0, \mathrm{e}^{\alpha\mathrm{x}}\mathrm{Hh_n}(\beta\mathrm{x} - \delta) \to 0$ as

$\mathrm{x} \to \infty$, thanks to (B5). Using integration by parts, we again have the same recursion, for $\mathrm{n} \geq 0, \mathrm{I_n} = -(\mathrm{e}^{\alpha\mathrm{c}}/\alpha)\mathrm{Hh_n}(\beta\mathrm{c} - \delta) + (\beta/\alpha)\mathrm{I_{n-1}}$, but with a different initial condition

$$\mathrm{I_{-1}} = \sqrt{2\pi}\int_{\mathrm{c}}^{\infty} \mathrm{e}^{\alpha\mathrm{x}}\varphi(-\beta\mathrm{x} + \delta)\mathrm{dx}$$

$$= -\frac{\sqrt{2\pi}}{\beta}\exp\{\frac{\alpha\delta}{\beta} + \frac{\alpha^2}{2\beta^2}\}\phi(\beta\mathrm{c} - \delta - \frac{\alpha}{\beta})$$

Solving it yields (B8), for $\mathrm{n} \geq -1$.

□

Finally, we sum the double exponential and the normal random variables

Proposition B.3.

Suppose $\{\xi_1, \xi_2, ...\}$ is a sequence of i.i.d. exponential random variables with rate $\eta > 0$, and Z is a normal variable with distribution $\mathrm{N}(0, \sigma^2)$. Then for every $\mathrm{n} \geq 1$, we have: (1) The density functions are given by:

$$\mathrm{f}_{\mathrm{Z}+\sum_{\mathrm{i}=1}^{\mathrm{n}} \xi_{\mathrm{i}}}(\mathrm{t}) = (\sigma\eta)^{\mathrm{n}} \frac{\mathrm{e}^{(\sigma\eta)^2/2}}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\mathrm{t}\eta} \mathrm{Hh}_{\mathrm{n}-1}(-\frac{\mathrm{t}}{\sigma} + \sigma\eta)$$

$$\mathrm{f}_{\mathrm{Z}-\sum_{\mathrm{i}=1}^{\mathrm{n}} \xi_{\mathrm{i}}}(\mathrm{t}) = (\sigma\eta)^{\mathrm{n}} \frac{\mathrm{e}^{(\sigma\eta)^2/2}}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\mathrm{t}\eta} \mathrm{Hh}_{\mathrm{n}-1}(\frac{\mathrm{t}}{\sigma} + \sigma\eta)$$

(2) The tail probabilities are given by

$$\mathrm{P}(\mathrm{Z} + \sum_{\mathrm{i}=1}^{\mathrm{n}} \xi_{\mathrm{i}} \geq \mathrm{x}) = (\sigma\eta)^{\mathrm{n}} \frac{\mathrm{e}^{(\sigma\eta)^2/2}}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\mathrm{t}\eta} \mathrm{I}_{\mathrm{n}-1}(\mathrm{x}; -\eta, -\frac{1}{\sigma}, -\sigma\eta)$$

$$\mathrm{P}(\mathrm{Z} - \sum_{\mathrm{i}=1}^{\mathrm{n}} \xi_{\mathrm{i}} \geq \mathrm{x}) = (\sigma\eta)^{\mathrm{n}} \frac{\mathrm{e}^{(\sigma\eta)^2/2}}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\mathrm{t}\eta} \mathrm{I}_{\mathrm{n}-1}(\mathrm{x}; \eta, \frac{1}{\sigma}, -\sigma\eta)$$

Proof. Case 1. The densities of $\mathrm{Z} + \sum_{\mathrm{i}=1}^{\mathrm{n}} \xi_{\mathrm{i}}$, and $\mathrm{Z} - \sum_{\mathrm{i}=1}^{\mathrm{n}} \xi_{\mathrm{i}}$. We have

$$\mathrm{f}_{\mathrm{Z}+\sum_{\mathrm{i}=1}^{\mathrm{n}} \xi_{\mathrm{i}}}(\mathrm{t}) = \int_{-\infty}^{\infty} \mathrm{f}_{\sum_{\mathrm{i}=1}^{\mathrm{n}} \xi_{\mathrm{i}}}(\mathrm{t}-\mathrm{x}) \mathrm{f}_{\mathrm{Z}}(\mathrm{x}) \mathrm{dx}$$

$$= \mathrm{e}^{-\mathrm{t}\eta}(\eta^{\mathrm{n}}) \int_{-\infty} \mathrm{t} \frac{\mathrm{e}^{\mathrm{x}\eta}(\mathrm{t}-\mathrm{x})^{\mathrm{n}-1}}{(\mathrm{n}-1)!} \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\mathrm{x}^2/(2\sigma^2)} \mathrm{dx}$$

$$= \mathrm{e}^{-\mathrm{t}\eta}(\eta^{\mathrm{n}}) \mathrm{e}^{(\sigma\eta)^2/(2)} \int_{-\infty} \mathrm{t} \frac{(\mathrm{t}-\mathrm{x})^{\mathrm{n}-1}}{(\mathrm{n}-1)!} \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-(\mathrm{x}-\sigma^2\eta)^2/(2\sigma^2)} \mathrm{dx}$$

Letting $\mathrm{y} = (\mathrm{x} - \sigma^2\eta)/\sigma$ yields

$$\mathrm{f}_{\mathrm{Z}+\sum_{\mathrm{i}=1}^{\mathrm{n}} \xi_{\mathrm{i}}}(\mathrm{t}) = \mathrm{e}^{-\mathrm{t}\eta}(\eta^{\mathrm{n}}) \mathrm{e}^{(\sigma\eta)^2/(2)} \sigma^{\mathrm{n}-1}$$

$$\times \int_{-\infty}^{\mathrm{t}/\sigma-\sigma\eta} \frac{(\mathrm{t}/\sigma - \mathrm{y} - \sigma\eta)^{\mathrm{n}-1}}{(\mathrm{n}-1)!} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\mathrm{y}^2/2} \mathrm{dy}$$

$$= \frac{e^{(\sigma\eta)^2/2}}{\sqrt{2\pi}}(\sigma^{n-1}\eta^n)e^{-t\eta}Hh_{n-1}(-t/\sigma+\sigma\eta)$$

because $(1/(n-1)!)\int_{-\infty} a(a-y)^{n-1}e^{-y^2/2}dy = Hh_{n-1}(a)$. *The derivation of* $f_{Z+\sum_{i=1}^n \xi_i}(t)$ *is similar.*

Case 2. $P(Z+\sum_{i=1}^n \xi_i \geq x)$ *and* $P(Z-\sum_{i=1}^n \xi_i \geq x)$. *From (B9), it is clear that*

$$P(Z+\sum_{i=1}^n \xi_i \geq x) = \frac{(\sigma\eta)^n e^{(\sigma\eta)^2/2}}{\sigma\sqrt{2\pi}}\int_x^\infty e^{(-i\eta)}Hh_{n-1}(-\frac{t}{\sigma}+\sigma\eta)dt$$

$$= \frac{(\sigma\eta)^n e^{(\sigma\eta)^2/2}}{\sigma\sqrt{2\pi}} I_{n-1}(x; -\eta, -\frac{1}{\sigma}, -\sigma\eta)dt$$

by (B6). We can compute $P(Z-\sum_{i=1}^n \xi_i \geq x)$ *similarly.*

Theorem 4.1. *Theorem With* $\pi_n := P(N(t)=n) = e^{-\lambda T}(\lambda T)^n/n!$ *and* I_n *in Proposition 2-2.*

, we have

$$P(Z(T) \geq a) = \frac{e^{(\sigma\eta_1)^2 T/2}}{\sigma\sqrt{2\pi T}}\sum_{n=1}^\infty \pi_n \sum_{k=1}^n P_{n,k}(\sigma\sqrt{T}\eta_1)^k \times I_{k-1}(a-\mu T; -\eta_1, -\frac{1}{\sigma\sqrt{T}}, -\sigma\eta_1\sqrt{T})$$

$$+\frac{e^{(\sigma\eta_2)^2 T/2}}{\sigma\sqrt{2\pi T}}\sum_{n=1}^\infty \pi_n \sum_{k=1}^n Q_{n,k}(\sigma\sqrt{T}\eta_2)^k$$

$$\times I_{k-1}(a-\mu T; \eta_2, \frac{1}{\sigma\sqrt{T}}, -\sigma\eta_2\sqrt{T})$$

$$+\pi_0\phi(-\frac{a-\mu T}{\sigma\sqrt{T}})$$

Proof by the decomposition (B2)

$$P(Z(T) \geq a) = \sum_{n=0}^\infty \pi_n P(\mu T + \sigma\sqrt{T}Z + \sum_{j=1}^n Y_j \geq a)$$

$$= \pi_0 \mathrm{P}(\mu \mathrm{T} + \sigma\sqrt{\mathrm{T}}\mathrm{Z} \geq \mathrm{a})$$

$$+ \sum_{\mathrm{n}=1}^{\infty} \pi_{\mathrm{n}} \sum_{\mathrm{k}=1}^{\mathrm{n}} \mathrm{P}_{\mathrm{n,k}} \mathrm{P}(\mu \mathrm{T} + \sigma\sqrt{\mathrm{T}}\mathrm{Z} + \sum_{\mathrm{j}=1}^{\mathrm{n}} \xi_{\mathrm{j}}^{+} \geq \mathrm{a})$$

$$+ \sum_{\mathrm{n}=1}^{\infty} \pi_{\mathrm{n}} \sum_{\mathrm{k}=1}^{\mathrm{n}} \mathrm{Q}_{\mathrm{n,k}} \mathrm{P}(\mu \mathrm{T} + \sigma\sqrt{\mathrm{T}}\mathrm{Z} - \sum_{\mathrm{j}=1}^{\mathrm{n}} \xi_{\mathrm{j}}^{-} \geq \mathrm{a})$$

The result now follows via (B11) and (B12) for $\eta_1 > 1$ and $\eta_2 > 0$.

CHAPTER 5
SUMMARY AND CONCLUSIONS

5.1 Non Porttitor Tellus

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5.1.1.1 Ut pellentesque velit sede

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APPENDIX A
THIS IS THE FIRST APPENDIX

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APPENDIX B
AN EXAMPLE OF A HALF TITLE PAGE

$\LaTeX\,2_{\varepsilon}$

Figure B-1. $\LaTeX 2_{\epsilon}$ logo

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APPENDIX C
DERIVATION OF THE Υ FUNCTION

Proposition C.1. *The Upsilon Function*

(1) If $\beta > 0$ *and* $\alpha \neq 0$*, then for all* $\mathrm{n} \geq -1$*,*

$$\mathrm{I_n}(\mathrm{c};\alpha;\beta;\delta) = -\frac{\mathrm{e}^{\alpha\mathrm{c}}}{\alpha}\sum_{\mathrm{i}=0}^{\mathrm{n}}(\frac{\beta}{\alpha})^{\mathrm{n-i}}\mathrm{Hh_i}(\beta\mathrm{c}-\delta)$$

$$+(\frac{\beta}{\alpha})^{\mathrm{n}+1}\frac{\sqrt{2\pi}}{\beta}\mathrm{e}^{\frac{\alpha\delta}{\beta}+\frac{\alpha^2}{2\beta^2}}\phi(-\beta\mathrm{c}+\delta+\frac{\alpha}{\beta})$$

(2) If $\beta < 0$ *and* $\alpha < 0$*, then for all* $\mathrm{x} \geq -1$

$$\mathrm{I_n}(\mathrm{c};\alpha;\beta;\delta) = -\frac{\mathrm{e}^{\alpha\mathrm{c}}}{\alpha}\sum_{\mathrm{i}=0}^{\mathrm{n}}(\frac{\beta}{\alpha})^{\mathrm{n-i}}\mathrm{Hh_i}(\beta\mathrm{c}-\delta)$$

$$-(\frac{\beta}{\alpha})^{\mathrm{n}+1}\frac{\sqrt{2\pi}}{\beta}\mathrm{e}^{\frac{\alpha\delta}{\beta}+\frac{\alpha^2}{2\beta^2}}\phi(\beta\mathrm{c}-\delta-\frac{\alpha}{\beta})$$

Proof. Case 1.

$\beta > 0$ and $\alpha \neq 0$. Since, for any constant α and $\mathrm{n} \geq 0$, $\mathrm{e}^{\alpha\mathrm{x}}\mathrm{Hh_n}(\beta\mathrm{x}-\delta) \to 0$ as $\mathrm{x} \to \infty$ thanks to (B4), integration by parts leads to

$$\mathrm{I_n} = -\frac{1}{\alpha}\mathrm{Hh}(\beta\mathrm{c}-\delta)\mathrm{e}^{\alpha\mathrm{c}} + \frac{\beta}{\alpha}\int_{\mathrm{c}}^{\infty}\mathrm{e}^{\alpha\mathrm{x}}\mathrm{Hh_{n-1}}(\beta\mathrm{c}-\delta)\mathrm{dx}$$

In other words, we have a recursion, for $\mathrm{n} \geq 0$, $\mathrm{I_n} = -(\mathrm{e}^{\alpha\mathrm{c}}\alpha)\mathrm{Hh_n}(\beta\mathrm{c}-\delta) + (\frac{\beta}{\alpha})\mathrm{I_{n-1}}$ with

$$\mathrm{I_{-1}} = \sqrt{2\pi}\int_{\mathrm{c}}\infty\mathrm{e}^{\alpha\mathrm{x}}\varphi(-\beta\mathrm{x}+\delta)\mathrm{dx}$$

$$= \frac{\sqrt{2\pi}}{\beta}\mathrm{e}^{\frac{\alpha\delta}{\beta}+\frac{\alpha^2}{2\beta^2}}\phi(-\beta\mathrm{c}+\delta+\frac{\alpha}{\beta})$$

Solving it yields, for $\mathrm{n} \geq -1$,

$$\mathrm{I_n} = -\frac{\mathrm{e}^{\alpha \mathrm{c}}}{\alpha} \sum_{\mathrm{i}=0}^{\mathrm{n}} (\frac{\beta}{\alpha})^{\mathrm{i}} \mathrm{Hh_{n-i}}(\beta \mathrm{c} + \delta) + (\frac{\beta}{\alpha})^{\mathrm{n}+1} \mathrm{I}_{-1}$$

$$= -\frac{\mathrm{e}^{\alpha \mathrm{c}}}{\alpha} \sum_{\mathrm{i}=0}^{\mathrm{n}} (\frac{\beta}{\alpha})^{\mathrm{n-i}} \mathrm{Hh_i}(\beta \mathrm{c} + \delta)$$

$$+(\frac{\beta}{\alpha})^{\mathrm{n}+1} \frac{\sqrt{2\pi}}{\beta} \mathrm{e}^{\frac{\alpha\delta}{\beta} + \frac{\alpha^2}{2\beta^2}} \phi(-\beta \mathrm{c} + \delta + \frac{\alpha}{\beta})$$

where the sum over an empty set is defined to be zero. □

Case2. $\beta < 0$ *and* $\alpha < 0$. *In this case, we must also have, for* $\mathrm{n} \geq 0$ *and any constant* $\alpha < 0, \mathrm{e}^{\alpha \mathrm{x}} \mathrm{Hh_n}(\beta \mathrm{x} - \delta) \to 0$ *as*

$\mathrm{x} \to \infty$, *thanks to (B5). Using integration by parts, we again have the same recursion, for* $\mathrm{n} \geq 0, \mathrm{I_n} = -(\mathrm{e}^{\alpha \mathrm{c}}/\alpha) \mathrm{Hh_n}(\beta \mathrm{c} - \delta) + (\beta/\alpha) \mathrm{I_{n-1}}$, *but with a different initial condition*

$$\mathrm{I}_{-1} = \sqrt{2\pi} \int_{\mathrm{c}}^{\infty} \mathrm{e}^{\alpha \mathrm{x}} \varphi(-\beta \mathrm{x} + \delta) \mathrm{dx}$$

$$= -\frac{\sqrt{2\pi}}{\beta} \exp\{\frac{\alpha\delta}{\beta} + \frac{\alpha^2}{2\beta^2}\} \phi(\beta \mathrm{c} - \delta - \frac{\alpha}{\beta})$$

Solving it yields (B8), for $\mathrm{n} \geq -1$.

Finally, we sum the double exponential and the normal random variables

Proposition B.3.

Suppose $\{\xi_1, \xi_2, ...\}$ *is a sequence of i.i.d. exponential random variables with rate* $\eta > 0$, *and Z is a normal variable with distribution* $\mathrm{N}(0, \sigma^2)$. *Then for every* $\mathrm{n} \geq 1$, *we have: (1) The density functions are given by:*

$$\mathrm{f}_{\mathrm{Z} + \sum_{\mathrm{i}=1}^{\mathrm{n}} \xi_{\mathrm{i}}}(\mathrm{t}) = (\sigma\eta)^{\mathrm{n}} \frac{\mathrm{e}^{(\sigma\eta)^2/2}}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\mathrm{t}\eta} \mathrm{Hh_{n-1}}(-\frac{\mathrm{t}}{\sigma} + \sigma\eta)$$

$$\mathrm{f}_{\mathrm{Z} - \sum_{\mathrm{i}=1}^{\mathrm{n}} \xi_{\mathrm{i}}}(\mathrm{t}) = (\sigma\eta)^{\mathrm{n}} \frac{\mathrm{e}^{(\sigma\eta)^2/2}}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\mathrm{t}\eta} \mathrm{Hh_{n-1}}(\frac{\mathrm{t}}{\sigma} + \sigma\eta)$$

(2) The tail probabilities are given by

$$P(Z+\sum_{i=1}^{n}\xi_i \geq x) = (\sigma\eta)^n \frac{e^{(\sigma\eta)^2/2}}{\sigma\sqrt{2\pi}} e^{-t\eta} I_{n-1}(x; -\eta, -\frac{1}{\sigma}, -\sigma\eta)$$

$$P(Z-\sum_{i=1}^{n}\xi_i \geq x) = (\sigma\eta)^n \frac{e^{(\sigma\eta)^2/2}}{\sigma\sqrt{2\pi}} e^{-t\eta} I_{n-1}(x; \eta, \frac{1}{\sigma}, -\sigma\eta)$$

Proof. Case 1. The densities of $Z+\sum_{i=1}^{n}\xi_i$*, and* $Z-\sum_{i=1}^{n}\xi_i$*. We have*

$$f_{Z+\sum_{i=1}^{n}\xi_i}(t) = \int_{-\infty}^{\infty} f_{\sum_{i=1}^{n}\xi_i}(t-x) f_Z(x) dx$$

$$= e^{-t\eta}(\eta^n) \int_{-\infty} t \frac{e^{x\eta}(t-x)^{n-1}}{(n-1)!} \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/(2\sigma^2)} dx$$

$$= e^{-t\eta}(\eta^n) e^{(\sigma\eta)^2/(2)} \int_{-\infty} t \frac{(t-x)^{n-1}}{(n-1)!} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\sigma^2\eta)^2/(2\sigma^2)} dx$$

Letting $y = (x-\sigma^2\eta)/\sigma$ *yields*

$$f_{Z+\sum_{i=1}^{n}\xi_i}(t) = e^{-t\eta}(\eta^n) e^{(\sigma\eta)^2/(2)} \sigma^{n-1}$$

$$\times \int_{-\infty}^{t/\sigma-\sigma\eta} \frac{(t/\sigma - y - \sigma\eta)^{n-1}}{(n-1)!} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$$

$$= \frac{e^{(\sigma\eta)^2/2}}{\sqrt{2\pi}} (\sigma^{n-1}\eta^n) e^{-t\eta} Hh_{n-1}(-t/\sigma + \sigma\eta)$$

because $(1/(n-1)!) \int_{-\infty} a(a-y)^{n-1} e^{-y^2/2} dy = Hh_{n-1}(a)$*. The derivation of* $f_{Z+\sum_{i=1}^{n}\xi_i}(t)$ *is similar.*

Case 2. $P(Z+\sum_{i=1}^{n}\xi_i \geq x)$ *and* $P(Z-\sum_{i=1}^{n}\xi_i \geq x)$*. From (B9), it is clear that*

$$P(Z+\sum_{i=1}^{n}\xi_i \geq x) = \frac{(\sigma\eta)^n e^{(\sigma\eta)^2/2}}{\sigma\sqrt{2\pi}} \int_x^{\infty} e^{(-i\eta)} Hh_{n-1}(-\frac{t}{\sigma} + \sigma\eta) dt$$

$$= \frac{(\sigma\eta)^n e^{(\sigma\eta)^2/2}}{\sigma\sqrt{2\pi}} I_{n-1}(x; -\eta, -\frac{1}{\sigma}, -\sigma\eta)dt$$

by (B6). We can compute $P(Z - \sum_{i=1}^{n} \xi_i \geq x)$ *similarly.*

Theorem C.1. *Theorem With* $\pi_n := P(N(t) = n) = e^{-\lambda T}(\lambda T)^n/n!$ *and* I_n *in Proposition 2-2.*

, we have

$$P(Z(T) \geq a) = \frac{e^{(\sigma\eta_1)^2 T/2}}{\sigma\sqrt{2\pi T}} \sum_{n=1}^{\infty} \pi_n \sum_{k=1}^{n} P_{n,k}(\sigma\sqrt{T}\eta_1)^k \times I_{k-1}(a - \mu T; -\eta_1, -\frac{1}{\sigma\sqrt{T}}, -\sigma\eta_1\sqrt{T})$$

$$+\frac{e^{(\sigma\eta_2)^2 T/2}}{\sigma\sqrt{2\pi T}} \sum_{n=1}^{\infty} \pi_n \sum_{k=1}^{n} Q_{n,k}(\sigma\sqrt{T}\eta_2)^k$$

$$\times I_{k-1}(a - \mu T; \eta_2, \frac{1}{\sigma\sqrt{T}}, -\sigma\eta_2\sqrt{T})$$

$$+\pi_0\phi(-\frac{a - \mu T}{\sigma\sqrt{T}})$$

Proof by the decomposition (B2)

$$P(Z(T) \geq a) = \sum_{n=0}^{\infty} \pi_n P(\mu T + \sigma\sqrt{T}Z + \sum_{j=1}^{n} Y_j \geq a)$$

$$= \pi_0 P(\mu T + \sigma\sqrt{T}Z \geq a)$$

$$+\sum_{n=1}^{\infty} \pi_n \sum_{k=1}^{n} P_{n,k} P(\mu T + \sigma\sqrt{T}Z + \sum_{j=1}^{n} \xi_j^+ \geq a)$$

$$+\sum_{n=1}^{\infty} \pi_n \sum_{k=1}^{n} Q_{n,k} P(\mu T + \sigma\sqrt{T}Z - \sum_{j=1}^{n} \xi_j^- \geq a)$$

The result now follows via (B11) and (B12) for $\eta_1 > 1$ *and* $\eta_2 > 0$*.*

APPENDIX D
DERIVATION OF THE Υ FUNCTION

We first decompose the sum of the double exponential random variables.

The memoryless property of exponential random variables yields $(\xi^+ - \xi^- | \xi^+ > \xi^-) =^{\mathrm{d}} \xi^+$ and $(\xi^+ - \xi^- | \xi^+ < \xi^-) =^{\mathrm{d}} -\xi^-$, thus leading to the conclusion that

$$\xi^+ - \xi^- = \left\{ \begin{array}{cl} \xi^+ & \text{with probability } \eta_2/(\eta_1+\eta_2) \\ -\xi^- & \text{with probability } \eta_1/(\eta_1+\eta_2) \end{array} \right\}.$$

because the probabilities of the events $\xi^+ > \xi^-$ and $\xi^+ < \xi^-$ are $\eta_2/(\eta_1+\eta_2)$ and $\eta_1/(\eta_1+\eta_2)$, respectively. The following proposition extends (B.1.)

Proposition B.1. For every $\mathrm{n} \geq 1$, we have the following decomposition

$$\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{Y_i} =^{\mathrm{d}} \left\{ \begin{array}{cl} \sum_{\mathrm{i}=1}^{\mathrm{k}} \xi_{\mathrm{i}}^+ & \text{with probability } \mathrm{P_{n,k}}, \mathrm{k} = 1, 2, ..., \mathrm{n} \\ -\sum_{\mathrm{i}=1}^{\mathrm{k}} \xi_{\mathrm{i}}^- & \text{with probability } \mathrm{Q_{n,k}}, \mathrm{k} = 1, 2, ..., \mathrm{n} \end{array} \right\}.$$

where $\mathrm{P_{n,k}}$ and $\mathrm{Q_{n,k}}$ are given by

$$\mathrm{P_{n,k}} = \sum_{\mathrm{i=k}}^{\mathrm{n}-1} \binom{\mathrm{n-k-1}}{\mathrm{i-k}} \binom{\mathrm{n}}{\mathrm{i}} \left(\frac{\eta_1}{\eta_1+\eta_2}\right)^{\mathrm{i-k}} \left(\frac{\eta_2}{\eta_1+\eta_2}\right)^{\mathrm{n-i}} \mathrm{p^i q^{n-i}}$$

$$1 \leq \mathrm{k} \leq \mathrm{n} - 1$$

$$\mathrm{Q_{n,k}} = \sum_{\mathrm{i=k}}^{\mathrm{n}-1} \binom{\mathrm{n-k-1}}{\mathrm{i-k}} \binom{\mathrm{n}}{\mathrm{i}} \left(\frac{\eta_1}{\eta_1+\eta_2}\right)^{\mathrm{n-i}} \left(\frac{\eta_2}{\eta_1+\eta_2}\right)^{\mathrm{i-k}} \mathrm{p^{n-i} q^i}$$

$$1 \leq \mathrm{k} \leq \mathrm{n} - 1, \mathrm{P_{n,n}} = \mathrm{p^n}, \mathrm{Q_{n,n}} = \mathrm{q^n}$$

and $\binom{0}{0}$ is defined to be one. Hence ξ_{i}^+ and ξ_{i}^- are i.i.d. exponential random variables with rates η_1 and η_2, respectively.

As a key step in deriving closed-form solutions for call and put options, this proposition indicates that the sum of the i.i.d. double exponential random variable can be written, in distribution, as a randomly mixed gamma random variable. To prove Proposition B.1, the following lemma is needed.

Lemma B.1.

$$\sum_{i=1}^{n} \xi_i^+ - \sum_{i=1}^{n} \xi_i^-$$

$$=^d \left\{ \begin{array}{ll} \sum_{i=1}^{k} \xi_i & \text{with probability } \binom{n-k+m-1}{m-1}(\frac{\eta_1}{\eta_1+\eta_2})^{n-k}(\frac{\eta_2}{\eta_1+\eta_2})^m, k = 1, ..., n \\ -\sum_{i=1}^{l} \xi_i & \text{with probability } \binom{n-l+m-1}{n-1}(\frac{\eta_1}{\eta_1+\eta_2})^{n}(\frac{\eta_2}{\eta_1+\eta_2})^{m-l}, l = 1, ..., m \end{array} \right\}.$$

We prove it by introducing the random variables $A(n,m) = \sum_{i=1}^{n} \xi_i - sum_{j=1}^{m} \tilde{\xi}_j$ Then

$$A(n,m) =^d \left\{ \begin{array}{ll} A(n-1,m-1) + \xi^+ & \text{with probability } \eta_2/(\eta_1+\eta_2) \\ A(n-1,m-1) - \xi^- & \text{with probability } \eta_1/(\eta_1+\eta_2) \end{array} \right\}.$$

$$=^d \left\{ \begin{array}{ll} A(n,m-1) & \text{with probability } \eta_2/(\eta_1+\eta_2) \\ A(n-1,m) & \text{with probability } \eta_1/(\eta_1+\eta_2) \end{array} \right\}.$$

via B.1.. Now suppose horizontal axis that are representing the number of $\{\zeta_i^+\}$ and vertical axis representing the number of $\{\zeta_i^-\}$. Suppose we have a random walk on the integer lattice points. Starting from any point $(n,m), n, m \geq 1$, the random walk goes either one step to the left with probability $\eta_1/(\eta_1+\eta_2)$ or one step down with probability $\eta_2/(\eta_1+\eta_2)$, and the random walks stops once it reaches the horizontal or vertical axis. For any path from (n,m) to (k,0) , $1 \geq k \geq n$, it must reach (k,1) first before it makes a final move to (k,0). Furthermore, all the paths going from (n,m) to (k,1) must have exactly n-k lefts and m-1 downs, whence the total number of such paths is $\binom{n-k+m-1}{m-1}$. Similarly the total number of paths from (n,m) to (0,l) , $1 \geq l \geq m$, is $\binom{n-l+m-1}{n-1}$. Thus

$$A(n,m) =^{d} \left\{ \begin{array}{ll} \sum_{i=1}^{k} \xi_i & \text{with probability } \binom{n-k+m-1}{m-1}(\frac{\eta_1}{\eta_1+\eta_2})^{n-k}(\frac{\eta_2}{\eta_1+\eta_2})^{m}, k = 1, ..., n \\ -\sum_{i=1}^{l} \xi_i & \text{with probability } \binom{n-l+m-1}{n-1}(\frac{\eta_1}{\eta_1+\eta_2})^{n}(\frac{\eta_2}{\eta_1+\eta_2})^{m-l}, l = 1, ..., m \end{array} \right\}.$$

and the lemma is proven.

Now, let's prove the proposition B.1. By the same analogy used in Lemma B.1 to compute probability $P_{n,m}, 1 \geq k \geq n$, the probability weight assigned to $\sum_{i=1}^{k} \xi_i^{+}$ when we decompose $\sum_{i=1}^{k} Y_i$, it is equivalent to consider the probability of the random walk ever reach (k,0) starting from the point (i,n-i) being $\binom{n}{i} p^i q^{n-i}$. Note that the point (k,0) can only be reached from point (i,n-i) such that $k \geq i \geq n-1$, because the random walk can only go left or down, and stops once it reaches the horizontal axis. Therefore, for $1 \geq k \geq n-1$, (B3) leads to

$$P_{n,k} = \sum_{i=k} n - 1P(\text{goingfrom}(i, n-i)\text{to}(k,0)).P(\text{startingfrom}(i, n-i))$$

$$= \sum_{i=k}^{n-1} \binom{i + (n-i) - k - 1}{(n-i) - 1} \binom{n}{i} (\frac{\eta_1}{\eta_1 + \eta_2})^{i-k} (\frac{\eta_2}{\eta_1 + \eta_2})^{n-i} p^i q^{n-i}$$

$$= \sum_{i=k}^{n-1} \binom{n-k-1}{n-i-1} \binom{n}{i} (\frac{\eta_1}{\eta_1 + \eta_2})^{i-k} (\frac{\eta_2}{\eta_1 + \eta_2})^{n-i} p^i q^{n-i}$$

$$= \sum_{i=k}^{n-1} \binom{n-k-1}{i-k} \binom{n}{i} (\frac{\eta_1}{\eta_1 + \eta_2})^{i-k} (\frac{\eta_2}{\eta_1 + \eta_2})^{n-i} p^i q^{n-i}$$

Of course $P_{n,n} = p^n$. Similarly, we can compute $Q_{n,k}$:

$$Q_{n,k} = \sum_{i=k} n - 1P(\text{goingfrom}(n-i, i)\text{to}(0,k)).P(\text{startingfrom}(n-i, i))$$

$$= \sum_{i=k}^{n-1} \binom{i + (n-i) - k - 1}{(n-i) - 1} \binom{n}{n-i} (\frac{\eta_1}{\eta_1 + \eta_2})^{n-i} (\frac{\eta_2}{\eta_1 + \eta_2})^{i-k} p^{n-i} q^{i}$$

$$= \sum_{i=k}^{n-1} \binom{n-k-1}{i-k} \binom{n}{i} (\frac{\eta_1}{\eta_1+\eta_2})^{n-i} (\frac{\eta_2}{\eta_1+\eta_2})^{i-k} p^{n-i} q^i$$

with $Q_{n,n} = q^n$. Incidentally, we have also got $\sum k = 1n(P_{n,k} + Q_{n,k}) = 1$

B.2. Let's develop now the results on Hh functions. First of all, note that $Hh_n(x) \to 0$, as $x \to \infty$, for $n \geq -1$; and $Hh_n(x) \to \infty$, as $x \to -\infty$, for $n \geq -1$; and $Hh_0(x) = \sqrt{2\pi}\phi(-x) \to \sqrt{2\pi}$, as $x \to -\infty$. Also, for every $n \geq -1$, as $x \to \infty$,

$$\lim Hh_n(x) / \{\frac{1}{x^{n+1}} e^{-\frac{x^2}{2}}\} = 1$$

and as $x \to \infty$

$$Hh_n(x) = O(|x|^n)$$

Here (B4) is clearly true for $n = -1$, while for $n \geq 0$ note that as $x \to_{\infty}$,

$$Hh_n(x) = \frac{1}{n!} \int_x \infty (t-x)^n e^{-\frac{t^2}{2}} dt$$

$$\leq \frac{2^n}{n!} \int_{-\infty}^{\infty} |t|^n e^{-t^2} 2dt + \frac{2^n}{n!} \int -\infty\infty |x|^n e^{-t^2} 2dt = O(|x|^n)$$

For option pricing it is important to evaluate the integral $I_n(c; \alpha; \beta; \delta)$,

$$I_n(c; \alpha; \beta; \delta) = \int_c \infty e^{\alpha x} Hh_n(\beta x - \delta) dx, n \geq 0$$

for arbitrary constants α, c and β.

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BIOGRAPHICAL SKETCH

This section is where your biographical sketch is typed in the bio.tex file. It should be in third person, past tense. Do not put personal details such as your birthday in the file. Again, to make a full paragraph you must write at least three sentences.