M2 RO

Optimisation globale déterministe

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Dans ce travail pratique, pour chacun des problèmes donnés, il est demandé de :

(i) formuler le problème en utilisant la programmation mathématique (si formulation pas fournie), et analyser la formulation pour établir à quelle classe de problèmes d'optimisation elle appartient.

(ii) (si ne pas fournie) implémenter la formulation dans l'environnement AMPL

(iii) calculer une solution optimale (globale) en utilisant un ou plusieurs solveurs d'optimisation appropriés

(iv) si la formulation est non linéaire, la reformuler pour qu'elle devienne linéaire, implémenter la reformulation en AMPL et résoudre le problème reformulé en utilisant un solveur approprié

(v) identifier le type de reformulation utilisé et comparer les solutions du problème original et reformulé.

1 Model s324 from K. Schittkowski's benchmark

Let us consider the model "s324" described by K. Schittkowski and available in classical optimization benchmarks:
the AMPL model is available in the folder "`models/nlmodels`" contained in the archive `ampl.linux-intel64.tar` (under Linux) / `ampl.mswin64.zip` (under Windows).

- Analyze the formulation and identify nonlinearities.
  Compute an optimal global solution by selecting an appropriate optimization solver.
  If multiple appropriate solvers are available, solve the problem with different solvers and compare the obtained output.
- The formulation is nonlinear. Reformulate it, to obtain a linear formulation.
  Note: you may consider (and add to the model) that the second variable is lower bounded by zero, and both variables are upper bounded by 10^3.
  Which kind of reformulation do you apply?
- Write an AMPL implementation of your reformulated model (files `.mod` and `.run`), and solve the reformulated problem to global optimality selecting an appropriate solver.
- How the solution of the linearized problem compares to that of the original problem? Comment on the quality of the solution of the reformulated problem.

2 Graph partitioning

Let us consider the following graph partitioning problem.

Given a weighted undirected graph $G = (V, E, c)$, where
V is the set of vertices of G,
E is the set of edges of G,
c is the set of weights eventually assigned to the edges,
and given an integer $k_{max} \leq |V|$,
find a partition of $k \leq k_{max}$ subsets of V (clusters) minimizing the total weight of edges between different clusters.

The objective can be formulated as:

$$\min_x \frac{1}{2} \sum_{k \neq l \leq k_{max}} \sum_{(u,v) \in E} c_{uv} x_{uk} x_{vl}$$

where x_{uk}, $\forall u \in V, k \leq k_{max}$, are assignment variables:

$$x_{uk} = \begin{cases} 1 & \text{if } u \in k^{th} \text{ cluster} \\ 0 & \text{otherwise} \end{cases}$$

- Provide a complete mathematical programming formulation for the problem, formulating the following constraints:
  - each vertex must be assigned to only one cluster
  - the trivial solution (all the vertices into one cluster) must be excluded.

  To which class of optimization problems the formulation belongs?

- Write an AMPL implementation of your model (file `.mod`), and test it on the data provided in file `graph_partitioning_s.dat`.
  Write a `.run` file, selecting an appropriate solver, and solve the problem to global optimality.

  Notes on AMPL syntax: in AMPL,
  $E \subset V \times V$ can be defined as `set E within {V,V}`
  a condition like $\forall u \in V, v \in V : (u, v) \in E$ can be expressed as `{u in V, v in V : (u,v) in E}`

- If the formulation is nonlinear, which kind of nonlinearities can you identify?
  Provide a linear reformulation. Which kind of reformulation do you apply?

- Write an AMPL implementation of your reformulated model (files `.mod` and `.run`), and solve the reformulated problem to global optimality selecting an appropriate solver.

- Compare the computed solutions, in terms of objective function value, CPU time and number of B&B nodes.