DELHI UNIVERSITY
GENERIC ELECTIVE FOR HONS. COURSES – 2015
PAPER : GE – I CALCULUS
SEMESTER I

Time : 3 Hours *Maximum Marks : 75*

Do any five questions from each of the three sections.
Each question is of five marks.

1. Using ε - δ definition to show that :

$$\lim\left(\frac{3}{2}x-1\right)=\frac{1}{2} \text{ as } x \to 1.$$

2. Find a linearization of : $f(x)=\dfrac{1}{1-x}$ at $x=0$.

3. For the function : $f(x) = x^4 + 2x^3$

   (*i*) Find the intervals over which f is increasing and the intervals over which f is decreasing.

   (*ii*) Also find points of inflexion, if any.

4. Find the asymptotes of the following function : $f(x)=\dfrac{x^2-3}{2x-4}$.

5. The radius of a circle is increased from 2 m to 2.02 m :

   (*i*) Estimate the resulting change in area using differentials.

   (*ii*) Express the estimate as a percentage of the circle's original area.

6. Use L'Hôpital's rule to find : $\lim\left(\dfrac{\sin x-\cos x}{x-\frac{\pi}{4}}\right)$ as $x \to \dfrac{\pi}{4}$.

Section II

7. The circle $x^2 + y^2 = a^2$ is rotated about the x-axis to generate a sphere. Find its volume.
8. Find the length of the curve $y = x^3,\quad 0 \le x \le 1$.
9. Evaluate : $\displaystyle\int_0^3 \frac{dx}{(x-1)^{2/3}}$.
10. Graph the curve $r = 1 - \cos q$ and identify its symmetries.
11. A person on a hang glider is spiraling upward due to rapidly rising air on a path having position vector :

$$\boldsymbol{r}(t) = 3\cos t\boldsymbol{i} + 3\sin t\boldsymbol{j} + t^2\boldsymbol{k}$$

Find :

(*i*) the velocity and acceleration vectors

(*ii*) the glider's speed at any time t.

(*iii*) the times, if any, when the glider's acceleration is orthogonal to its velocity.

12. Solve the initial value problem :

Differential equation : $\frac{dr}{dt} = (t^3 + 4t)\boldsymbol{i} + (t)\boldsymbol{j} + \left(2t^2\right)\boldsymbol{k}$

Initial value : $\boldsymbol{r}(0) = \boldsymbol{i} + \boldsymbol{j}$

Section III

13. Without finding T and N write the acceleration of motion, when :

$$\boldsymbol{r}(t) = (\sin t - t\cos t)\boldsymbol{i} + (\cos t + t\sin t)\boldsymbol{j}, \quad t>0$$

in the form :

$$\boldsymbol{a} = a_T \boldsymbol{T} + a_N \boldsymbol{N}$$

14. Show that the function : $f(x,y) = \dfrac{2x^2 y}{x^4 + y^2}$ has no limit as $(x, y) \to (0, 0)$.

15. Express $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial s}$ in terms of r and s if :

$$w = x + 2y + z^2, \; x = \frac{r}{s}, \; y = r^2 + \ln s \text{ and } z = 2r.$$

16. Find equations for :

(*a*) Tangent Plane

(*b*) Normal line at the point $P_0(1, -1, 3)$, to the given surface :

$$x^2 + 2xy - y^2 + z^2 = 7.$$

17. Find the directions in which the function :

$$f(x, y, z) = \ln(xy) + \ln(yz) + \ln(xz)$$

increases, decreases most rapidly at $P_0(1, 1, 1)$. Then find the directional derivatives of the function in these directions.

18. Find the absolute maximum and minimum values of the function

$$f(x,y) = 2x^2 - 4x + y^2 - 4y + 1$$

on the triangular region in the first quadrant, bounded by the lines :

$$x = 0, \; y = 2, \; y = 2x.$$

DELHI UNIVERSITY

GENERIC ELECTIVE FOR HONS. COURSES – 2016

PAPER : GE – I CALCULUS

SEMESTER I

Time : 3 Hours *Maximum Marks : 75*

Do any five questions from each of the three sections.
Each question is of five marks.

1. Use ε-δ definition to show that $\lim_{x \to 3}(3x - 7) = 2$

2. Find the equations of the asymptotes for the curve $f(x) = \dfrac{x^3 + 1}{x^3}$.

3. Find the linearization of $f(x) = \sin x$ at $x = p$.

4. For $f(x) = x^3 - 3x + 3$
   (*i*) Identify where the extrema of 'f' occur.
   (*ii*) Find where the graph is concave up and where it is concave down.

5. Use L'Hôpital's rule to find $\lim_{x \to \pi/2} \dfrac{1 - \sin x}{1 + \cos 2x}$

6. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

7. Find the length of the astroid $x = \cos^3 t$, $y = \sin^3 t$, $0 \leq t \leq 2\pi$.

Section II

8. State Limit Comparison Test. Using the Limit Comparison Test, discuss the convergence of $\int_1^\infty \dfrac{dx}{1 + x^2}$.

9. Identify the symmetries of the curve and then sketch the graph of $r = \sin 2q$.

10. Solve the initial value problem for $\boldsymbol{r}$ as a vector function of t

    Differential equation : $\dfrac{d^2\boldsymbol{r}}{dt^2} = 32\boldsymbol{k}$

    Initial Conditions : $\boldsymbol{r}(0) = 100\boldsymbol{k}$ and $\left(\dfrac{d\boldsymbol{r}}{dt}\right)_{t=0} = 8\boldsymbol{i} + 8\boldsymbol{j}$

11. Find the curvature of the helix :

$$\boldsymbol{r}(t) = (a\cos t)\boldsymbol{i} + (a\sin t)\boldsymbol{i} + bt\,\boldsymbol{k}, \quad a, b \geq 0, \; a^2 + b^2 \neq 0$$

12. Write the acceleration vector $\boldsymbol{a} = a_T\boldsymbol{T} + a_N\boldsymbol{N}$ at the given value of t without finding $\boldsymbol{T}$ and $\boldsymbol{N}$ for the position vector given by

$$\boldsymbol{r}(t) = (t\cos t)\boldsymbol{i} + (t\sin t)\boldsymbol{i} + t^2\,\boldsymbol{k}, \quad t = 0$$

13. Show that $f(x,y)=\begin{cases}\dfrac{2xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

is continuous at every point except at the origin.

14. If $f(x,y)=\dfrac{x-y+2\sqrt{x}-2\sqrt{y}}{\sqrt{x}-\sqrt{y}}$

(*i*) Find the domain of the given function $f(x,y)$.

(*ii*) Evaluate $\lim\limits_{(x,y)\to(0,0)} f(x,y)$.

Section III

15. If $z=5\tan^{-1}x$ and $x=e^u+\ln v$, find $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$ using chain rule, when $u=\ln 2$, $v=1$.

16. Find the directions in which the given function f increase and decrease most rapidly at the given point P_0. Then, find the derivative of the function in those directions.

$$f(x,y,z)=\frac{x}{y}-yz,\quad P_0(4,1,1).$$

17. Find the parametric equations for the line tangent to the curve of intersection of the given surfaces at the given point.

Surfaces : $x+y^2+2z=4,\ x=1$

Point : $(1, 1, 1)$.

18. Find equations for the

(*a*) Tangent plane and

(*b*) Normal line at the point P_0 on the given surface

$$z^2-2x^2-2y^2-12=0;\quad P_0\,(1,-1,4)$$

19. Find the absolute maxima and minima of the function $f(x,y)=x^2+y^2$ on the closed triangular plate bounded by the lines $x=0$, $y=0$, $y+2x=2$ in the first quadrant.

20. If $f(x,y)=x\cos y+ye^x$, find $\dfrac{\partial^2 f}{\partial x^2}$, $\dfrac{\partial^2 f}{\partial y\partial x}$, $\dfrac{\partial^2 f}{\partial y^2}$ and $\dfrac{\partial^2 f}{\partial x\partial y}$.

21. If $f(x,y)=x-y$ and $g(x,y)=3y$. Show that

(*i*) $\nabla(fg)=g\nabla f+f\nabla g$

(*ii*) $\nabla\left(\dfrac{f}{g}\right)=\dfrac{g\nabla f-f\nabla g}{g^2}$.