Image restoration with adaptive local noise reduction filter using pixel adjacency relation

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***Abstract*—Image restoration is the process of removing degradation of any kind from a deteriorated image, and techniques within it use the statistics of pixels of the neighborhood encompassed by the filter. While the response of these ordered statistics filters is spatially based on the ordering and ranking among pixels contained in that neighborhood, adaptive filters consider this response based on the measure of central dispersion of pixel intensity levels. Adaptive local noise reduction filter is a statistical method that uses mean and standard deviation of pixel intensities and corrects the image based on the neighborhoods within. It uses the signal-noise ratio of the image in comparison with a filter, and later applying this factor with a deviation of pixel intensities from the mean can estimate the relative noise level intensity of pixels. The approach discussed in this paper can assess this noise level much more precisely by utilizing the m-adjacency of pixels within an image and identifying the connected components for an exact calculation of the global noise variance. Traditionally the adaptive filters are superior in their performance, and the following approach builds on top of that.**

***Index Terms*—image, restoration, adaptive, local, filter, adjacency, connected, noise, reduction statistics, mean, variance, ration, neighborhood, periodic, gaussian, rayleigh, erlang**

I. Introduction

Within image enhancement, the primary goal of its restoration is to improve the image close to its actuality in the physical world. This process involves improving upon any degradation while digitizing or during transmission and storage. The implementation of this process, also known as restoration techniques, aims to achieve a function or a model that reverses this degradation by applying inverse strategies of what was subjected.

$$g(x, y) = (h * f)(x, y) + \eta(x, y) \tag{1}$$

$$G(u, v) = H(u, v)F(u, v) + N(u, v) \tag{2}$$

Establishing the degradation model [Fig. 1] is essential for the image restoration process. The degradation of an image is modeled in the spatial domain as indicated in the above equation. In this equation [eq. (1)] g(x,y) indicates the degraded image, h(x,y) represents the degradation function, f(x,y) represents the actual near-perfect image we desire, and η(x,y) represents the additive noise. The h*f indicates a convolution process of f with h. The same described in the frequency domain is shown in equation [eq. (2)]. In both these instances, our goal is to obtain a near-perfect estimate for f(x,y) and an estimate close to the original image; it's essential to precisely know the degradation function and the additive noise. H(u,v)F(u,v) here represents multiplication here.

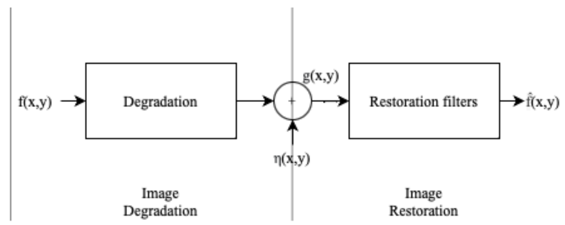

Fig. 1. Degradation and Restoration System

II. Noise

Noise within the images is natural and may arise during image acquisition, transmission, or storage retrieval. For instance, digitizing a picture with an underperforming sensor can affect the appearance to deviate from the physical scene. Or during transmission of these acquired digital images, the interference in the channel and external atmospheric conditions can affect the carried information, resulting in a corrupted image. There are a variety of noises that can be categorized into different types based on the statistics of the intensity of these noise components.

We use the random variables to mathematically simulate the noise into images, characterized by the probability density function(PDF). The noise component in the above equation η(x,y), is of the same size as the original image, in which a random value follows a specific function. A few of the noise components PDFs are discussed below.

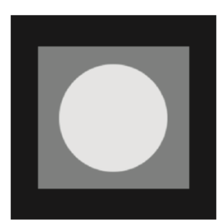

Fig. 2. Test pattern image to denote PDF of different noises

A. Gaussian Noise

Gaussian noise models are usually defined by PDF for gaussian random variable z, and the below expression defines it [eq. (3)]. z represents intensity and $\bar{z}$ is mean value and σ is standard deviation. The images below result from a sample image [Fig. 2] added with Gaussian noise [Fig. 3]. The image usually has three varying intensity levels. However, simulating the gaussian noise, the image's histogram follows the Gaussian distribution.

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{z-\bar{z}}{2\sigma^2}} \qquad -\infty < z < \infty \quad (3)$$

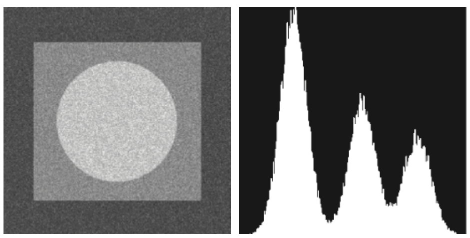

Fig. 3. Gaussian noise image and it's PDF

B. Salt and Pepper Noise

The salt and pepper noise model, also called bipolar impulse noise, is modeled with the below PDF function. It's giveIn by the equation [eq. (4)] where V is any integer value in the range $0 <$ V $< 2^k-1$. If P_s is 0, it's called black noise, also called data drop-out noise, and if P_b is 0, then it's called white noise, also known as spike noise. When this noise model is applied to an image, the range of possible intensity values is either 0 or 2^k. As a result, an 8-bit image lies at either of the values 0 or 255. The sum of the probability of data drop noise and spike noise gives the total probability of this bipolar impulse noise. For instance, if the image [Fig. 4] is modeled with salt and pepper noise of a total P noise density value of 0.05, then the combined noise points of black and white would equal 5% of the whole image pixels that are corrupted with noises.

$$z_m(t) = \begin{cases} P_s & \text{for } z = < 2^k - 1, \\ P_p & \text{for } z = 0, \\ 1 - (P_s + P_p) & \text{for } z = V \end{cases} \quad (4)$$

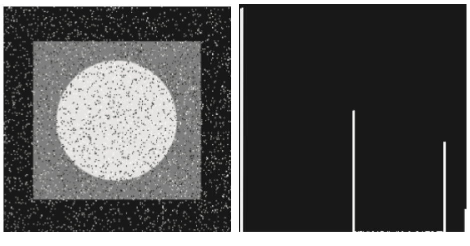

Fig. 4. Salt and Pepper noise image and it's PDF

III. Image Restoration using Spatial Filtering

For image enhancement using the below restoration techniques, let's assume that our digitization of images from physical world is perfect and the degradation system doesn't exist. This results in images only degraded with additive noise. The equations[eq. (1,2)] for spatial domain and frequency domain becomes the following respectively [eq. (5,6)].

$$g(x, y) = f(x, y) + \eta(x, y) \quad (5)$$

$$G(u, v) = F(u, v) + N(u, v) \quad (6)$$

Noise terms observed in the equation [eq. (5,6)]are unknown and subtracting the noised image with actual image is not as straight forward as it seems. While it may be possible to estimate η(x,y) from spectrum of g(x,y), spatial filtering method is the method of choice for estimating f(x,y) with denoising of image g(x,y). Below are examples of noise reduction filters.

A. Mean Filter

- Arithmetic Mean Filter
- Geometric Mean Filter
- Harmonic Mean Filter
- Contraharmonic Mean Filter

Mean filters are a special type of spatial filter whose response is based on the specific type of mean performed on the sub-image window or neighborhood S_{xy} of size (m,n) centered around the point (x,y). Image restoration is achieved with various types of mean, such as arithmetic, geometric, harmonic mean, etc. The simplest among them is the arithmetic mean filter which computes average among the elements in the window resulting in the effect of smooths local variations in an image and noise is reduced as result of blurring.

B. Ordered Statistic Filters

- Median Filter
- Max Min Filter
- Midpoint Filter
- Alpha Trimmed Mean Filter

These types of filters are spatial filters whose response is based on ordering or ranking values of pixels in the neighborhood S_{xy} encompassed by the filter. With various types of filtering techniques, its ranking determines the filter's response. There are many types of ordered statistic filters, such as median filter, min-max filter, midpoint filter, etc. All these operations provide excellent noise reduction capabilities for a certain type of random noise with considerably less blurring than linear smoothing. It's extremely effective for unipolar and bipolar noise.

C. Adaptive Filters

All the above discussed Mean Filters and Ordered Statistic filters apply the techniques discussed in a purely mathematical sense, and neither considers the relations of how pixels vary. Adaptive filters look for those behavioral changes based on statistical characters of the image inside the filter region defined by m x n rectangular neighborhood S_{xy}. These adaptive filters are far superior to the ones observed above, and this comes with the price of additional computation and increased complexity.

IV. Adaptive Local Noise Reduction Filter

The statistical characteristics used in this filtering technique are the mean and variance. These parameters are reasonable to base the adaptive filter on as they dictate the relation to neighboring pixels within and outside the sub-components. While the mean provides us with a measure of the average intensity in a region, the variance measures the deviation of the pixel from the mean value, letting us information about the image contrast region.

In this filtering technique, similar to other operations we have seen above, we run the entire image with a neighborhood S_{xy}, with point (x,y) centered in the neighborhood. The response of this filter operation at (x,y) is based on g(x,y), the pixel value of the noisy image at (x,y), ${\sigma^2}_\eta$ the variance of noise, and $\bar{Z}_{S_{xy}}$ is the local average intensity value in the neighborhood S_{xy} and ${\sigma^2}_{S_{xy}}$ is the local variance of the intensities of pixels in S_{xy}. The behavior of the filter is defined as the following

1) If ${\sigma^2}_\eta$ is zero, the filter should return a value of g(x,y). This indicates that the value of the pixel has low chance of getting corrupted with noise, as pixel values within a neighborhood are the same. Its a trivial zero noise case in which g is equal to f at (x,y)
2) If local variance ${\sigma^2}_{S_{xy}}$ is high relative to ${\sigma^2}_\eta$, the filter should return a value close to g at (x,y). Such a high local variance indicates an edge or rapid changes within a neighborhood, indicating that that value should be preserved with minimal change.
3) When variances are equal, suggesting that the entire noise variance is the same as the neighborhood variance value, the filter should return the arithmetic mean value of pixels in S_{xy}. Such situation indicates that local area has the same properties as an overall image, and local noise can be reduced by averaging.

The above rules dictate the equation for adaptive local mean filtering to be as follows.

$$\hat{f}(x,y) = g(x,y) - \frac{{\sigma^2}_\eta}{{\sigma^2}_{S_{xy}}}\left[g(x,y) - \bar{Z}_{S_{xy}}\right] \tag{7}$$

$$\sigma^2 = \sum_{i=0}^{L-1}(r_i - m)^2 p(r_i) \tag{8}$$

In the equation [eq. (8)], our limits of summation run along all the pixel intensities of the image and m is mean of all pixel intensities, r_i is pixel intensity at i and $p(r_i)$ denotes probability of intensity r_i.

With all these calculations, the key finding to be known prior is ${\sigma^2}_\eta$, the variance of noise corrupting the image f(x,y). This value can be estimated from eq of variance calculation of the image using equation [eq. (7)]. Other parameters of S(x,y) can be computed by using equation [eq. (8)]. The thing to watch out for is the S_{xy} neighborhood, as this filter is incomplete along the edges or boundaries of the image. Hence padding is required to fulfill the conditions that have been discussed above.

Another assumption with adaptive local mean filter operation is that ${\sigma^2}_\eta <= {\sigma^2}_{S_{xy}}$. This is because additive noise is position independent, and it's practically possible in some conditions neighborhood has less value than global noise variance. As a result, the filter is nonlinear, and to avoid the negative numbers induced by the filtering technique, we set the ratio to 1 during such cases. Rescale is also an option instead of this condition; however, it leads to a loss of dynamic range of the final output image range.

The filtered image results are usually on par with the outputs of arithmetic and geometric mean filters. However, the image filtered with an adaptive local mean filter appears sharper and much clearer than the mean filter outputs because the latter tends to smoothen the image with an effect of blur.

Estimating noise variance is crucial for the adaptive local mean filtering technique. If known noise variance values are utilized, the resultant output image is sharp and clear, better performing than the mean or median filters; however, this comes at a price of additional added complexity. Once the noise variance is unknown, there can be consequences. If the estimate is low, the output image is close to the original noised image, and the low ratio corresponds to lower pixel corrections. But if the estimate is high, the variance ratio would be clipped to 1, and it's as straightforward as applying a mean filtering technique. If clipping is not an option, the severe image corrections will lead to negative pixel values, when rescaled, can reduce the dynamic range of the image.

V. Pixel relationship and m-adjacency

The pixel at coordinates (x,y) has two horizontal and two vertical neighbors with coordinates. This set of pixels is called 4-neighbors of p and is denoted as N4(p). These 4 neighbors are denoted in equation [eq. (9)]below.

$$(x+1,y),(x-1,y),(x,y+1),(x,y-1) \tag{9}$$

If V defines a set of intensity values used to define adjacency, then in an image, two pixels p and q belong to set V if p has q is in set N4(p). This is called 4-adjacency, and the pixels are said to be connected.

VI. Overview

The variance value of the image is essential for working out the adaptive local noise reduction filter. The exact value of $\sigma^2{}_\eta$ can be extremely effective against noise and work best in image restoration. However, the algorithm requires this $\sigma^2{}_\eta$ value to be known prior. Hence the added complexity of this algorithm to find this value. Five types of adaptive local noise reduction filters are discussed below, which tend to work best in cleaning noisy images. A note for all the algorithms below is that all assume 0 padding for the neighborhood relations to work.

- Variance levels of noise
- Global noise variance
- Mean of local window noise variances
- Connected noise variance
- Connected noise variance with neighborhood bounds

A. *Variance levels of noise*

Usually, when a specific gaussian noise is simulated into an image that follows a gaussian probability density function, it is required to specify mean and variance values of the random number z of the equation [eq. (3)]. With this technique, an adaptive local noise reduction filter can work with this variance of the gaussian PDF as our $\sigma^2{}_\eta$ to remove the noise and obtain a restored image.

B. *Global noise variance*

When considering the scope of the variance, global means considering every pixel intensity value within an image, and local indicates something within a region. In this method, to solve the [eq. (7)], by using the ways of calculating the variance as in [eq. (8)], we try to find the global variance of all the pixel intensities. This methodology considers this global variance of all pixel values as $\sigma^2{}_\eta$, and using this adaptive local noise reduction filter is applied to remove noise and obtain the restored image.

C. *Mean of local window noise variance*

Under this method, we operate on the noise variance levels of all the neighborhoods S_{xy} within the image, capturing these values and obtaining the average among them. With this averaged estimation as the $\sigma^2{}_\eta$ value, we work out the equation [eq. (7)] and identify the noise level, thereby correcting and restoring it. The output of this technique is sharp looking images at rapid intensity changes, unlike the smoothing or blurring observed in the average filtering process.

D. *Connected noise variance*

We have considered the mean of all captured local window neighborhood S_{xy} variances in the previous method. However, these values are the combinations of all rapidly changing frequencies or intensities. The idea is to estimate the $\sigma^2{}_\eta$ precisely based on only considering the variances of segmented objects separately but not combining all sub-components. As a result, this process requires the m-adjacency to individual segment components of the image and identify the different independent $\sigma^2{}_\eta$ values. Later while working out the equation [eq. (7)] used to remove the noise, the neighborhood S_{xy} picks its necessary variance value when running over that part of the image component. The output in this method closely resembles what is observed while restoring the image with the mean of local windows noise variance.

E. *Connected noise variance with neighborhood bounds*

This method also uses a similar technique of $\sigma^2{}_\eta$ value that is used in connected noise variance. However, the difference lies in how the neighborhood S_{xy} runs over the image and calculates the $\sigma^2{}_{S_{xy}}$ variance value within each neighborhood. With the ability of the connectivity that we have established, it provides us with ways to drive the window operations alongside the image sub-components. Usually, variance along a neighborhood defines how deviating all the pixels are apart from the average value, but this value is also quite affected when a rapid frequency change occurs. This is typical when the filter operations are performed along the boundary edges of the zero-padded image and also most frequent when the neighborhood window passes over two or more segmented components within an image. With the knowledge of component segmentation, this technique allows us to define a clear boundary that can shrink and expand when passing over such rapid frequency-changing intensities, allowing us to calculate the $\sigma^2{}_{S_{xy}}$ values precisely. This precision is used when solving equation [eq. (7)] lead to noiseless images as outputs with much sharper segment component edges than observed in the mean local window approach and connected noise variances approach.

VII. Results

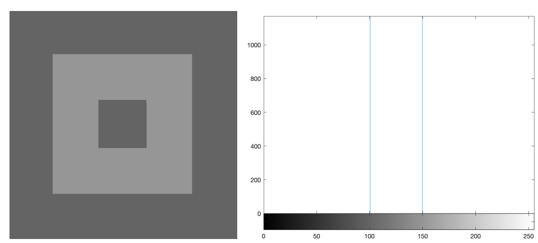

Fig. 5. Original simulated image and its PDF

With all the different approaches discussed above in the overview, this section covers the results observed with a simulated noisy image with their PDFs. Neighborhood window S_{xy} has size of $3 * 3$ Figure [Fig. 5] contains two images, the original simulated image, and its PDF.

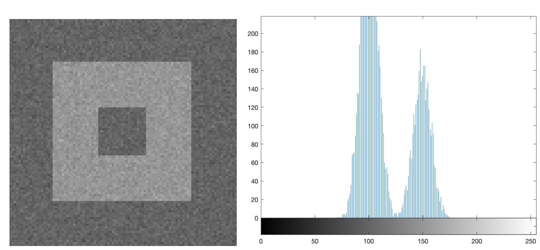

Fig. 6. Gaussuan noised image and its PDF

Figure [Fig. 6] contains the simulated image with added Gaussian noise of mean=0 and sigma=0.001. It also displays its PDF, which shows a clear Gaussian distribution.

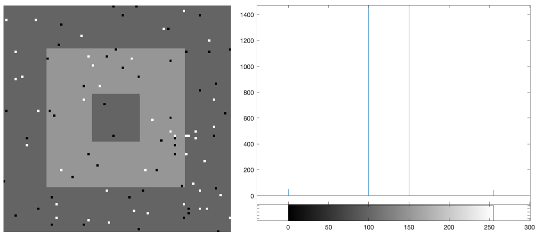

Fig. 7. Salt and Pepper noised image and its PDF

Figure [Fig. 7] shows the simulated image with Salt and Pepper noise of noise density 0.01. It also shows the PDF of this image which resembles the general histogram of Salt and Pepper noise type.

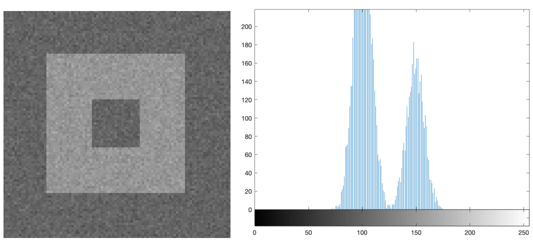

Fig. 8. Adaptive noise reduction filter using variance levels of noise(0.001)

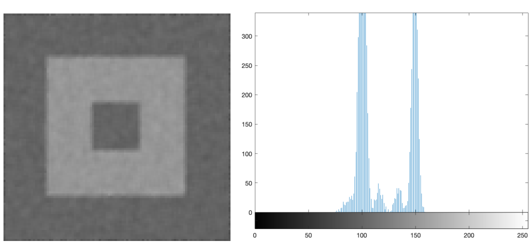

Fig. 9. Adaptive noise reduction filter using global noise variance

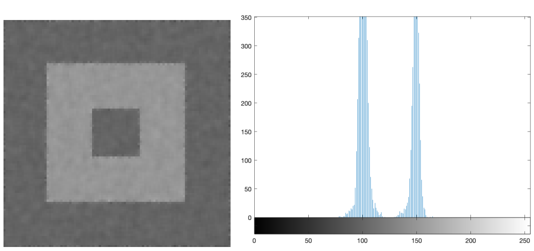

Fig. 10. Adaptive noise reduction filter using mean of local window noise variance

Figure [Fig. 8,9,10] shows the results of denoised images and their respective PDFs. Adaptive local noise filters and the respective methods remove the noise in all these instances, and their PDFs are gradually reduced to match the original, as seen in [Fig. 5].

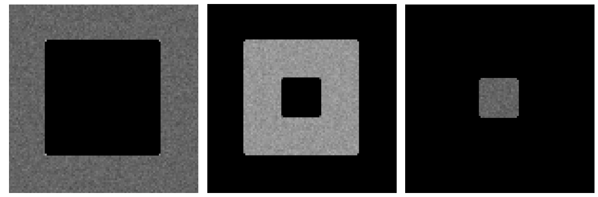

Fig. 11. Finding all 4-adjacent sub components within noised image

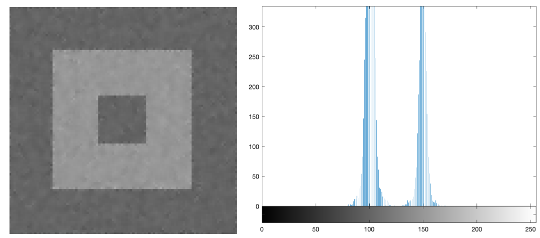

Fig. 12. Adaptive noise reduction filter using connected noise varaince σ_η^2

This figure [Fig. 11] use the knowledge of 4-adjacency, and the variance of connected components is calculated using that knowledge. The results are the figures [Fig. 12,13], which uses adaptive local noise reduction using the connected ${\sigma^2}_\eta$ and flexible ${\sigma^2}_{S_{xy}}$, which avoids the rapid boundary changes and calculates neighborhood variances only within the connected component.

TABLE I
ADAPTIVE NOISE REDUCTION FILTER RESULTS - GAUSSIAN NOISED IMAGE

Filter Type	**Noise Residual and De-noise Percentage**	
	Residual	***Percentage***
Noisless Image	0	NA
Noisy Gaussian image	32599	100%
Variance levels of noise(0.001)	32599	100%
Global noise variance	18802	57.68%
Mean of local window noise variances	14002	42.95%
Connected noise variance	14792	45.38%
Connected noise variance with neighborhood bounds	13744	42.16%
Arithmetic Mean Filters	28806	88.36%
Ordered Statistic Median Filters	16795	51.52%

The table [Tab. 1] contains the results of all variants of adaptive local noise filtering against images with Gaussian noise. It can be observed that all these types have a good performance except for the first type, which uses noise variance value. But other types perform better than mean and median filtering. The mean of local window variances performs better than traditional global noise. It is also evident that the connected variances method performs similarly to this global variance method. Finally, the connected noise variance with the neighborhood bounds performs even better for the fact that the variance values are carefully considered when the window crosses over two or more sub-components of the image. It also results in sharper glances of the sub-component

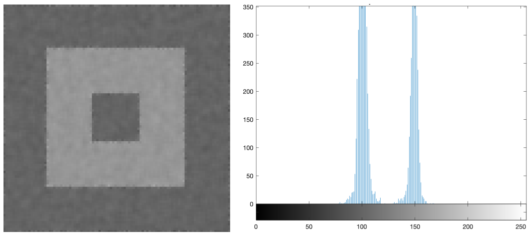

Fig. 13. Adaptive noise reduction filter using connected noise varaince and neighborhood bounds

image.

TABLE II
ADAPTIVE NOISE REDUCTION FILTER RESULTS - SALT AND PEPPER NOISED IMAGE

Filter Type	**Noise Residual and De-noise Percentage**	
	Residual	***Percentage***
Noisless Image	0	NA
Noisy Salt and Pepper image	6050	100%
Variance levels of noise	6050	100%
Global noise variance	15012	248.13%
Mean of local window noise variances	10021	165.64%
Connected noise variance	8712	144%
Connected noise variance with neighborhood bounds	9469	42.16%
Arithmetic Mean Filters	24007	396.80%
Ordered Statistic Median Filters	396.81	9.92%

The table [Tab. 2] contains the results of all variants of adaptive local noise filtering against images with Salt and Pepper noise. The first variant doesn't improve or reduce any kind of noise as the noise ratio is very low to restore the image. The arithmetic filter is the worst, which corrupts the image much more instead of removing noise. The median filter works best for salt and pepper noise, and the same can be observed here. Comparing the remaining 3 adaptive local noise reduction variations, it can be seen that, although they don't corrupt the image as worst as global noise variance or the arithmetic mean filter, they consider the variances and only pollute the image by little.

VIII. CONCLUSION

Images with periodic noises of gaussian noise are restored better by adaptive local noise reduction filters and all their variants seen above. Traditional global noise reduction filter works fine but considering there is a chance this can be influenced greatly by specific types of images with many frequencies changing. These can be images that have a lot of different sub-components with varying gray level intensities. Comparing this global noise reduction filter with the mean variances of all neighborhoods is a better way to avoid this problem.

Adaptive local noise reduction filter with connected noise variances or 4-adjacent components works on par with the third approach of filtering technique that uses mean of local window variances. The last type of filter, which also runs the neighborhood with careful boundaries, is most effective in restoring the image, which builds upon the 4th type by considering the connected component's noise variance levels instead of a mean or the global value.

Random noise, such as salt and pepper noise, is better off when worked with a median filter rather than a mean filter or adaptive local noise reduction method, but the variance relation didn't contaminate the image as a mean filter.

IX. FUTURE WORK

More data will have to be gathered, and additional work needs to be done on how these adaptive local noise reduction filtering techniques work against other images with periodic noises, like Rayleigh and Erlang noises. Additionally, there is a need for improvement on how the neighborhood filter needs to be calculating the variance value for the m-adjacent edges. Currently, only the connected values are chosen within the window, and their variance is estimated; however, this needs improvement.

REFERENCES

[1] Rafael C. Gonzalez and Richard E. Woods, " Digital Image Processing - Fourth Edition ".