PHYS 5027 INTRODUCTION TO COMPUTATIONAL METHODS

Term Project II Report

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Project 2: Simulating Fluid Dynamics

1 Introduction

In this project, the fluid dynamics equations were solved to create a simulation of the Kelvin-Helmholtz instability between two fluid layers moving at different velocities, such as the Great Red Spot on the surface of Jupiter. A sound wave propagation in the fluid is demonstrated as a test case in the first part. In the second part, the Kelvin-Helmholtz instability is simulated for three cases with various initial parameters.

2 Methods

With `C++`, Euler equations were solved which are the reduced form of the equations describing the fluid dynamics originally known as the Navier-Stokes equations [1]. The equations were solved for 2D inside a box with dimensions $L_x \times L_y$ discretized into $N_x \times N_y$ grid and the periodic boundary conditions were applied on all sides. Utilizing the 2^{nd} *Order Finite Volume* method, the fluxes inside of each cell were computed to evaluate the conserved quantities for fluid as cell-averaged values.

The second-order method was chosen for its relatively higher accuracy compared to the first-order method. It is important to note that the CFL condition has an important role in the stability of the method, by determining a Δt which prohibits the signal propagation of more than one cell in each step [1]. The value of CFL will be argued in the case-specific parts.

The Euler equations are evaluated by the `class fluid_solver_2d` in the header file `fluid_solver_2d.h`. The *FTCS* method is applied by the function `void primitive_update` while updating primitive variables using Euler equations. `void extrapolate_to_interface()` and `void compute_fluxes()` the other functions exploits the *Finite Volume* method

For the first task, `sound_waves.cpp` was provided to test the header file `fluid_solver_2d.h` which introduces a sound wave in a fluid. For the second task, `kelvin_helmholtz.cpp` was scripted to simulate the *Kelvin-Helmholtz vortices* in the fluid using the same header file.

`plot.py` script was used to plot the time evolution of the fluid inside the box using the `matplotlib` library, and movies were created using `ffmpeg` package.

3 Task I: Sound Waves in a Fluid

A sound wave propagating in x direction in the form of a sine wave was simulated in a fluid.

3.1 Wave Speed Estimation

The initial condition for v_x on the `sound_wave.cpp` is computed by `vx[idx] = c * amplitude * wave / rho0`, where `amplitude = 0.001`, `c =` $(5/3)^{1/2}$, `rho0 = 1` and `wave` is a sine wave with the average value can be taken as 0.64 [2], therefore it can be approximated as $v_x = (5/3)^{1/2} \times 0.64 \times 0.001 \approx 0.000813$ $[L_x/\Delta t]$. Here, $L_x = 1.0$ andif Δt is printed during computation, it gives $\delta t \approx 0.0006042$ so that $v_x = (5/3)^{1/2} \times 0.64 \times 0.001 \approx 0.000813 \times 1/0.0006042 \approx 1.3656801$. When 1 and 2 are compared, it can be seen that for the time elapsed $t = 0.12$, the wave propagates for $L_x = 0.2$, then $v_{x_{estimate}} = 0.2/0.12 = 1.6666667$. The difference between the estimated value and the theoretical value may be a result of the approximations made and the error in the visual measurement by eye.

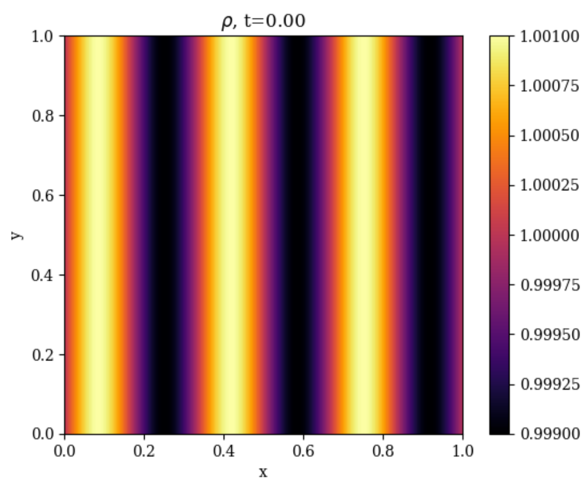


Figure 1: Sound wave propagation with $v_x =\approx 0.000813$ $[L_x/\Delta t]$ at t = 0.0.

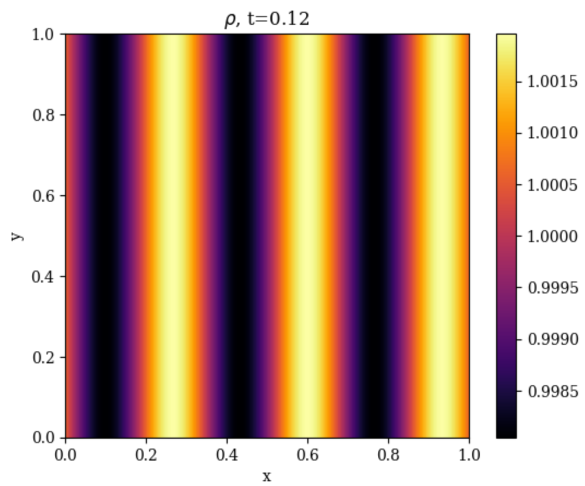


Figure 2: Sound wave propagation with $v_x =\approx 0.000813$ $[L_x/\Delta t]$ at t = 0.12.

3.2 CFL Condition

During the time step evaluation, the CFL value is crucial to keep the system stable. The first value attempted for the CFL constant was for $CFL = 0.4$. Although there was no ambiguity in the first iterations, the system became unstable before reaching $t = 1.0$, as seen in Figure 3

Since $\Delta t \propto CFL$, and considering that numerical methods work with higher precision for smaller time steps, a CFL constant was decreased to stabilize the system, shown in Figure 4.

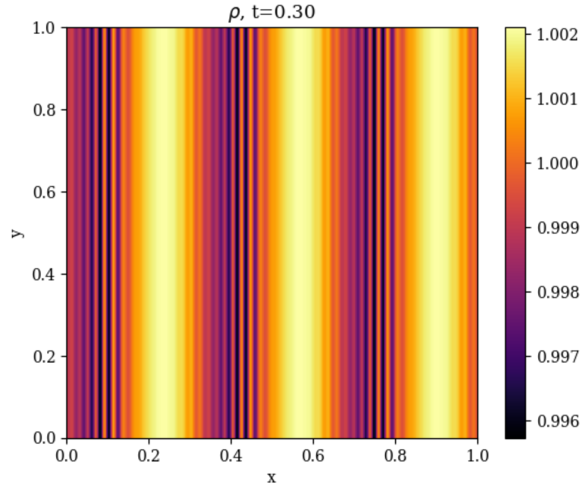


Figure 3: Sound wave propagation with $CFL = 0.4$.

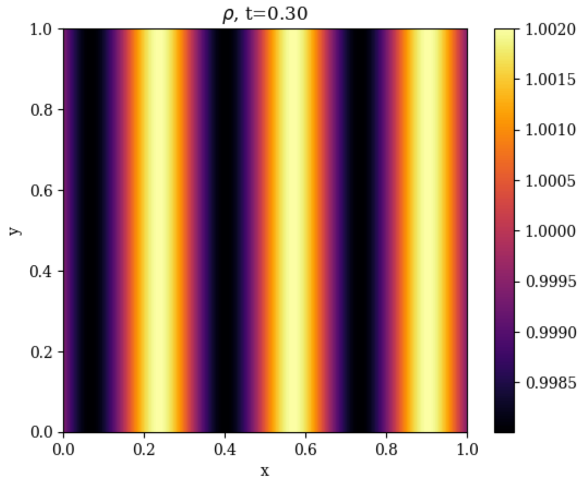


Figure 4: Sound wave propagation with $CFL = 0.2$.

4 Task II: Kelvin-Helmholtz Instability in 2D

In this part, Kelvin-Helmholtz's instability and vortex formations which occurs due to the motion of fluids with different density and velocities were simulated in the same numerical scheme. A new script was written to initialize different kinds of fluids with various parameters on the box mesh on which the periodic boundary conditions were applied. The instability in the y direction was introduced to the system by a small velocity profile near the interfaces of the fluids, provided by the project description [1].

For that task, a smaller CFL condition was used to ensure the stability of the system in a more complicated environment, $CFL = 0.1$. For higher CFL values, the system becomes unstabilized before completing its time evolution for t=4.0.

4.1 Case I: k=6π vs. k=12π

This case demonstrates the effect of the wave number k inside of the velocity profile function introduced to trigger the instability. In Figure 5 and 6, the system of Fluid 1 (background) and Fluid 2 (stripe) varying in two initial parameters $\rho_1 = 1.0$, $vx_1 = 0.5$, $\rho_2 = 1.0$, $vx_2 = 0.5$ was simulated with for $k = 6\pi$ and $k = 12\pi$. It can be seen that k determines the number of the vortices that appear during the time evolution.

4.2 Case II: $v_{small} = 0.01$ vs. $v_{small} = 0.05$

This case demonstrates the effect of the magnitude of the parameter v_{small} multiplying the velocity profile function. In Figure 7 and 8, again the system of Fluid 1 (background) and Fluid 2 (stripe) varying in two initial

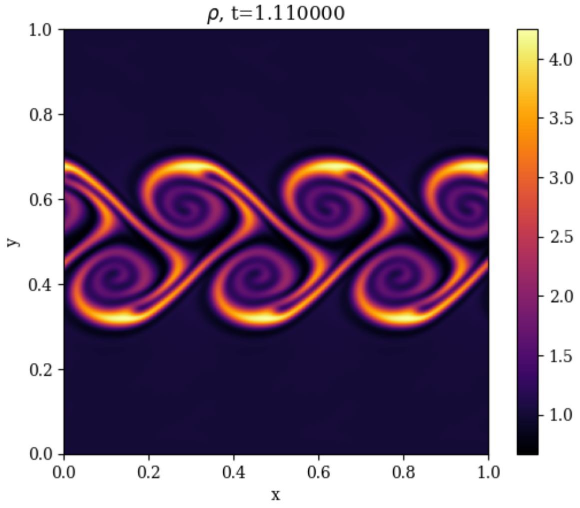


Figure 5: Kelvin-Helmholtz's vortices for k=6π

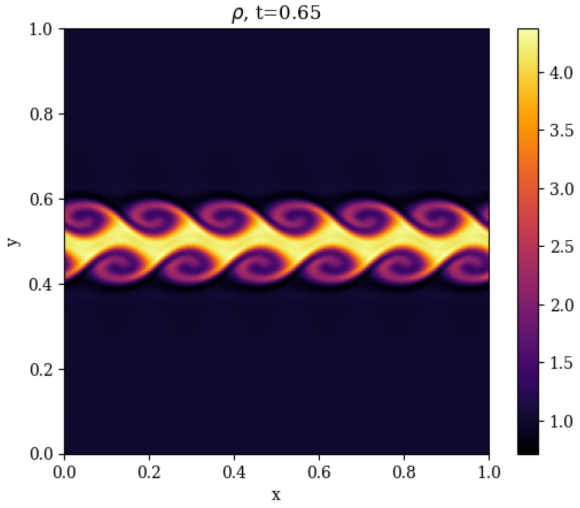


Figure 6: Kelvin-Helmholtz's vortices for k=12π

parameters $\rho_1 = 1.0$, $vx_1 = 0.5$, $\rho_2 = 1.0$, $vx_2 = 0.5$ was simulated with for $v_{small} = 0.01$ and $v_{small} = 0.05$. The figures show the same points during the time evolution for both cases. As expected, Figure 7 with the lower amplitude for the instability introducing function has more defined features having a lower density distribution on average. When the magnitude of the perturbation increases, the vortex patterns become saturated and have a higher density profile with respect to the background, as shown in Figure 8.

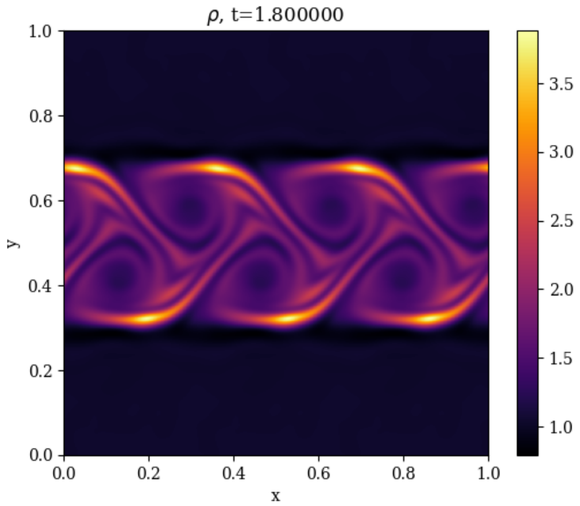


Figure 7: Kelvin-Helmholtz's vortices for $v_{small} = 0.01$

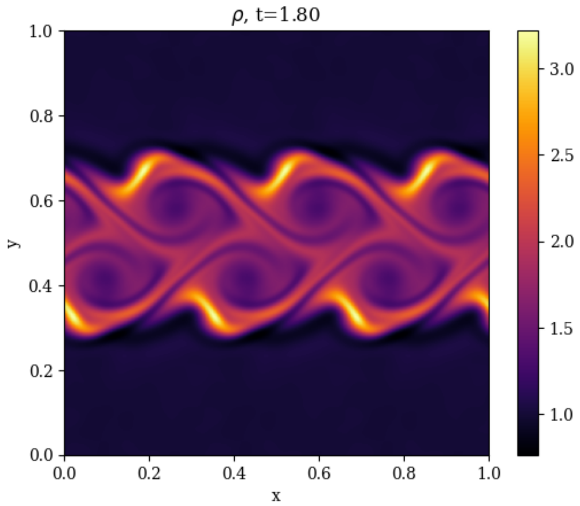


Figure 8: Kelvin-Helmholtz's vortices for $v_{small} = 0.05$

4.3 Case III: Multiple Stripes and Fluids

This case demonstrates how multiple stripes affect each other's flow. Through Figures 9, 10, 11, the time evolution of the same system in the previous cases with three fluid streams of Fluid 2, identical except the middle stream has twice the initial velocity in x direction the top and the bottom stream have. The role of the distance between the streams can be observed in Figure 9, and the co-evolution of the vortex structures. Figure 10 shows that stable vortices occur eventually.

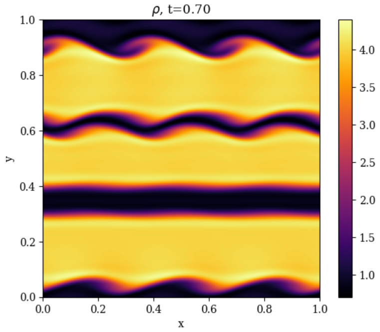


Figure 9

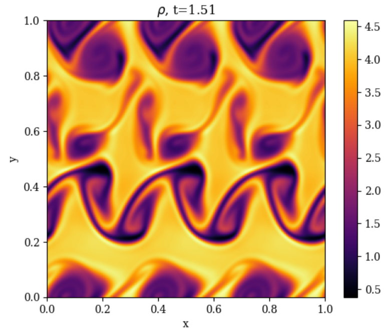


Figure 10

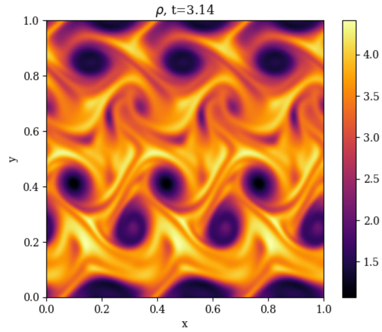


Figure 11

On the other hand, if streams of different fluids with densities $\rho_1 = 4.0$ (top), $\rho_2 = 1.2$ (middle), $\rho_3 = 2.0$ (bottom) are simulated, Figures 12-14 show more chaotic structures occur, but still stabilize in the end.

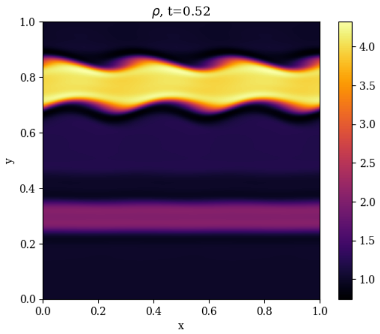


Figure 12

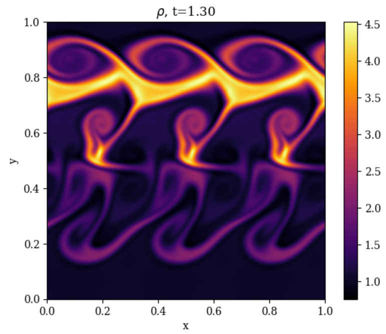


Figure 13

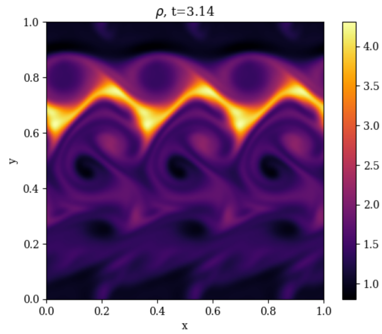
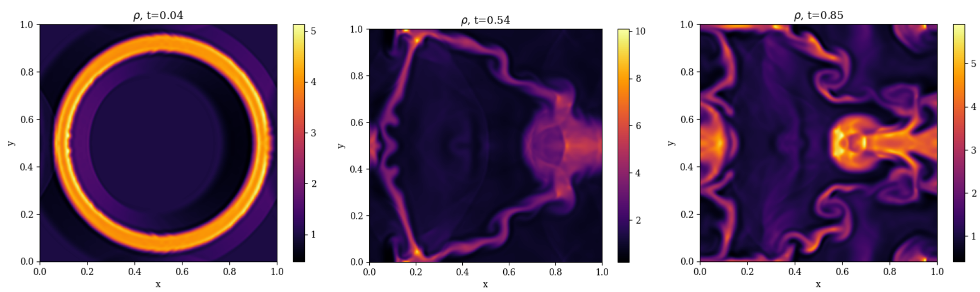
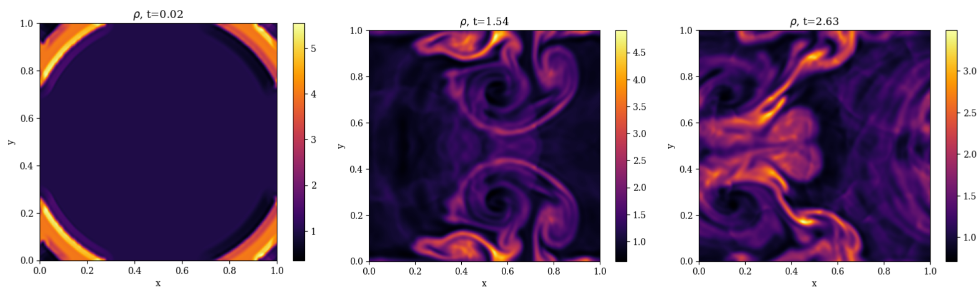


Figure 14

4.4 Bonus :)

Just wanted to share these for different geometries because I think they are *beautiful*.





References

[1] A. Chen. "Project 2: Simulating fluid dynamics," Washington University in St. Louis. (2023).

[2] "8.4 average value of a function." (), [Online]. Available: https://www.whitman.edu/mathematics/calculus_late_online/section08.04.html.