Exercise 1

1.

The unobserved variable X_t represents the weather state at time t. It can take two values:

- Rain (R): It is raining at time t
- No Rain (NR): It is not raining at time t

2.

The observable variable E_t represents whether an umbrella is observed at time t. It can take two values:

- Umbrella (U): An umbrella is carried/observed
- No Umbrella (NU): No umbrella is carried/observed

3.

Dynamic Model $P(X_t \mid X_{t-1})$

The transition matrix A, where rows are X_{t-1} and columns are X_t:

X_{t-1} \ X_t	Rain (R)	No Rain (NR)
Rain (R)	0.7	0.3
No Rain (NR)	0.3	0.7

Observation Model $P(E_t \mid X_t)$

The Observation matrix B, where rows are X_t and columns are E_t:

X_t \ E_t	Umbrella (U)	No Umbrella (NU)
Rain (R)	0.9	0.1
No Rain (NR)	0.2	0.8

4.

The model assumes a discrete-time framework with time slices (0, 1, 2, ...) where the interval Δ between slices is constant. Each slice contains the same set of variables: unobservable state variables X_t and observable evidence variables E_t. In the umbrella world, X_t represents whether it is raining ($Rain_t$), and E_t represents whether an umbrella is observed ($Umbrella_t$). The state sequence starts at t=0, with evidence from t=1 onward.

These assumptions are reasonable as they model temporal processes discretely, allowing probabilistic inference about hidden weather states from umbrella observations over time.

Exercise 2

Verification

```
Verification: P(X_2 | umbrella on day 1 and 2)
Normalized f_1:1 = [0.81818182 0.18181818]
Normalized f_1:2 = [0.88335704 0.11664296]
P(Rain at day 2) = 0.883
```

The code results verify P(X_2 | e_{1:2}) = 0.883.

Forward Messages for Sequence e_{1:5}

```
Sequence: e_{1:5} = {Umbrella1=True, Umbrella2=True, Umbrella3=False, Umbrella4=True, Umbrella5=True}
Normalized f_1:1 = [0.81818182 0.18181818]
Normalized f_1:2 = [0.88335704 0.11664296]
Normalized f_1:3 = [0.19066794 0.80933206]
Normalized f_1:4 = [0.730794 0.269206]
Normalized f_1:5 = [0.86733889 0.13266111]

Final P(Rain at day 5) = 0.867
```

The code results for sequence e_{1:5} calculate P(X_5 | e_{1:5}) = 0.867.