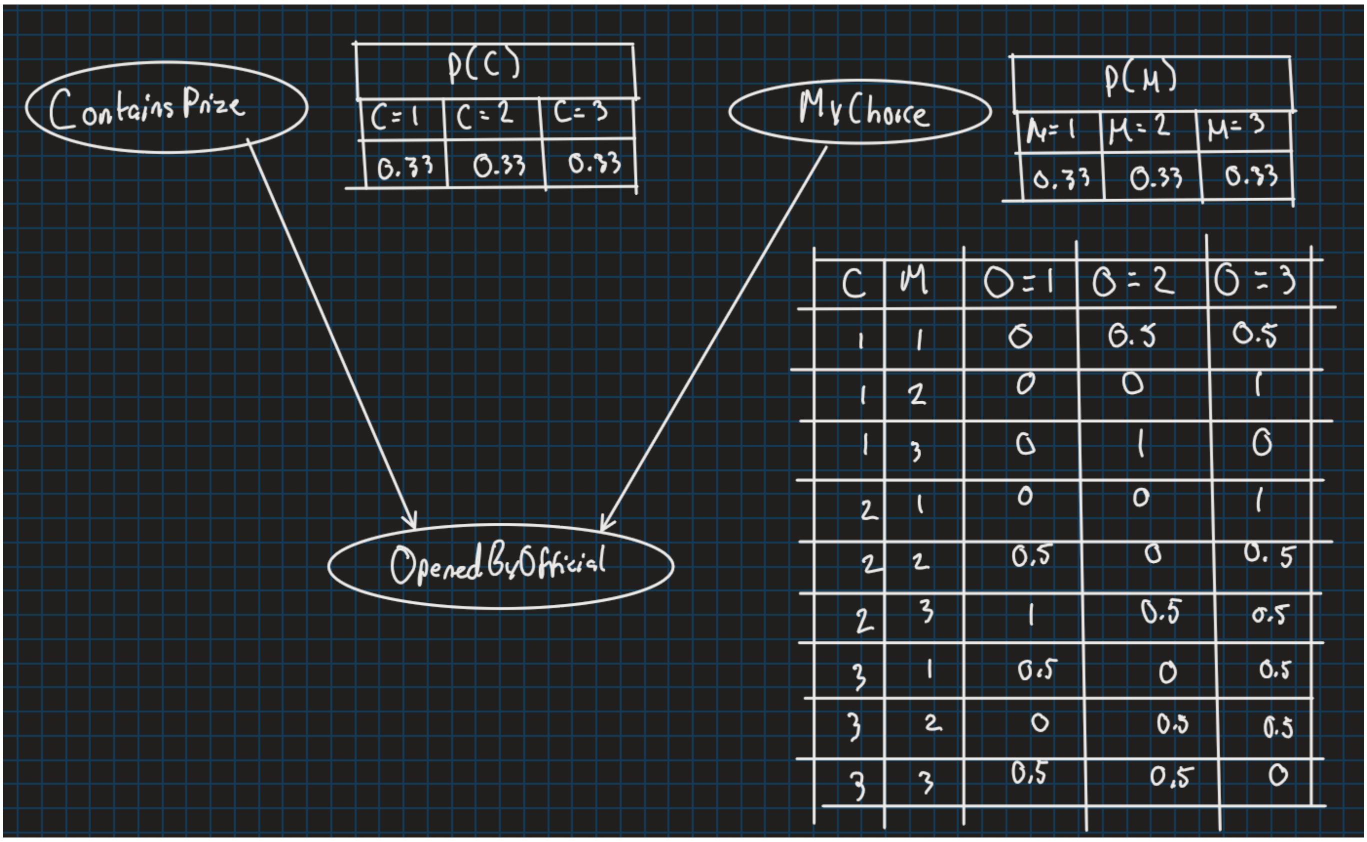
The Monty Hall Problem

Bayesian network



Bayesian network of Monty Hall Problem with conditonal dependencies and probability tables

Numerical evidence:

We calculate the probability in the case where $MyChoice = U$ and $OpenedByOfficial = V$.

We find the probability of $ContainsPrize = U$ (staying) versus $ContainsPrize = W$ (switching).

We use Bayes' Theorem:

$$P(C \mid O, M) = \frac{P(O \mid C, M) \cdot P(C \mid M)}{P(O \mid M)}$$

We find the denominator by summing all cases of C:

$$P(O = V \mid M = U)$$

$$= 0.5 \times \frac{1}{3} + 0 \times \frac{1}{3} + 1 \times \frac{1}{3} = \frac{1}{2}$$

Staying:

$$P(C = U \mid O = V, M = U)$$

$$= \frac{P(O = V \mid C = U, M = U) \cdot P(C = U \mid M = U)}{P(O = V \mid M = U)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{3}}{\frac{1}{2}} = \frac{1}{3}$$

Switching:

$$P(C = W \mid O = V, M = U)$$
$$= \frac{P(O = V \mid C = W, M = U) \cdot P(C = W \mid M = U)}{P(O = V \mid M = U)}$$
$$= \frac{1 \times \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

Conclusion:

The probability of winning if you switch to door W is $2/3$ whereas if you stay on door U the probability is $1/3$.

Thus if you would like to win, you should switch!!