

Exercise 1

a)

The number of distinct 5-card hands is the number of ways to choose 5 cards out of 52:

$$C(52, 5) = 52! / (5! 47!) = 2,598,960$$

b)

Each atomic event has equal probability:

$$1 / C(52, 5) = 1 / 2,598,960 \approx 3.85 \cdot 10^{-7}$$

c)

Royal straight flush:

- There are 4 suits -> 4 such hands

$$\text{Probability} = 4 / C(52, 5) = 1 / 649,740 \approx 1.539 \cdot 10^{-6}$$

- **Four of a kind:**

- Choose rank for the quadruple: 13 ways
- Choose remaining card: any of the other 48 cards

$$\text{Count} = 13 \cdot 48 = 624$$

$$\text{Probability} = 624 / C(52, 5) = 1 / 4,165 \approx 2.401 \cdot 10^{-4}$$

Exercise 2

a)

All 64 outcomes (4^3) are equally likely. Counting winning outcomes:

- **BAR/BAR/BAR**: 1 outcome -> 20
- **BELL/BELL/BELL**: 1 outcome -> 15
- **LEMON/LEMON/LEMON**: 1 outcome -> 5
- **CHERRY/CHERRY/CHERRY**: 1 outcome -> 3
- **CHERRY/CHERRY/?**: 3 outcomes -> 2
- **CHERRY/?/?**: $3 \cdot 4 = 12$ outcomes -> 1

$$\text{Total payout over all outcomes} = 20 + 15 + 5 + 3 + 3 \cdot 2 + 12 \cdot 1 = 61$$

$$\text{Expected return per play} = 61 / 64 \approx 0.953125 = 95.31\% \text{ payback}$$

b)

$$\text{Number of winning outcomes} = 1 + 1 + 1 + 1 + 3 + 12 = 19$$

$$\text{Probability(win)} = 19 / 64 \approx 0.296875 = 29.69\%$$

c)

```
python solution.py --slot
```

Summary of results (20,000 trials):

- Trials: 20000
- Starting coins: 10
- Mean plays until broke: 219.199
- Median plays until broke: 21

Exercise 3

Part 1

a)

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python solution.py --birthday
```

b)

Summary of results (20,000 trials):

- The smallest **N** such that the probability is at least 50% is 23 ($P \approx 0.506$ in the simulation).
- The proportion of **N** in the interval $[10, 50]$ for which the event happens with at least 50% chance is $28/41 \approx 0.683$.

Part 2

a)

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python solution.py --cover
```

Summary of results (20,000 trials):

- Mean number of people required: 2366.689
- Median number required: 2290