CSCI 340, Winter 2025
Math HW03

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1. Express the homogeneous 3D transformation defined by the matrix

$$M = \begin{pmatrix} 0 & -1 & 0 & 2 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

As a sequence of transformations in the following ways. Express your answer in English as well as two simpler transformation matrices.

(a) A rotation followed by a translation.

Answer:

Let, the given matrix be M and we want to express M as the product of a rotation followed by a translation where the rightmost matrix is applied first:

$$M = T(2,3,4)\,R_{\text{hom}}$$

where

$$R_{\text{hom}} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad T(2,3,4) = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$M = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 & 2 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Here, we first rotate every point by $+90°$ about the z-axis, then translate **the rotated point** by $(2,3,4)$

(b) A translation followed by a rotation.

Answer:

In this case,

$$M = R_{\text{hom}}\, T(\mathbf{t}')$$

where,

$$R\,t' = t$$

Since the inverse of R is its transpose,

$$R^{-1} = R^T = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

we compute

$$t' = R^{-1}\,t = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$$

Thus,

$$T(3,-2,4) = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

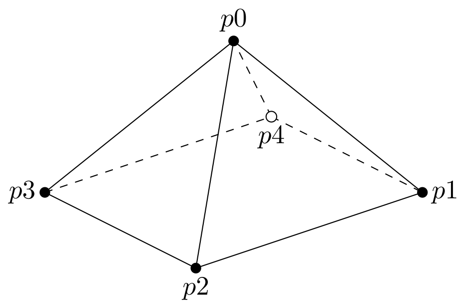
Now,

$$M = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 & 2 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Here, we first translate every point by $(3,-2,4)$, then rotate **the translated point** by $+90°$ about the z-axis.

2. The pyramid shown is represented by the following vertices:

$$\begin{aligned} p0 &: (0,1,0,0) \\ p1 &: (1,0,0,0) \\ p2 &: (0,0,1,0) \\ p3 &: (-1,0,0,0) \\ p4 &: (0,0,-1,0) \end{aligned}$$



We assume a standard OpenGL frame.

(a) Give an elements list that describes the top four triangles, being careful to make each of them face outward.

Answer: Using right-hand rule:

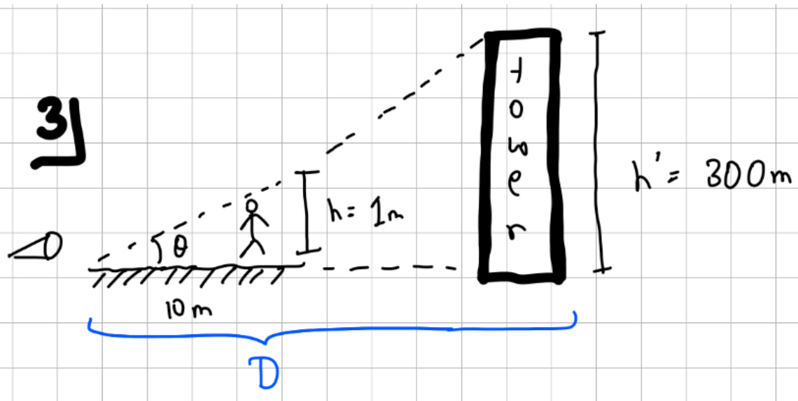
- **T1:** $[p0, p2, p1]$
- **T2:** $[p0, p3, p2]$
- **T3:** $[p0, p4, p3]$
- **T4:** $[p0, p1, p4]$

(b) Give an elements list that describes the bottom square as two triangles, both of which include the point numbered 1. Again, make sure they face outward (in this case, down).

Answer: Using right-hand rule:

- **T5:** $[p1, p2, p3]$
- **T6:** $[p1, p3, p4]$

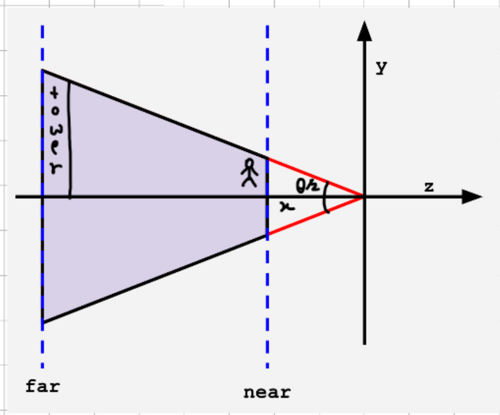
3]



$$\therefore \frac{1}{10} = \frac{300}{D}$$

$$\Rightarrow \boxed{D = 3000 \text{ m}} \quad [\text{Ans.}]$$

For open GL,



$$\therefore \tan\left(\frac{\theta}{2}\right) = \frac{1}{x}$$

$$\Rightarrow \frac{\theta}{2} = \tan^{-1}(1/x)$$

$$\Rightarrow \theta = 2\tan^{-1}(1/x) \quad [\text{Ans.}]$$

3. A camera is placed at $(5,0,0)$ looking at the origin. It's *up* vector is the y axis. What is the camera's *view* matrix?

Answer:

The view matrix derivation is explained in detail in [2]. From this, we get:

$$
\begin{aligned}
\mathbf{V} &= \mathbf{R}^{\mathrm{T}} \times \mathbf{T}^{-1} \\
\mathbf{V} &= \begin{bmatrix} r_x & r_y & r_z & 0 \\ u_x & u_y & u_z & 0 \\ f_x & f_y & f_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & -p_x \\ 0 & 1 & 0 & -p_y \\ 0 & 0 & 1 & -p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Figure 1: General formula for the view matrix

Here,

- $\mathbf{p} = (5,0,0)$
- $\mathbf{o} = (0,0,0)$
- $\mathbf{u} = (0,1,0)$

Now, to compute forward and right vector, from our textbook [1] we know:

```
def makeLookAt(position, target):
    worldUp = [0, 1, 0]
    forward = subtract( target, position )
    right = cross( forward, worldUp )
```

Figure 2: Computations for fwd and right vector

$$\mathbf{f} = norm(o - p) = (-1,0,0)$$

$$right, r = \mathbf{f} \times \mathbf{u} = (0,0,-1)$$

Placing all the above values in the general formula, after doing the matrix multiplication we get,

$$V = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

References

[1] Stemkoski, L., & Pascale, M. *Developing Graphics Frameworks with Python and OpenGL*.

[2] *View Matrix Derivation*, Two Dee Blog. Available at: `https://twodee.org/blog/17560`.