# CSCI 340, Winter 2025
# Math HW01

**Name**: *Abid Jeem*

Show your work. Explain, in each case, how you got a particular numerical result. You may, of course, use a computer for numeric computations, but which expressions you used to get each result must be made clear. For example, you can't just put down 12, you must put down $2^2+2^2+2^2 = 12$. You can't just put down 4.443, you must put down $\pi \cdot \sqrt{2} = 4.443$

If you have trouble writing neat math, consider learning and using LaTeX.

1. Let $v = \langle 2, 2, 1\rangle$ and $w = \langle 1, -2, 0\rangle$. Find the following:

   (a) $v \cdot w$

   **Answer:** Computing dot product we get —

   $$(2 * 1) + (2 * -2) + (1 * 0) = \boxed{-2} \tag{1}$$

   (b) The vector projection of $w$ on $v$.

   **Answer:** Computing projection formula—

   $$\left(\frac{w \cdot v}{v \cdot v}\right) v = \left(\frac{-2}{2^2 + 2^2 + 1^2}\right) \langle 2, 2, 1\rangle = \left(\frac{-2}{9}\right) \langle 2, 2, 1\rangle = \boxed{\langle -\tfrac{4}{9}, -\tfrac{4}{9}, -\tfrac{2}{9}\rangle} \tag{2}$$

2. Normalize each of the following vectors, in other words, make each vector unit length.

   (a) $v_1 = \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\rangle$

   **Answer:**

   $$\frac{v_1}{\|v_1\|} = \frac{\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\rangle}{\sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 + 0^2}} = \frac{\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\rangle}{1} = \boxed{\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\rangle} \tag{3}$$

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(b) $v_2 = \langle -1, 1, -1 \rangle$

**Answer:**

$$\frac{v_2}{\|v_2\|} = \frac{\langle -1, 1, -1 \rangle}{\sqrt{(-1)^2 + (1)^2 + (-1)^2}} = \frac{\langle -1, 1, -1 \rangle}{\sqrt{3}} = \boxed{\langle -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \rangle} \quad (4)$$

(c) $v_3 = \langle 0, -2, -2 \rangle$

**Answer:**

$$\frac{v_3}{\|v_3\|} = \frac{\langle 0, -2, -2 \rangle}{\sqrt{(0)^2 + (-2)^2 + (-2)^2}} = \frac{\langle 0, -2, -2 \rangle}{\sqrt{8}} = \boxed{\langle 0, -\frac{2}{\sqrt{8}}, -\frac{2}{\sqrt{8}} \rangle} \quad (5)$$

3. Find the cosine of the angle between the vectors $v = \langle 1, 2, 3 \rangle$ and $w = \langle 3, 2, 1 \rangle$

**Answer:**

Here, from the dot product equation

$$\cos(\theta) = \frac{v \cdot w}{\|v\|\|w\|} = \frac{(1)(3) + (2)(2) + (3)(1)}{\sqrt{(1)^2 + (2)^2 + (3)^2}\sqrt{(3)^2 + (2)^2 + (1)^2}} = \frac{10}{\sqrt{14}\sqrt{14}} = \frac{10}{14} = \boxed{\frac{5}{7}} \quad (6)$$

4. From the diagram, we get—

$$
\begin{aligned}
p &= (3,7) \\
q &= (9,3) \\
u &= (4-2, 1-1) = (2,0) \\
v &= (9-5, 5-9) = (4,-4) \\
w &= (7-5, 5-3) = (2,2)
\end{aligned}
$$

(a) The coordinates of $q$ in the f $= \langle p, u, v \rangle$:

$$
q = p + au + bv \quad \Rightarrow \quad \begin{aligned} q_x &= p_x + au_x + bv_x \\ q_y &= p_y + au_y + bv_y \end{aligned}
$$

Substituting the values:

$$
\begin{aligned}
q_x &= 9,\ p_x = 3,\ u_x = 2,\ v_x = 4 \\
q_y &= 3,\ p_y = 7,\ u_y = 0,\ v_y = -4
\end{aligned}
$$

Step-by-step calculations:

$$
\begin{aligned}
q_x = p_x + au_x + bv_x \quad &\Rightarrow \quad 9 = 3 + 2a + 4b \quad \Rightarrow \quad 6 = 2a + 4b \quad \Rightarrow \quad a + 2b = 3 \\
q_y = p_y + au_y + bv_y \quad &\Rightarrow \quad 3 = 7 + 0a - 4b \quad \Rightarrow \quad -4 = -4b \quad \Rightarrow \quad b = 1
\end{aligned}
$$

Substituting $b = 1$ into $a + 2b = 3$:

$$
a = 3 - 2(1) = 1
$$

Final result:

$$
\boxed{q = (1,1)_{\langle p,u,v \rangle}}
$$

(b) The coordinates of $q$ in the frame $\langle p, u, w \rangle$

$$
q = p + au + bw \quad \Rightarrow \quad \begin{aligned} q_x &= p_x + au_x + bw_x \\ q_y &= p_y + au_y + bw_y \end{aligned}
$$

Substituting the values:

$$
\begin{aligned}
q_x &= 9,\ p_x = 3,\ u_x = 2,\ w_x = 2 \\
q_y &= 3,\ p_y = 7,\ u_y = 0,\ w_y = 2
\end{aligned}
$$

Step-by-step calculations:

$$
\begin{aligned}
q_x = p_x + au_x + bw_x \quad &\Rightarrow \quad 9 = 3 + 2a + 2b \quad \Rightarrow \quad 6 = 2a + 2b \quad \Rightarrow \quad a + b = 3 \\
q_y = p_y + au_y + bw_y \quad &\Rightarrow \quad 3 = 7 + 0a + 2b \quad \Rightarrow \quad -4 = 2b \quad \Rightarrow \quad b = -2
\end{aligned}
$$

Substituting $b = -2$ into $a + b = 3$:

$$
a = 3 - (-2) = 3 + 2 = 5
$$

Final result:

$$
\boxed{q = (5,-2)_{\langle p,u,w \rangle}}
$$

(c) The coordinates of $q$ in the frame $\langle p, v, w \rangle$

$$q = p + av + bw \quad \Rightarrow \quad \begin{aligned} q_x &= p_x + av_x + bw_x \\ q_y &= p_y + av_y + bw_y \end{aligned}$$

Substituting the values:

$$\begin{aligned} &q_x = 9,\ p_x = 3,\ v_x = 4,\ w_x = 2 \\ &q_y = 3,\ p_y = 7,\ v_y = -4,\ w_y = 2 \end{aligned}$$

Step-by-step calculations:

$$\begin{aligned} q_x &= p_x + av_x + bw_x \quad &\Rightarrow \quad 9 = 3 + 4a + 2b \quad &\Rightarrow \quad 6 = 4a + 2b \quad &\Rightarrow \quad 2a + b = 3 \\ q_y &= p_y + av_y + bw_y \quad &\Rightarrow \quad 3 = 7 - 4a + 2b \quad &\Rightarrow \quad -4 = -4a + 2b \quad &\Rightarrow \quad 2a - b = 2 \end{aligned}$$

Adding the equations:

$$(2a + b) + (2a - b) = 3 + 2 \quad \Rightarrow \quad 4a = 5 \quad \Rightarrow \quad a = \frac{5}{4}$$

Substituting $a = \frac{5}{4}$ into $2a + b = 3$:

$$2\left(\frac{5}{4}\right) + b = 3 \quad \Rightarrow \quad \frac{10}{4} + b = 3 \quad \Rightarrow \quad b = 3 - \frac{10}{4} = \frac{12}{4} - \frac{10}{4} = \frac{2}{4} = \frac{1}{2}$$

Final result:

$$\boxed{q = \left(\frac{5}{4}, \frac{1}{2}\right)_{\langle p, v, w \rangle}}$$

(d) The coordinates of $p$ in the frame $\langle q, u, v \rangle$

$$p = q + au + bv \quad \Rightarrow \quad \begin{aligned} p_x &= q_x + au_x + bv_x \\ p_y &= q_y + au_y + bv_y \end{aligned}$$

Substituting the values:

$$\begin{aligned} &p_x = 3,\ q_x = 9,\ u_x = 2,\ v_x = 4 \\ &p_y = 7,\ q_y = 3,\ u_y = 0,\ v_y = -4 \end{aligned}$$

Step-by-step calculations:

$$\begin{aligned} p_x &= q_x + au_x + bv_x \quad &\Rightarrow \quad 3 = 9 + 2a + 4b \quad &\Rightarrow \quad -6 = 2a + 4b \quad &\Rightarrow \quad a + 2b = -3 \\ p_y &= q_y + au_y + bv_y \quad &\Rightarrow \quad 7 = 3 + 0a - 4b \quad &\Rightarrow \quad 4 = -4b \quad &\Rightarrow \quad b = -1 \end{aligned}$$

Substituting $b = -1$ into $a + 2b = -3$:

$$a + 2(-1) = -3 \quad \Rightarrow \quad a - 2 = -3 \quad \Rightarrow \quad a = -1$$

Final result:

$$\boxed{p = (-1, -1)_{\langle q, u, v \rangle}}$$



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(e) The coordinates of $p$ in the frame $\langle q, u, w\rangle$

$$p = q + au + bw \quad \Rightarrow \quad \begin{aligned} p_x &= q_x + au_x + bw_x \\ p_y &= q_y + au_y + bw_y \end{aligned}$$

Substituting the values:

$$\begin{aligned} &p_x = 3,\ q_x = 9,\ u_x = 2,\ w_x = 2 \\ &p_y = 7,\ q_y = 3,\ u_y = 0,\ w_y = 2 \end{aligned}$$

Step-by-step calculations:

$$\begin{aligned} p_x = q_x + au_x + bw_x \quad &\Rightarrow \quad 3 = 9 + 2a + 2b \quad \Rightarrow \quad -6 = 2a + 2b \quad \Rightarrow \quad a + b = -3 \\ p_y = q_y + au_y + bw_y \quad &\Rightarrow \quad 7 = 3 + 0a + 2b \quad \Rightarrow \quad 4 = 2b \quad \Rightarrow \quad b = 2 \end{aligned}$$

Substituting $b = 2$ into $a + b = -3$:

$$a + 2 = -3 \quad \Rightarrow \quad a = -3 - 2 \quad \Rightarrow \quad a = -5$$

Final result:

$$\boxed{p = (-5, 2)_{\langle q,u,w\rangle}}$$

(f) The coordinates of $p$ in the frame $\langle q, v, w\rangle$

$$p = q + av + bw \quad \Rightarrow \quad \begin{aligned} p_x &= q_x + av_x + bw_x \\ p_y &= q_y + av_y + bw_y \end{aligned}$$

Substituting the values:

$$\begin{aligned} &p_x = 3,\ q_x = 9,\ v_x = 4,\ w_x = 2 \\ &p_y = 7,\ q_y = 3,\ v_y = -4,\ w_y = 2 \end{aligned}$$

Step-by-step calculations:

$$\begin{aligned} p_x = q_x + av_x + bw_x \quad &\Rightarrow \quad 3 = 9 + 4a + 2b \quad \Rightarrow \quad -6 = 4a + 2b \quad \Rightarrow \quad 2a + b = -3 \\ p_y = q_y + av_y + bw_y \quad &\Rightarrow \quad 7 = 3 - 4a + 2b \quad \Rightarrow \quad 4 = -4a + 2b \quad \Rightarrow \quad -2a + b = 2 \end{aligned}$$

Adding the equations:

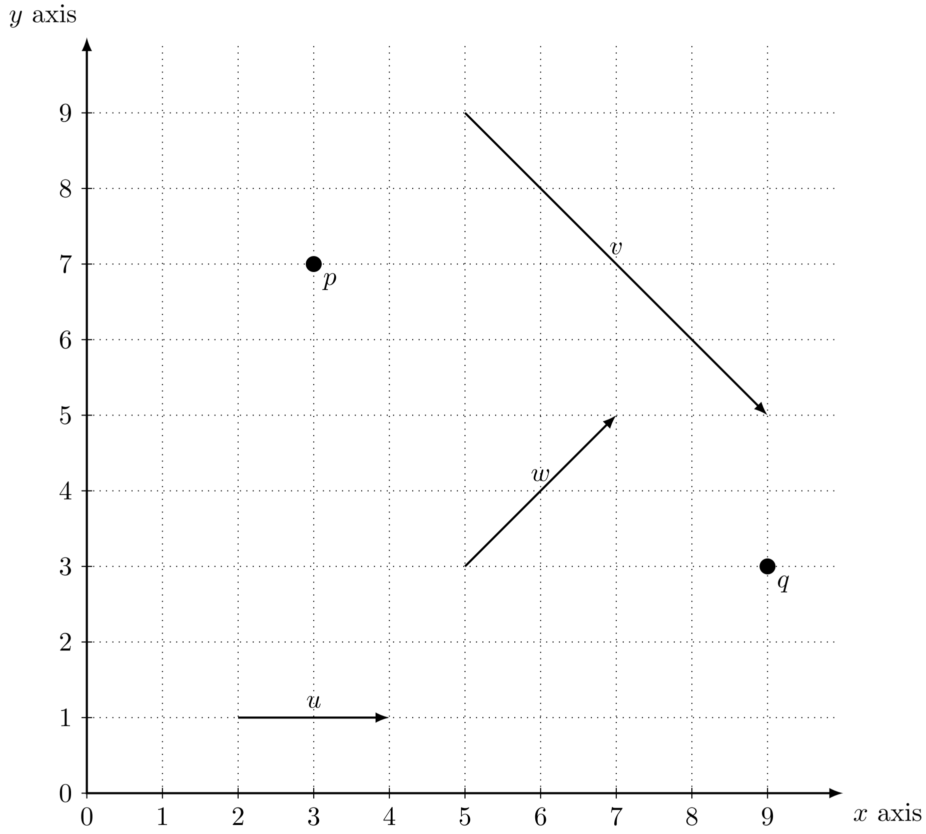
$$(2a + b) + (-2a + b) = -3 + 2 \quad \Rightarrow \quad 2b = -1 \quad \Rightarrow \quad b = -\frac{1}{2}$$

Substituting $b = -\frac{1}{2}$ into $2a + b = -3$:

$$2a - \frac{1}{2} = -3 \quad \Rightarrow \quad 2a = -3 + \frac{1}{2} \quad \Rightarrow \quad 2a = -\frac{6}{2} + \frac{1}{2} = -\frac{5}{2} \quad \Rightarrow \quad a = -\frac{5}{4}$$

Final result:

$$\boxed{p = \left(-\frac{5}{4}, -\frac{1}{2}\right)_{\langle q,v,w\rangle}}$$



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