Parochial by Construction: Dual Constructions for Cosmological Inference and Their Diagnostics

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Abstract

We develop a dual-construction framework for cosmological inference. *Temporal Evolution* (TE) treats the cosmos as a family of complete models indexed by age; *Causal Evolution* (CE) treats each observation as defining a causal completion with observer-indexed priors. We formalize CE as a context-penalized Gaussian prior, prove well-posedness, and derive closed-form shifts relative to TE in a rank-1 toy model. We define structural-zero diagnostics, give sensitivity estimates for six predictions accessible to present/future data, and outline simulation and multiple-testing protocols. The framework is falsifiable and useful whether the outcome is a null bound or a small, reproducible coupling.

Contents

1 Conceptual Core

Duality. TE and CE are non-reducible constructions. TE provides a parametrized family $U_{\mathrm{TE}}(\tau)$; CE provides $U_{\mathrm{CE}}(O)$ for observation O on worldline γ, with context encoded by templates and penalties. Local microphysics is shared; global reconstructions may differ.

2 Formal Framework

Let $a \in \mathbb{R}^n$ be primordial mode coefficients, $d = Ta + n$ with transfer T and noise $n \sim \mathcal{N}(0, N)$. A baseline prior is $a \sim \mathcal{N}(0, C_\tau)$.

2.1 TE posterior

$$\Sigma_{\mathrm{TE}} = (T^\top N^{-1} T + C_\tau^{-1})^{-1}, \qquad \mu_{\mathrm{TE}} = \Sigma_{\mathrm{TE}} T^\top N^{-1} d. \tag{1}$$

2.2 CE prior and posterior

Let $P_{\mathrm{ctx}} \succeq 0$ be the context projector (from Section 5). Define

$$\Pi_\gamma(a) \propto \exp\Big[-\tfrac{1}{2} a^\top (C_\tau^{-1} + \lambda_\gamma P_{\mathrm{ctx}}) a\Big]. \tag{2}$$

Then

$$\Sigma_{\mathrm{CE}} = (T^\top N^{-1} T + C_\tau^{-1} + \lambda_\gamma P_{\mathrm{ctx}})^{-1}, \qquad \mu_{\mathrm{CE}} = \Sigma_{\mathrm{CE}} T^\top N^{-1} d. \tag{3}$$

2.3 Well-posedness

Theorem 1 (Existence/Uniqueness). *If $N \succ 0$, $C_\tau \succ 0$, $P_{\mathrm{ctx}} \succeq 0$, and $\lambda_\gamma \geq 0$, the CE posterior exists and is unique up to standard gauge (e.g., monopole).*

Proof. The posterior precision matrix $T^\top N^{-1} T + C_\tau^{-1} + \lambda_\gamma P_{\mathrm{ctx}}$ is symmetric positive definite. Hence the Gaussian posterior is well-defined and unique. □

3 Toy Model: Rank-1 Context Prior (Explicit)

Let $P_{\mathrm{ctx}} = cc^\top / \|c\|^2$ with $c \neq 0$. Using Woodbury for a rank-1 update,

$$\Sigma_{\mathrm{CE}} = \Sigma_{\mathrm{TE}} - \frac{\lambda_\gamma\, \Sigma_{\mathrm{TE}} c c^\top \Sigma_{\mathrm{TE}}}{1 + \lambda_\gamma\, c^\top \Sigma_{\mathrm{TE}} c}, \tag{4}$$

$$\mu_{\mathrm{CE}} - \mu_{\mathrm{TE}} = (\Sigma_{\mathrm{CE}} - \Sigma_{\mathrm{TE}}) T^\top N^{-1} d = \lambda_\gamma\, \frac{(\mu_{\mathrm{TE}}^\top c)}{1 + \lambda_\gamma\, c^\top \Sigma_{\mathrm{TE}} c}\, \Sigma_{\mathrm{TE}} c. \tag{5}$$

Let $u = W\mu$ be an alignment statistic (e.g., low-ℓ projection). Then

$$\Delta u \equiv u_{\mathrm{CE}} - u_{\mathrm{TE}} = \lambda_\gamma\, \frac{(\mu_{\mathrm{TE}}^\top c)}{1 + \lambda_\gamma\, c^\top \Sigma_{\mathrm{TE}} c}\, W \Sigma_{\mathrm{TE}} c. \tag{6}$$

Thus CE departs from TE only along c, with amplitude $\mathcal{O}(\lambda_\gamma)$.

4 Information-Theoretic Derivation of the Penalty

Let $p_0(a) = \mathcal{N}(0, C_\tau)$ and T_{ctx} extract context features $z = T_{\mathrm{ctx}} a$. Consider

$$\min_p\ \mathrm{KL}(p \,\|\, p_0) + \lambda_\gamma\, \mathbb{E}_p \|T_{\mathrm{ctx}} a\|_2^2. \tag{7}$$

The solution in the Gaussian family is $p(a) = \mathcal{N}(0, \tilde{C})$ with precision

$$\tilde{C}^{-1} = C_\tau^{-1} + \lambda_\gamma\, T_{\mathrm{ctx}}^\top T_{\mathrm{ctx}}, \tag{8}$$

i.e., a quadratic penalty in the context subspace. For rank-1 T_{ctx} we recover P_{ctx}. A parallel decision-theoretic derivation obtains the same form by minimizing worst-case quadratic loss under context mis-specification.

5 Context Template Construction

Given exposure maps $E(\hat{n})$, scan harmonics, zodiacal templates $Z(\hat{n})$, and masks:

1. Build a feature matrix from the first few ecliptic-aligned spherical harmonics of E and Z.
2. Orthogonalize against known systematics (beam asymmetries, far sidelobes, Galactic residuals).
3. Normalize columns to unit variance; set T_{ctx} as the resulting basis.
4. For a minimalist test, take the leading mode c and set $P_{\mathrm{ctx}} = cc^\top/\|c\|^2$.

6 Diagnostics and Predictions with Sensitivity

P1 Polarization phase-locking (low-ℓ). Nonzero correlation between TE/EE phases and C_γ after cleaning. Sensitivity: Planck $\sim 10^{-2}$, LiteBIRD $\sim 10^{-3}$.

P2 Lensing–ISW commutator. Order non-commutativity $\Delta_{\mathrm{comm}}^{\phi\times T}$ is zero in ΛCDM, $\mathcal{O}(\lambda_\gamma)$ in CE. Sensitivity: CMB-S4 $\sim 10^{-3}$ of ISW amplitude.

P3 Off-ecliptic sign flip. A tilted scan should flip the sign of S_γ after the same orthogonalization. Binary outcome; not cosmic-variance limited.

P4 Parameter-path sensitivity. Context derivatives $G_\gamma^{(\tau)} \neq 0$ while $G_\gamma^{(\Omega_b h^2)} = 0$. Next-gen polarization can test $\mathcal{O}(10^{-3})$ shifts.

P5 Galaxy 2-point anisotropy. Tiny quadrupolar leakage aligned with C_γ in ultra-large scales; DESI+Euclid noise $\sim 2\times 10^{-3}$.

P6 JWST field variance. Extra 1–2% aligned variance in high-z counts across fields; detectable with $\mathcal{O}(10)$ deep fields.

7 Implications for H_0 Pathways

Context coupling alters TE-anchored inferences via low-ℓ anchoring and θ_* response, nudging CMB-only $\widehat{H_0}$ by

$$\Delta H_0^{(\mathrm{TE})} \approx \left(\partial H_0/\partial\theta_*\right)\lambda_\gamma \,\frac{(\mu_{\mathrm{TE}}^\top c)}{1+\lambda_\gamma\, c^\top \Sigma_{\mathrm{TE}} c}\,(L\Sigma_{\mathrm{TE}} c), \tag{9}$$

with L the linear response of θ_* to primordial modes. CE-invariant probes (distance ladder, BBN) should remain unchanged to first order.

8 Simulation and Multiple-Testing Protocol

Null. ΛCDM skies + beams + noise + zodiacal + masks; run full pipeline to each statistic.

Injection. Add known λ_{true} along P_{ctx}; verify recovery curves and false-positive rate.

Multiplicity. Pre-register a single template family; apply FDR if scanning K nearby modes; hold out a validation split.

9 Conclusion

The dual-construction framework yields concrete, low-variance diagnostics for context sensitivity with explicit, pre-registrable predictions. Nulls certify robustness; positives quantify a single coupling λ_γ. Either outcome is scientifically useful.