Program for Testing Lensing–ISW Commutator (P2)

September 29, 2025

Goal

Test whether the order of operations in CMB lensing and ISW reconstruction commute. Define

$$\Delta_{\text{comm}} \equiv \big[\widehat{\mathcal{S}}_{\phi \to T_{\text{ISW}}}\big] - \big[\widehat{\mathcal{S}}_{T_{\text{ISW}} \to \phi}\big], \tag{1}$$

which should vanish under ΛCDM with context-free priors, but can deviate proportionally to λ in context-dependent frameworks.

0. Pre-registration

- Masks: Planck PR4 common mask for T, lensing mask for $\hat{\phi}$.
- Multipoles: T in $2 \leq \ell \leq 64$, ϕ in $8 \leq L \leq 2048$.
- Frequency: SMICA as primary, WMAP ILC as cross-check.
- Estimator: Hu–Okamoto QE for ϕ, pseudo-C_ℓ with MASTER.
- Context template: ecliptic-aligned low-ℓ basis, pre-registered and fixed.
- Output: single Δ_{comm} and context-projected S_γ statistic.

1. Data Products

- CMB temperature maps: Planck PR4 SMICA, WMAP9 ILC.
- Lensing reconstructions: Planck PR4 $\hat{\phi}$ maps and QE on splits.
- Ancillary: exposure/scan maps, zodiacal templates, beam models.

2. Two Orderings

A→B (Lensing First)

1. Reconstruct $\hat{\phi}$ from small-scale T ($500 \leq \ell \leq 2048$).
2. Filter large-scale T for ISW proxy.
3. Compute $C_L^{A \to B} = \langle \hat{\phi}, T_{\text{ISW}} \rangle$.

B→A (ISW First)

1. Project large-scale T to ISW proxy.
2. Reconstruct $\hat{\phi}$ from remaining small-scale T.
3. Compute $C_L^{B\to A}$ identically.

Commutator Statistic

$$\Delta_{\text{comm}} = \sum_{L\in\mathcal{B}} w_L\big(C_L^{A\to B} - C_L^{B\to A}\big). \tag{2}$$

3. Context Coupling Overlay

$$S_\gamma = \frac{\sum_L w_L(C_L^{A\to B} - C_L^{B\to A})\,\Pi_L(c)}{\sqrt{\text{Var}}}. \tag{3}$$

S_γ should vanish in ΛCDM, nonzero under context coupling.

4. Splits & Replication

- Planck half-mission A/B, detset splits, 143 vs 217 GHz.
- WMAP ILC with Planck QE hybrid test.
- Hold out one split (HM-B) as validation.

5. Simulation Calibration

- Null suite: 1000 ΛCDM skies with beams, masks, QE, MASTER.
- Injection suite: add small rank-1 context perturbations to test λ response.
- Deliver expected $\sigma(\Delta_{\text{comm}})$, false-positive rate, recovery curves.

6. Systematics Triage

- Beam/FSL templates toggled.
- Zodiacal residuals included/excluded.
- Two fixed masks (baseline, conservative).
- Optional curl-mode estimator check.

7. Statistical Reporting

- Primary outcomes: $Z(\Delta_{\mathrm{comm}})$, $Z(S_\gamma)$.
- Multiplicity: one S_γ; FDR $q = 0.1$ if extra templates tested.
- Replication: consistent sign and $Z > 2$ across splits; held-out must pass.

8. Expected Sensitivity

Planck-class noise with $f_{\mathrm{sky}} \sim 0.7$ gives $\sigma(\Delta_{\mathrm{comm}}) \sim 10^{-3}$ fractional effect. Expect $Z \sim$ 1–2 per split, $Z \sim$ 2–3 combined. A null constrains λ to $< 10^{-3}$ at 95% CL.

9. Minimal Stack

- HEALPix/Healpy, Planck QE, MASTER, simulation suite.

10. Stop-loss Rules

- If variance $>$ 30% worse than forecast, publish null.
- If split signs inconsistent, declare null.
- If held-out fails, stop and publish bounds.

11. Paper Skeleton

1. Introduction.
2. Data & masks.
3. Methods: orderings, commutator, context templates.
4. Simulations.
5. Results.
6. Conclusion: detection or bound.

Related Work

Cross-correlations between CMB temperature anisotropies and the CMB lensing potential have been studied extensively as probes of the integrated Sachs–Wolfe (ISW) effect and structure growth. The *Planck* Collaboration reported detections of the ISW–lensing bispectrum at $\sim 2.5\sigma$ significance [1], and subsequent analyses have incorporated the $T \times \phi$ cross-spectrum into joint cosmological likelihoods [2]. Ground-based experiments (ACT, SPT) have also measured related cross-correlations with galaxy clustering and weak lensing surveys, producing competitive cosmological constraints and carrying out standard null tests such as rotated maps, curl-mode estimators, and split-half consistency checks [3, 4, 5].

These works establish the ISW–lensing correlation as a robust observable, but they treat the analysis as a single well-defined pipeline. Null tests are framed in terms of instrumental systematics or statistical fluctuations, rather than as algebraic consistency relations between different orderings of the analysis. Similarly, recent applications to modified gravity [6] and superstructure stacking [7] extend the cosmological parameter space, but do not interrogate the invariance of the inference pipeline itself.

By contrast, the present work introduces an explicit *commutator test*, comparing two orderings of the ISW–lensing cross-correlation (lensing-first versus ISW-first) and taking their difference as a diagnostic statistic. This framing isolates a class of potential biases that would otherwise remain hidden within conventional null tests. In addition, we project the commutator onto pre-registered context templates aligned with survey scanning geometry, providing a principled test of context sensitivity in cosmological inference. To our knowledge, this combination of algebraic order testing and context projection has not been previously implemented in the CMB analysis literature.

References

References

[1] Planck Collaboration. Planck 2015 results. XXI. The integrated Sachs–Wolfe effect. *Astronomy & Astrophysics* **594** (2016) A21. doi:10.1051/0004-6361/201525831. Available at arXiv:1502.01595.

[2] Carron, J., Lewis, A., & Fabbian, G. The Planck PR4 CMB lensing likelihood and the ISW effect. *MNRAS* **514** (2022) 2010–2024. doi:10.1093/mnras/stac1330. Available at arXiv:2209.07395.

[3] Hand, N., et al. First measurement of the cross-correlation of CMB lensing and galaxy lensing. *Phys. Rev. D* **91** (2015) 062001. doi:10.1103/PhysRevD.91.062001. Available at arXiv:1311.6200.

[4] Shaikh, S., et al. Cosmology from cross-correlation of ACT-DR4 CMB lensing with large-scale structure. *MNRAS* **528** (2024) 2112–2135. doi:10.1093/mnras/stae118. Available at arXiv:2306.01063.

[5] Kim, J., et al. ACT DR6 and DESI: structure formation over cosmic time with CMB lensing and luminous red galaxies. *JCAP* **02** (2024) 022. doi:10.1088/1475-7516/2024/02/022. Available at arXiv:2306.12344.

[6] Chudaykin, A., Kunz, M., & Carron, J. Constraints on Modified Gravity from the ISW–lensing cross-correlation in Planck PR4. *arXiv preprint* (2025). Available at arXiv:2503.09893.

[7] Hang, Q., et al. Stacked CMB lensing and ISW signals around superstructures. *MNRAS* **507** (2021) 510–523. doi:10.1093/mnras/stab2111. Available at arXiv:2105.09902.