Context-Dependent Bayesian Inference in Cosmology: Detecting and Quantifying Hidden Parochial Biases

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Abstract

We propose a methodological extension of cosmological inference in which priors are explicitly context-dependent. Standard pipelines assume that prior distributions on primordial modes are universal and independent of observational setup. In practice, scanning geometries, sky cuts, and noise properties embed subtle context. We formalize a framework in which priors are observer-indexed and penalize information-theoretic dependence on context templates. This induces predictable perturbations in posterior inferences, which can be tested through structural-zero diagnostics and additional null observables. We derive a closed-form toy model, outline well-posedness, and present sensitivity estimates for six predictions accessible to current and next-generation experiments. The framework is falsifiable: null results certify robustness against parochial bias; positive results reveal systematic dependence of cosmological inference on observational context.

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1 Problem Statement

1.1 Hidden assumption in current pipelines

Cosmological analyses typically assume a context-free, statistically isotropic prior on primordial modes. Observational context (scan patterns, masks, zodiacal emission, anisotropic noise) is treated as nuisance to be removed or marginalized, not as a structured dimension along which priors could couple.

1.2 Proposal

We introduce observer-indexed, context-penalized priors that allow weak coupling to a pre-registered template basis constructed from housekeeping data. This elevates "parochial effects" from afterthoughts to formally testable objects.

2 Framework

We consider a linear data model

$$d = Ta + n, \qquad n \sim \mathcal{N}(0, N), \tag{1}$$

where $a \in \mathbb{R}^n$ are primordial modes (e.g., spherical-harmonic coefficients), T is a transfer operator, and N is the noise covariance. The baseline prior is $a \sim \mathcal{N}(0, C_\tau)$, with τ denoting age/expansion parameters.

2.1 Temporal baseline (TE)

The standard posterior is

$$\Sigma_{\mathrm{TE}} = (T^\top N^{-1} T + C_\tau^{-1})^{-1}, \qquad \mu_{\mathrm{TE}} = \Sigma_{\mathrm{TE}} T^\top N^{-1} d. \tag{2}$$

2.2 Context-extended inference (CE)

Let $T_{\mathrm{ctx}} \in \mathbb{R}^{k\times n}$ be a template operator built from context (Section 4). Introduce the prior

$$\Pi_\gamma(a) \propto \exp\Big[-\tfrac{1}{2}\, a^\top (C_\tau^{-1} + \lambda_\gamma\, P_{\mathrm{ctx}}) a\Big], \qquad P_{\mathrm{ctx}} \equiv T_{\mathrm{ctx}}^\top T_{\mathrm{ctx}}, \tag{3}$$

where γ indexes the observer/experiment and $\lambda_\gamma \geq 0$ controls context sensitivity. The CE posterior is

$$\Sigma_{\mathrm{CE}} = (T^\top N^{-1} T + C_\tau^{-1} + \lambda_\gamma P_{\mathrm{ctx}})^{-1}, \qquad \mu_{\mathrm{CE}} = \Sigma_{\mathrm{CE}} T^\top N^{-1} d. \tag{4}$$

2.3 Well-posedness (Gaussian case)

Theorem 1 (Existence and uniqueness). *If $N \succ 0$, $C_\tau \succ 0$, $P_{\mathrm{ctx}} \succeq 0$, and $\lambda_\gamma \geq 0$, then the CE posterior exists and is unique up to standard gauge degeneracies.*

Sketch. The posterior precision $T^\top N^{-1} T + C_\tau^{-1} + \lambda_\gamma P_{\mathrm{ctx}}$ is symmetric positive-definite on the observable subspace; standard constraints fix null modes (e.g., monopole).

3 Principled derivation of the penalty

3.1 Information-theoretic derivation (MaxEnt with context bound)

Let $p_0(a) = \mathcal{N}(0, C_\tau)$ be the baseline prior. We penalize information flow from context to modes by the functional

$$\min_p \; \mathrm{KL}(p \,\|\, p_0) + \lambda_\gamma \, \mathbb{E}_p[\|T_{\mathrm{ctx}} a\|_2^2]. \tag{5}$$

Within Gaussians, the unique solution is $p(a) = \mathcal{N}(0, \tilde{C})$ with precision

$$\tilde{C}^{-1} = C_\tau^{-1} + \lambda_\gamma \, T_{\mathrm{ctx}}^\top T_{\mathrm{ctx}} = C_\tau^{-1} + \lambda_\gamma P_{\mathrm{ctx}}, \tag{6}$$

i.e., a quadratic penalty in the context subspace.

3.2 Decision-theoretic derivation (minimax mis-specification)

Assume quadratic loss $L(a, \hat{a}) = \|W(a - \hat{a})\|_2^2$ and an adversary that perturbs a within an ellipsoid $\{a : \|T_{\mathrm{ctx}} a\|_2^2 \leq \kappa\}$. The Bayes estimator under a augmented Gaussian prior that minimizes worst-case posterior risk yields the same precision augmentation $+\lambda_\gamma P_{\mathrm{ctx}}$ with λ_γ the Lagrange multiplier of the adversary's budget.

4 Context template construction

4.1 Inputs

Housekeeping and survey artifacts: exposure maps $E(\hat{n})$, scan harmonics tied to the ecliptic, zodiacal templates $Z(\hat{n})$, standard masks, beam asymmetry and far-sidelobe templates, and Galactic residual maps.

4.2 Algorithm (pre-registered)

1. **Feature basis:** build a design matrix from low-ℓ spherical harmonics of $E(\hat{n})$ and $Z(\hat{n})$ in ecliptic coordinates (and their products up to a fixed order).
2. **Orthogonalization:** regress out beam-asymmetry and far-sidelobe templates; orthogonalize remaining columns against Galactic residuals on the unmasked sky.
3. **Normalization:** scale each column to unit variance on the analysis mask.
4. **Select basis:** set T_{ctx} to the first k left-singular vectors (or fix $k = 1$ for a rank-1 test). Define $P_{\mathrm{ctx}} = T_{\mathrm{ctx}}^\top T_{\mathrm{ctx}}$.
5. **Freeze:** publish T_{ctx} and code that reproduces it from raw housekeeping; no tuning on cosmology maps.

5 Toy model: explicit CE–TE difference

For rank-1 $P_{\rm ctx} = cc^\top/\|c\|^2$, Woodbury gives

$$\Sigma_{\rm CE} = \Sigma_{\rm TE} - \frac{\lambda_\gamma\, \Sigma_{\rm TE} cc^\top \Sigma_{\rm TE}}{1 + \lambda_\gamma\, c^\top \Sigma_{\rm TE} c}, \tag{7}$$

$$\mu_{\rm CE} - \mu_{\rm TE} = (\Sigma_{\rm CE} - \Sigma_{\rm TE}) T^\top N^{-1} d = \lambda_\gamma\, \frac{(\mu_{\rm TE}^\top c)}{1 + \lambda_\gamma\, c^\top \Sigma_{\rm TE} c}\, \Sigma_{\rm TE} c. \tag{8}$$

For a low-ℓ alignment statistic $u = W\mu$,

$$\Delta u = u_{\rm CE} - u_{\rm TE} = \lambda_\gamma\, \frac{(\mu_{\rm TE}^\top c)}{1 + \lambda_\gamma\, c^\top \Sigma_{\rm TE} c}\, W \Sigma_{\rm TE} c. \tag{9}$$

6 Diagnostics and predictions (with sensitivities)

We define statistics that vanish under ΛCDM but are generically $\mathcal{O}(\lambda_\gamma)$ under CE.

P1 Polarization phase-locking (low-ℓ): correlation between TE/EE phases and $T_{\rm ctx}$ after cleaning. Sensitivity: Planck $\sim 10^{-2}$, LiteBIRD $\sim 10^{-3}$.

P2 Lensing–ISW commutator: order non-commutativity $\Delta_{\rm comm}^{\phi\times T}$ zero in ΛCDM, $\mathcal{O}(\lambda_\gamma)$ in CE. Sensitivity: CMB-S4 $\sim 10^{-3}$ of ISW amplitude.

P3 Off-ecliptic sign flip: a tilted scan should flip the sign of a pre-registered S_γ; binary outcome, not cosmic-variance limited.

P4 Parameter-path sensitivity: context derivatives $G_\gamma^{(\tau)} \neq 0$ while $G_\gamma^{(\Omega_b h^2)} = 0$. Next-gen polarization can test $\mathcal{O}(10^{-3})$ shifts.

P5 Galaxy 2-point anisotropy: tiny quadrupolar leakage aligned with $T_{\rm ctx}$ on ultra-large scales; DESI+Euclid variance $\sim 2 \times 10^{-3}$.

P6 JWST field variance: extra 1–2% aligned variance in high-z counts across independent fields; detectable with $\mathcal{O}(10)$ fields.

7 Implications for H_0 pathways

CMB/BAO inference obtains H_0 via the acoustic angle $\theta_* = r_s(z_*)/D_A(z_*)$. Context coupling perturbs the low-ℓ anchoring and hence θ_*:

$$\Delta\theta_* \approx L\,(\mu_{\rm CE} - \mu_{\rm TE}) = \lambda_\gamma\, \frac{(\mu_{\rm TE}^\top c)}{1 + \lambda_\gamma\, c^\top \Sigma_{\rm TE} c}\, L \Sigma_{\rm TE} c, \tag{10}$$

where L is the linear response operator. Thus

$$\Delta H_0^{(\rm TE)} \approx \Big(\partial H_0/\partial\theta_*\Big)\, \Delta\theta_*, \tag{11}$$

while CE-invariant late-universe estimates (distance ladder, BBN) remain unchanged at first order.

8 Simulation demonstration and calibration

8.1 Null suite

Generate ΛCDM skies; add beams, anisotropic noise, zodiacal emission, masks; propagate through the full pipeline to each statistic. Use these to calibrate null distributions and Z-scores.

8.2 Injection suite

Inject a known λ_{true} along P_{ctx} at the primordial level; verify recovery of sign and amplitude for each statistic; estimate false-positive rate and uncertainty inflation when projecting out P_{ctx}. Release code and seeds.

9 Comparison to existing robustness diagnostics

Jackknifes, null maps, and cross-spectra catch gross systematics but lack a single, parameter-level coupling to context. The CE framework provides: (i) an explicit hyperparameter λ_γ, (ii) pre-registered templates, and (iii) low-variance diagnostics (commutator, sign-flip) that are difficult to fake by random mask choices.

10 Multiple testing control

Scanning many templates risks false positives. We adopt:

- Pre-registration of a single template family derived from housekeeping; no tuning on cosmology maps.
- If K nearby modes are tested, control family-wise error (Bonferroni) or use FDR with declared q.
- Hold out a validation split (e.g., half-mission B) untouched until final confirmation.

11 Empirical payoff

Nulls certify that pipelines are robust to context at the 10^{-3} level and yield upper bounds on λ_γ. Non-nulls reveal reproducible, sign-fixed context dependence, guiding re-analysis and instrument design (e.g., off-ecliptic scanning).

12 Conclusion

We recast cosmological inference to allow controlled, testable prior coupling to observational context. The program is modest in ontology, sharp in falsifiability, and useful regardless of outcome: either certify robustness or quantify a small, coherent bias with a single hyperparameter.