4. Выраз. через Эйлер. инт. и опр. область сущ.

$$\int_0^{+\infty} x^m e^{-x^n} dx = \left[\begin{matrix} x^n = t & dx = \frac{1}{n} t^{\frac{1}{n}-1} \\ x = t^{\frac{1}{n}} & \end{matrix}\right] =$$

$$= \frac{1}{n}\int_0^{+\infty} t^{\frac{m}{n}} e^{-t} \cdot t^{\frac{1}{n}-1} dt =$$

$$= \frac{1}{n}\int_0^{+\infty} t^{\frac{m+1}{n}-1} e^{-t} dt = \frac{1}{n}\Gamma\left(\frac{m+1}{n}\right)$$

$$\frac{m+1}{n} > 0 \quad \begin{cases} m+1 > 0 \\ n > 0 \end{cases} \text{ или } \begin{cases} m+1 < 0 \\ n < 0 \end{cases}$$

$$\begin{cases} m > -1 \\ n > 0 \end{cases} \text{ или } \begin{cases} m < -1 \\ n < 0 \end{cases}$$

5. $$\int_0^{+\infty} e^{-x^4} dx \cdot \int_0^{+\infty} x^2 e^{-x^4} dx = \left[\begin{matrix} x^4 = t, & dx = \frac{1}{4} t^{-\frac{3}{4}} \\ x = t^{\frac{1}{4}} & \end{matrix}\right]$$

$$= \frac{1}{16}\int_0^{+\infty} t^{-\frac{3}{4}} e^{-t} dt \cdot \int_0^{+\infty} t^{\frac{1}{2}-\frac{3}{4}} e^{-t} dt =$$

$$= \frac{1}{16}\int_0^{+\infty} t^{-\frac{3}{4}} e^{-t} dt \cdot \int_0^{+\infty} t^{-\frac{1}{4}} e^{-t} = \left[\begin{matrix} a_1 - 1 = -\frac{3}{4} \\ a_2 - 1 = -\frac{1}{4} \end{matrix}\right]$$

$$= \frac{1}{16} \cdot \Gamma\left(1-\frac{3}{4}\right) \cdot \Gamma\left(1-\frac{1}{4}\right) = \frac{1}{16}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) =$$

$$= \frac{1}{16} \frac{\pi}{\sin\frac{\pi}{4}} = \frac{\pi}{8\sqrt{2}}$$

1. Дз №20

$$I = \int_0^{+\infty} \frac{\sqrt[4]{x}}{(1+x)^2}\,dx = \int_0^{+\infty} \frac{x^{\frac{1}{4}}}{(1+x)^2}\,dx$$

$$\left[\begin{array}{ll} a-1=\frac{1}{4} & a=\frac{5}{4} \\ a+b=2 & b=2-\frac{5}{4}=\frac{3}{4} \end{array}\right] = B\left(\frac{5}{4};\frac{3}{4}\right) =$$

$$= \frac{1}{4}B\left(\frac{1}{4};\frac{3}{4}\right) = \frac{1}{4}\cdot\frac{\pi}{\sin\frac{1}{4}\pi} = \frac{1}{4}\cdot\pi\cdot\frac{2}{\sqrt{2}} = \frac{\pi}{2\sqrt{2}}$$

2. $$\int_0^1 \frac{dx}{\sqrt[n]{1-x^n}} = \left[\begin{array}{ll} t=x^n & dx=\frac{1}{n}\cdot t^{\frac{1}{n}-1}dt \\ x=\sqrt[n]{t} & \end{array}\right] =$$

$$= \frac{1}{n}\int_0^1 \frac{t^{\frac{1}{n}-1}}{(1-x)^{\frac{1}{n}}}\,dt = \frac{1}{n}\int_0^1 t^{\frac{1}{n}-1}(1-x)^{-\frac{1}{n}} = \left[\begin{array}{l} a=\frac{1}{n} \\ b-1=-\frac{1}{n} \\ b=1-\frac{1}{n} \end{array}\right]$$

$$\frac{1}{n}B\left(\frac{1}{n},1-\frac{1}{n}\right) = \frac{1}{n}\frac{\pi}{\sin\frac{\pi}{n}}$$

3. $$\int_0^1 \frac{dx}{\sqrt[n]{1-x^m}},\ m>0 \quad \left[\begin{array}{ll} x^m=t & dx=\frac{1}{m}t^{\frac{1}{m}-1} \\ x=t^{\frac{1}{m}} & \end{array}\right]$$

$$= \frac{1}{m}\int_0^1 t^{\frac{1}{m}-1}(1-t)^{-\frac{1}{n}}\,dt = \left[\begin{array}{ll} a=\frac{1}{m} & \\ b-1=-\frac{1}{n} & b=1-\frac{1}{n} \end{array}\right] =$$

$$= \frac{1}{m}B\left(\frac{1}{m},1-\frac{1}{n}\right) = \frac{1}{m}\cdot\frac{\Gamma\left(\frac{1}{m}\right)\cdot\Gamma\left(1-\frac{1}{n}\right)}{\Gamma\left(\frac{1}{m}+1-\frac{1}{n}\right)} =$$