$$\|x\|_\infty = 1 = \max |x_i|.$$

$$\|AB\| \le \|A\| \cdot \|B\|$$

$$\|A\| = \sup_{\|x\| \ne 0} \frac{\|Ax\|}{\|x\|}$$

$$\|A\|_1 = \max_j \left(\sum_{i=1}^{n} |a_{ij}| \right).$$ sum столбец

$$\|A\|_\infty = \max_i \left(\sum_{j=1}^{n} |a_{ij}| \right).$$ sum строка

Нормой вектора $x = (x_1 \dots x_n)^T$ наз. функционал:

1. $\|x\| > 0 \quad \forall x \ne 0$
2. $\|\alpha x\| = |\alpha| \cdot |x|. \quad \forall \alpha, x$
3. $\|x + y\| \le \|x\| + \|y\|. \quad \forall x, y$

$$\left\{\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 12 \\ 2 & 5 & 8 & 11 & 31 \\ 3 & 8 & 14 & 20 & 54 \\ 4 & 11 & 20 & 30 & 78 \end{array}\right.$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \end{pmatrix}$$

$1 \quad 2 \quad 3 \quad 4$

$=$

$$\left(\begin{array}{cccc|c} 2 & 1 & 3 & 1 & 6 \\ 4 & 5 & 8 & 3 & 22 \\ 6 & 9 & 14 & 7 & 40 \\ 8 & 13 & 20 & 12 & 59 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 2 & 1 & 3 & 1 & 6 \\ 0 & 3 & 2 & 1 & 10 \\ 0 & 0 & 1 & 2 & 7 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right)$$

$$\begin{array}{ccc} 2 & & \\ 3 & 2 & \\ 4 & 3 & 2 \end{array} \qquad x = \begin{pmatrix} 1 \\ 3 \\ 0 \\ 1 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 3 \\ 3 & 6 \end{pmatrix} \qquad \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} =$$

$$\left(\begin{array}{cccc|c} 2 & 1 & 3 & 1 & 5 \\ 6 & 6 & 11 & 4 & 22 \\ 2 & 7 & 8 & 5 & 21 \\ 2 & 7 & 9 & 8 & 24 \end{array}\right)$$

X − ?

L − ?

R − ?

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 2 & 2 & 1 \end{pmatrix}$$

$\downarrow$

$$\left(\begin{array}{cccc|c} 2 & 1 & 3 & 1 & 5 \\ 0 & 3 & 2 & 1 & 7 \\ 0 & 6 & 5 & 4 & 16 \\ 0 & 6 & 6 & 7 & 19 \end{array}\right) = \left(\begin{array}{cccc|c} 2 & 1 & 3 & 1 & 5 \\ 0 & 3 & 2 & 2 & \cancel{7} \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 2 & 3 & 5 \end{array}\right)$$

3 $\downarrow$

$$= \left(\begin{array}{cccc|c} 2 & 1 & 3 & 1 & 5 \\ 0 & 3 & 2 & 2 & 7 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 3 & \cancel{2}\ 1 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 2 & 1 & 3 & 1 & 5 \\ 0 & 3 & 2 & 1 & 7 \\ 0 & 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 5 & 5 \end{array}\right)$$

$$x = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}$$

$\downarrow$

0 0 0 1

$$\begin{array}{ccccc} 2 & 1 & 3 & 1 & 5 \\ 0 & 3 & 2 & 1 & 7 \\ 0 & 0 & 1 & 2 & 2 \end{array}$$

$A = QR$

$$\begin{pmatrix} 3 & 1 & -1 \\ 1 & 2 & -1 \\ -1 & -1 & 4 \end{pmatrix}$$

$$b_1 = a_1 = \begin{vmatrix} 3 \\ 1 \\ -1 \end{vmatrix}$$

$$b_2 = a_2 - \frac{a_2 \cdot b_1}{b_1 \cdot b_1}$$

1) $a_k = \sum_{j=1}^{k} r_{jk} \cdot q_j$

2) $a_1 \neq 0 \Rightarrow q_1 = a_1 / |a_1| = a_1 / r_1$

3) $q_j \in (1; k-1)$ $\Rightarrow$ 3) $\boxed{r_{kk} q_k = a_k - \sum_{j=1} r_{jk} q_j}$

$= b_k$

$r_{jk} = q_j^T \cdot a_k$

4)

$$r_{kk} = |b_k|;$$

$$q_k = \frac{b_k}{r_{kk}}.$$

$$q_1 = \begin{vmatrix} 3 \\ 1 \\ -1 \end{vmatrix} / \sqrt{11} = \frac{1}{\sqrt{11}} \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$$

$$r_{12} = q_1^T \cdot a_2 = \frac{1}{\sqrt{11}} \left(\cancel{3 \; 1 \; -1} \right) \times \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \frac{6}{\sqrt{11}}$$

$$b_2 = a_2 - r_{12} q_1 = \begin{pmatrix}1\\2\\-1\end{pmatrix} - \frac{6}{11}\begin{pmatrix}3\\1\\-1\end{pmatrix} =$$

$$= \frac{1}{11}\begin{pmatrix}-7\\16\\-5\end{pmatrix}$$

$$|b_2| = \frac{1}{11}\sqrt{330}$$

$$q_2 = \frac{1}{\sqrt{330}}\begin{pmatrix}-7\\16\\-5\end{pmatrix}$$

$$r_{13} = q_1^T \cdot a_3 = \frac{1}{\sqrt{11}}\begin{pmatrix}3\\1\\-1\end{pmatrix}^T \cdot \begin{pmatrix}-1\\-1\\4\end{pmatrix} =$$

$$= \frac{-8}{\sqrt{11}}$$

$$r_{23} = q_2^T a_3 = \frac{1}{\sqrt{330}}\begin{pmatrix}-7 & 16 & -5\end{pmatrix} \times \begin{pmatrix}-1\\-1\\4\end{pmatrix} =$$

$$= \frac{-29}{\sqrt{330}}$$

$$b_3 = r_{33} \cdot q_3 = a_3 - r_{13} q_1 - r_{23} q_2$$

$$\begin{pmatrix}-1\\-1\\4\end{pmatrix} + \frac{8}{11}\begin{pmatrix}3\\1\\-1\end{pmatrix} + \frac{29}{\sqrt{330}}\begin{pmatrix}-7\\16\\-5\end{pmatrix} =$$

$$= \frac{1}{330}\left(\begin{pmatrix}-330\\-330\\1320\end{pmatrix} + \begin{pmatrix}\overset{720}{\cancel{840}}\\240\\-240\end{pmatrix} + \begin{pmatrix}203\\464\\-175\end{pmatrix}\right) =$$

$$= \frac{1}{330}\begin{pmatrix}\cancel{713}\\374\\935\end{pmatrix}^{593} \cdot \quad r_3 = |b_3| = \frac{\sqrt{1522470}}{330}$$

$$q_3 = \frac{1}{\sqrt{}}\begin{pmatrix}713\\374\\935\end{pmatrix}$$

$$\sum_{n=1}^{\infty} \frac{n}{5^n} = \frac{1}{5} + \frac{2}{5^2} + \frac{3}{5^3} + \dots$$

рассмотрим

$$f(x) = \sum n x^{n-1}.$$

[redacted] и вычислим $f(1/5)$.

$$\int f(x)\,dx = \sum \int n x^{n-1}\,dx = \sum_{n=1}^{\infty} x^n + C.$$

$$f(1/5) = \sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n$$

$$A = \begin{pmatrix} 6 & 6 & 1 \\ \underline{3} & 6 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

QR — разл

плоские вращения

$$\Omega^{(1,2)} = \begin{pmatrix} \cos & \cos & 0 \\ \sin & -\sin & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$p = 1$

$q = 2$

$a_{pp} = 6$

$a_{qp} = 3$

$\sin\ 1/\sqrt{5}$

$$\cos = \frac{a_{pp}}{\sqrt{a_{pp}^2 + a_{qp}^2}}$$

$$\sin = \frac{a_{qp}}{}$$

$$\cos = \frac{6}{\sqrt{45}} = 2/\sqrt{5}$$

$\Omega^{12} \cdot A$

$\cancel{6} \quad \cancel{6} \quad \cancel{1}$

$\cancel{3}$

$$
\begin{array}{ccc}
\frac{2}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \\
\multicolumn{3}{c}{\begin{pmatrix} 4 & 0 & 5 \\ 4 & 6 & 2 \\ -2 & 3 & 5 \end{pmatrix}}
\end{array}
$$

$$a_{pp} = a_{qp} = 4$$

$$c = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}} = s$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 & 5 \\ 4 & 6 & 2 \\ -2 & 3 & 5 \end{pmatrix} =$$

$$= \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 8 & 6 & 7 \\ 0 & 6 & -3 \\ -2\sqrt{2} & 3\sqrt{2} & 5\sqrt{2} \end{pmatrix}.$$

$$a_{pp} = 8/\sqrt{2} \qquad c = \frac{8/\sqrt{2}}{\sqrt{32+4}} = \frac{8}{6\sqrt{2}} = \frac{4}{3\sqrt{2}}$$

$$a_{qp} = -2 \qquad s = \frac{-2}{6} = -\frac{1}{3}$$

$$\frac{1}{3} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} \cancel{4/\sqrt{2}}\ 2\sqrt{2} & \cancel{-1} & 0 \\ \cancel{1} & \cancel{4/\sqrt{2}} & 0 \\ 0 & 0 & 3 \end{pmatrix} \cdot \begin{pmatrix} 8 & 6 & 7 \\ 0 & 6 & -3 \\ -2\sqrt{2} & 3\sqrt{2} & 5\sqrt{2} \end{pmatrix} = \Bigl(.$$

$$\frac{1}{3}\,\frac{1}{\sqrt{2}}\begin{pmatrix} 2\sqrt{2} & 0 & -1 \\ 0 & 3 & 0 \\ 1 & 0 & 2\sqrt{2} \end{pmatrix}\begin{pmatrix} 8 & 6 & 7 \\ 0 & 6 & -3 \\ -2\sqrt{2} & 3\sqrt{2} & 5\sqrt{2} \end{pmatrix} =$$

$$= \frac{1}{6}\begin{pmatrix} \overset{6}{\cancel{}}\sqrt{2} & \overset{3}{\cancel{9}}\sqrt{2} & \overset{3}{\cancel{9}}\sqrt{2} \\ 0 & 6 & -3 \\ 0 & 6 & 9 \end{pmatrix}$$