AE588 Assignment 4 - akshatdy

4.1

Maximize

$$f(x_1, x_2) = x_1 x_2$$

Subject to

$$h(x_1, x_2) = 2x_1 + 2x_2 - p = 0$$

Lagrangian of the problem:

$$\mathcal{L}(x_1, x_2, \lambda) = x_1 x_2 - \lambda(2x_1 + 2x_2 - p)$$

Differentiating this to get the first-order optimality conditions

$$\frac{\partial \mathcal{L}}{\partial x_1} = x_2 - 2\lambda = 0 \frac{\partial \mathcal{L}}{\partial x_2} = x_1 - 2\lambda = 0 \frac{\partial \mathcal{L}}{\partial \lambda} = -2x_1 - 2x_2 + p = 0$$

Solving these three equations for the three unknowns (x_1, x_2, λ), we obtain:

$$x_A = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} p/4 \\ p/4 \end{bmatrix}, \lambda_A = \frac{p}{8}$$

Checking the Hessian

$$H = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

2nd order check passes with the Hessian

The units of the lambda is m^2/m, which is the change of area (in m^2) of the rectangle for a given a change in the perimeter constraint in m

4.2

Constants:

$$
\begin{aligned}
h &= 250mm = .25m \\
b &= 125mm = .125m \\
\sigma_{yield} &= 200MPa = 200000000Pa \\
\tau_{yield} &= 116MPa = 116000000Pa \\
P &= 100kN = 100000N \\
l &= 1m
\end{aligned}
$$

Variables

$$t_b = x_1 t_w = x_2$$

Minimize

$$f(x_1, x_2) = 2bx_1 + hx_2$$

Given second moment of area

$$
\begin{aligned}
I &= \frac{h^3}{12}x_2 + \frac{b}{6}x_1^3 + \frac{h^2 b}{2}x_1 \\
\implies I &= \frac{.25^3}{12}x_2 + \frac{.125}{6}x_1^3 + \frac{.25^2.125}{2}x_1 \\
\implies I &= \frac{x_2}{768} + \frac{x_1^3}{48} + \frac{x_1}{256} \\
\implies I &= \frac{x_2 + 16x_1^3 + 3x_1}{768} \\
\implies \frac{1}{I} &= \frac{768}{3x_1 + 16x_1^3 + x_2}
\end{aligned}
$$

Subject to

$$
\begin{aligned}
g_1(x_1, x_2) &= \frac{Plh}{2I} - \sigma_{yield} \leq 0 \\
g_2(x_2) &= \frac{1.5P}{hx_2} - \tau_{yield} \leq 0
\end{aligned}
$$

The Lagrangian for this problem is

$$\mathcal{L}(x, \sigma, s) = 2bx_1 + hx_2 + \sigma_1(\frac{Plh}{2I} - \sigma_{yield} + s_1^2) + \sigma_2(\frac{1.5P}{hx_2} - \tau_{yield} + s_2^2)$$

Differentiating the Lagrangian with respect to all the variables, we get the first-order optimality conditions,

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x_1} &= 2b - \frac{384\sigma_1 Plh(3+48x_1^2)}{(3x_1+16x_1^3+x_2)^2} &&= 0\\
\frac{\partial \mathcal{L}}{\partial x_2} &= h - \frac{384\sigma_1 Plh}{(3x_1+16x_1^3+x_2)^2} - \frac{1.5\sigma_2 P}{hx_2^2} &&= 0\\
\frac{\partial \mathcal{L}}{\partial \sigma_1} &= \frac{384Plh}{3x_1+16x_1^3+x_2} - \sigma_{yield} + s_1^2 &&= 0\\
\frac{\partial \mathcal{L}}{\partial \sigma_2} &= \frac{1.5P}{hx_2} - \tau_{yield} + s_2^2 &&= 0\\
\frac{\partial \mathcal{L}}{\partial s_1} &= 2\sigma_1 s_1 &&= 0\\
\frac{\partial \mathcal{L}}{\partial s_2} &= 2\sigma_2 s_2 &&= 0
\end{aligned}
$$

Roots of the equation found with scipy.optimize.fsolve, with initial guesses of $t_b(x_1), t_w(x_2)$ as $10cm(0.01m)$ by eyeballing the diagram, and letting $\sigma_1 = \sigma_2 = s_1 = s_2 = 0$

$$
\begin{aligned}
t_b &= 1.42603955e-02 &&\approx 14cm\\
t_w &= 5.17241379e-03 &&\approx 5cm\\
\sigma_1 &= 1.99351362e-11 &&\approx 0\\
\sigma_2 &= 7.44367855e-12 &&\approx 0\\
s1 & &&= 0\\
s2 & &&= 0
\end{aligned}
$$

Graphical verification

Figure 1: 4.2 Graph

4.3

4.3.a

Exterior:

- Max Penalty: 2167979.329
- Distance to optimum: 1.17e-06
- Optimal point: Optimial point is slightly infeasible
- Starting points tried (all managed to converge)
  - [-2, -1]
  - [-2, -2]
  - [2, 1]
  - [2, 2]

Interior:

- Min Penalty parameter: 1.79e-07
- Closeness to optimum: 2.53e-07
- Optimal point: Optimial point is feasible
- Starting points tried (all managed to converge)
  - [-1, 0]
  - [0, 0.5]
  - [-1, -0.5]
  - [0, 0]

4.3.b

Solutions:

- Exterior: ~0.014 (14cm)
- Interior: ~0.005 (5cm)

Solutions are the same as the analytical solutions

4.4

I was not able to implement a working Quasi-Newton SQP algorithm, it would get stuck after a couple of iterations, even though the first iteration hessian and lambdas matched example 5.12 exactly.

here is the optimization path for Example 5.4

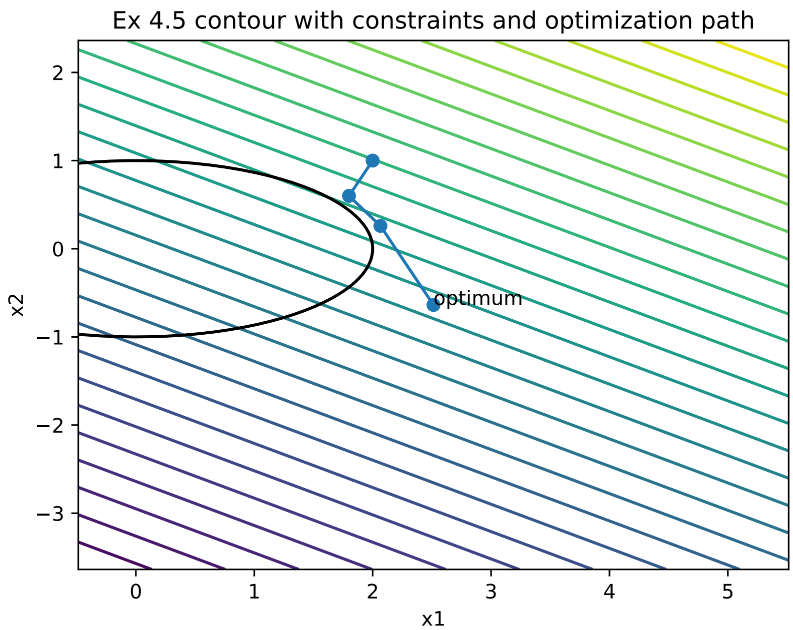

Figure 1: 4.2 Graph