# AE588 Assignment 4 - akshatdy

### 4.1

Maximize

$$f(x_1, x_2) = x_1 x_2$$

Subject to

$$h(x_1, x_2) = 2x_1 + 2x_2 - p = 0$$

Lagrangian of the problem:

$$\mathcal{L}(x_1, x_2, \lambda) = x_1 x_2 - \lambda(2x_1 + 2x_2 - p)$$

Differentiating this to get the first-order optimality conditions

$$\frac{\partial \mathcal{L}}{\partial x_1} = x_2 - 2\lambda = 0 \frac{\partial \mathcal{L}}{\partial x_2} = x_1 - 2\lambda = 0 \frac{\partial \mathcal{L}}{\partial \lambda} = -2x_1 - 2x_2 + p = 0$$

Solving these three equations for the three unknowns $(x_1, x_2, \lambda)$, we obtain:

$$x_A = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} p/4 \\ p/4 \end{bmatrix}, \lambda_A = \frac{p}{8}$$

Checking the Hessian

$$H = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

2nd order check passes with the Hessian

The units of the lambda is $m^2/m$, which is the change of area (in $m^2$) of the rectangle for a given a change in the perimeter constraint in $m$