4.2

Constants:

$$\begin{aligned} h &= 250mm = .25m \\ b &= 125mm = .125m \\ \sigma_{yield} &= 200MPa = 200000000Pa \\ \tau_{yield} &= 116MPa = 116000000Pa \\ P &= 100kN = 100000N \\ l &= 1m \end{aligned}$$

Variables

$$t_b = x_1 t_w = x_2$$

Minimize

$$f(x_1, x_2) = 2bx_1 + hx_2$$

Given second moment of area

$$\begin{aligned} I &= \frac{h^3}{12}x_2 + \frac{b}{6}x_1^3 + \frac{h^2 b}{2}x_1 \\ \implies I &= \frac{.25^3}{12}x_2 + \frac{.125}{6}x_1^3 + \frac{.25^2.125}{2}x_1 \\ \implies I &= \frac{x_2}{768} + \frac{x_1^3}{48} + \frac{x_1}{256} \\ \implies I &= \frac{x_2 + 16x_1^3 + 3x_1}{768} \\ \implies \frac{1}{I} &= \frac{768}{3x_1 + 16x_1^3 + x_2} \end{aligned}$$

Subject to

$$\begin{aligned} g_1(x_1, x_2) &= \frac{Plh}{2I} - \sigma_{yield} \leq 0 \\ g_2(x_2) &= \frac{1.5P}{hx_2} - \tau_{yield} \leq 0 \end{aligned}$$

The Lagrangian for this problem is

$$\mathcal{L}(x, \sigma, s) = 2bx_1 + hx_2 + \sigma_1(\frac{Plh}{2I} - \sigma_{yield} + s_1^2) + \sigma_2(\frac{1.5P}{hx_2} - \tau_{yield} + s_2^2)$$

Differentiating the Lagrangian with respect to all the variables, we get the first-order optimality conditions,

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x_1} &= 2b - \frac{384\sigma_1 Plh(3 + 48x_1^2)}{(3x_1 + 16x_1^3 + x_2)^2} &&= 0 \\
\frac{\partial \mathcal{L}}{\partial x_2} &= h - \frac{384\sigma_1 Plh}{(3x_1 + 16x_1^3 + x_2)^2} - \frac{1.5\sigma_2 P}{hx_2^2} &&= 0 \\
\frac{\partial \mathcal{L}}{\partial \sigma_1} &= \frac{384Plh}{3x_1 + 16x_1^3 + x_2} - \sigma_{yield} + s_1^2 &&= 0 \\
\frac{\partial \mathcal{L}}{\partial \sigma_2} &= \frac{1.5P}{hx_2} - \tau_{yield} + s_2^2 &&= 0 \\
\frac{\partial \mathcal{L}}{\partial s_1} &= 2\sigma_1 s_1 &&= 0 \\
\frac{\partial \mathcal{L}}{\partial s_2} &= 2\sigma_2 s_2 &&= 0
\end{aligned}$$

Roots of the equation found with scipy.optimize.fsolve, with initial guesses of $t_b(x_1), t_w(x_2)$ as $10cm(0.01m)$ by eyeballing the diagram, and letting $\sigma_1 = \sigma_2 = s_1 = s_2 = 0$

$$\begin{aligned}
t_b &= 1.42603955e-02 &&\approx 14cm \\
t_w &= 5.17241379e-03 &&\approx 5cm \\
\sigma_1 &= 1.99351362e-11 &&\approx 0 \\
\sigma_2 &= 7.44367855e-12 &&\approx 0 \\
s1 & &&= 0 \\
s2 & &&= 0
\end{aligned}$$

Graphical verification

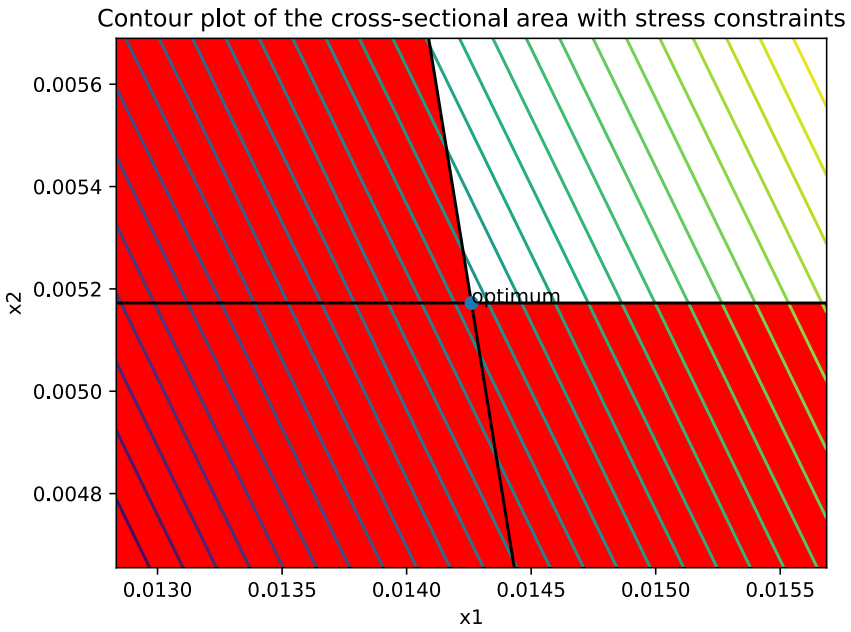

Figure 1: 4.2 Graph