Cs 5050

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Lower Bounds "No better than"

Example 1 Sorting real numbers - problem n numbers

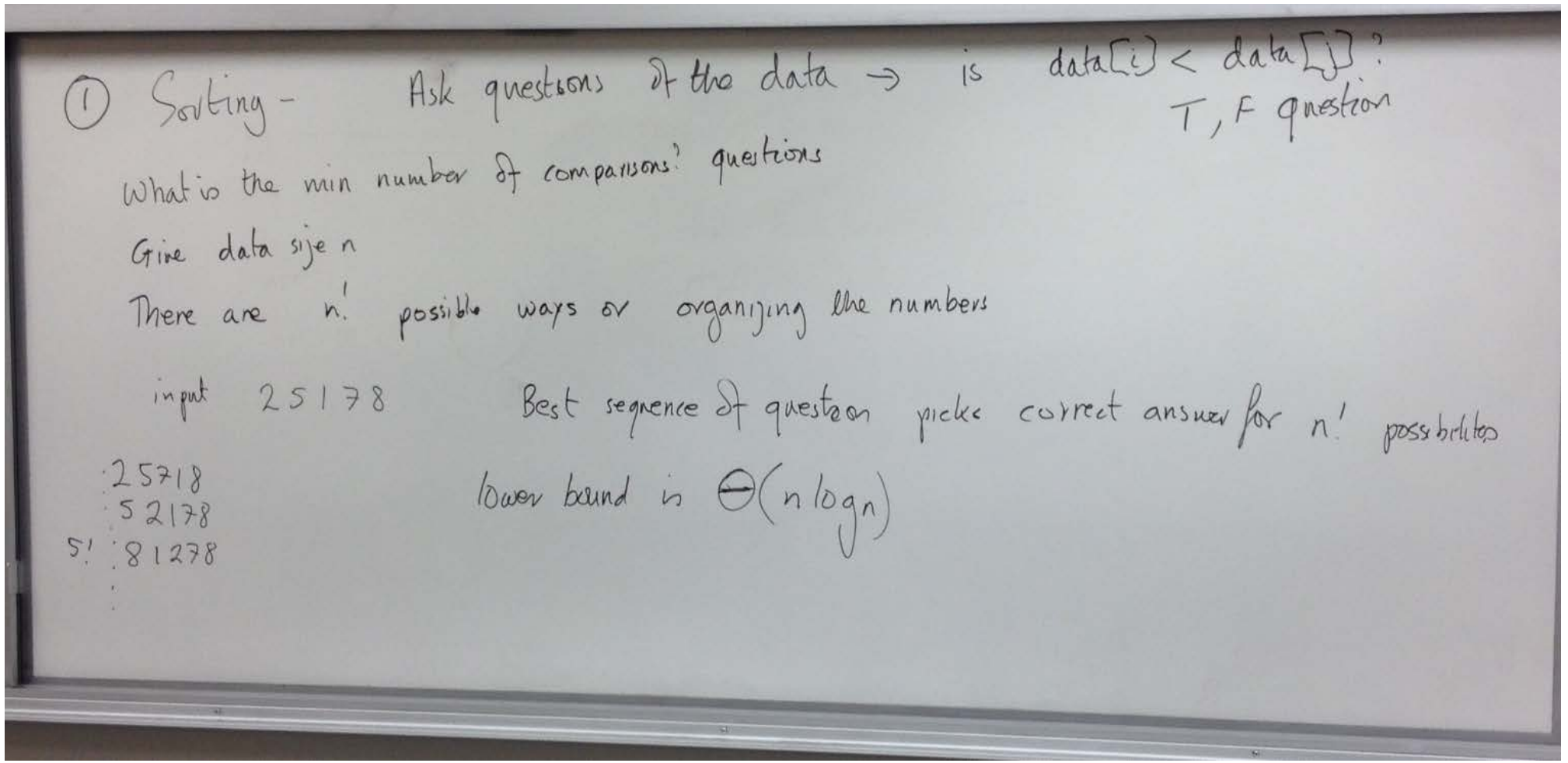
$O(n \log n)$

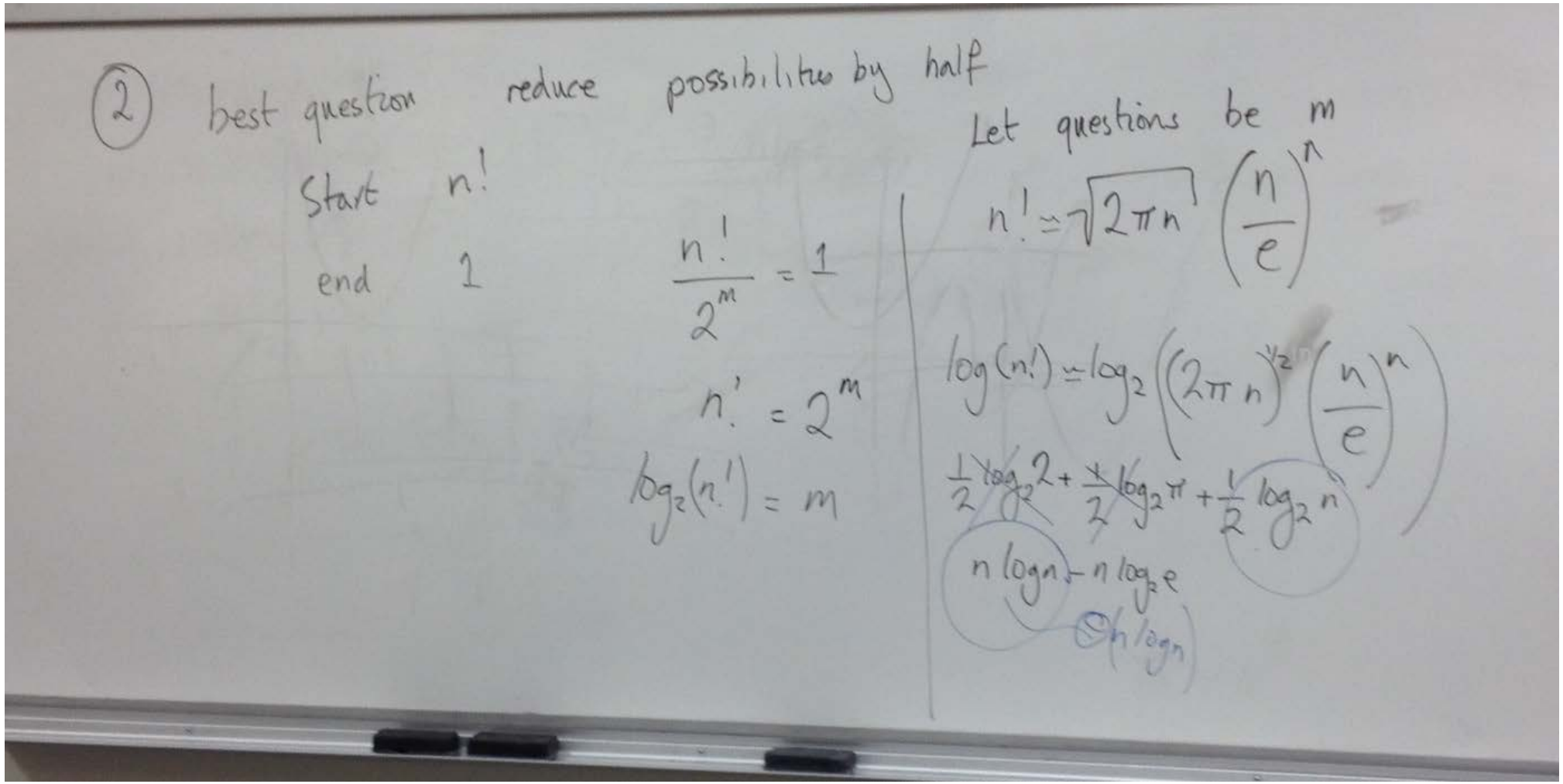
$O(n^2)$

There exists no algorithm better than $O(n \log n)$

$\exists$ for general sorting

Example SAT


① Sorting - Ask questions of the data → is data[i] < data[j]?
T, F question
What is the min number of comparisons? questions
Give data size n
There are n! possible ways or organizing the numbers
input 25178
Best sequence of question picks correct answer for n! possibilities
25718
52178
5! 81278
lower bound is Θ(n log n)


(2) best question reduce possibilities by half
Start n!
end 1
$\frac{n!}{2^m} = 1$
$n! = 2^m$
$\log_2(n!) = m$
Let questions be m
$n! \simeq \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$
$\log(n!) \simeq \log_2\left((2\pi n)^{1/2}\left(\frac{n}{e}\right)^n\right)$
$\frac{1}{2}\log_2 2 + \frac{1}{2}\log_2 \pi + \frac{1}{2}\log_2 n$
$n\log n - n\log_2 e$
$O(n\log n)$