6 Optimizations

a Use indexing into X rather than squaring x

know that $X[i] = \omega^i$

so $X[i] * X[i] = X[2i]$

$(\omega^i)^2 = \omega^{2i}$

so pass power integer down - x2 each time

keep x "global" call it Omega

Only place used is in solution construction

old code $X[i]$ new code is

p is the power Omega[i * p]

write new recursive calls

write new solution construction.

7 Polynomial multiplication

two polys n size P Q

evaluate at $2n$ points

multiply values together $P[i] \times Q[i] = PQ[i]$

interpolate using - Inverse FFT

Given $P[0 \ldots n-1]$ and $Q[0 \ldots n-1]$

as doubles

Write code to (a) pad by n and copy into Complex type

Write code to call FFT on each new polynomial

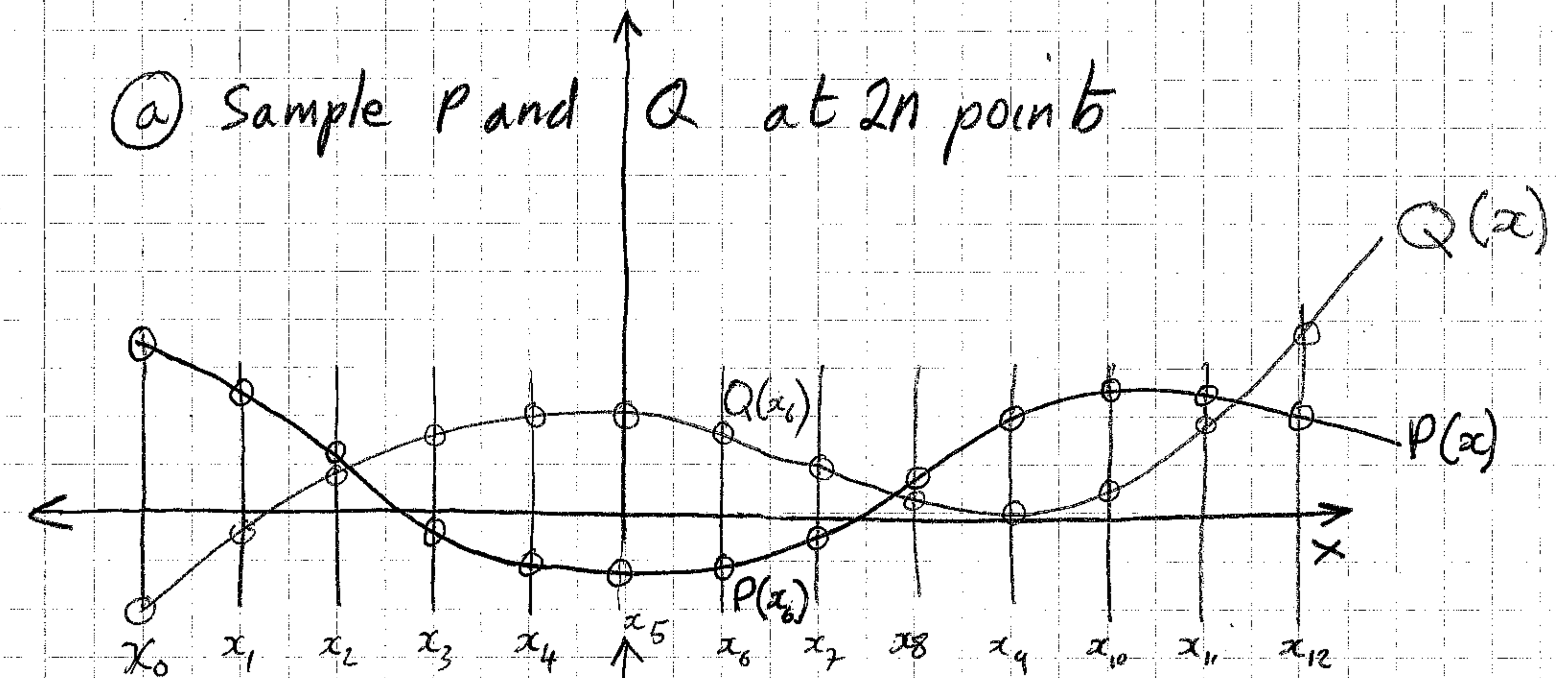
Sol P =

Sol Q =

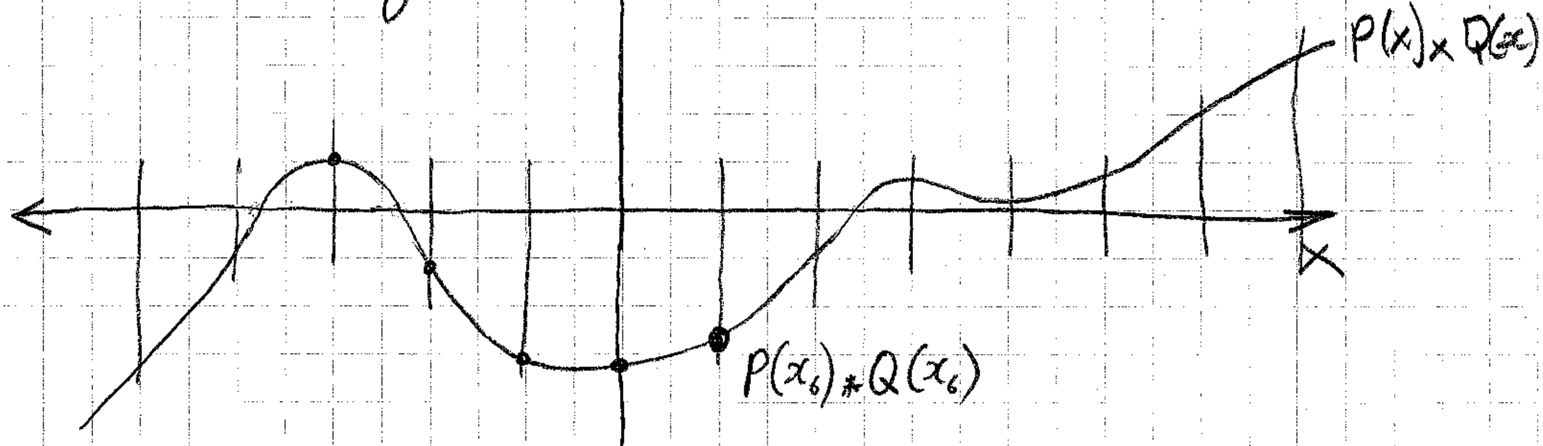
these will be arrays of complex

8 So far

ⓐ Sample P and Q at $2n$ points



ⓑ multiply $P(x_i) \times Q(x_i)$



ⓒ Interpolate

Given m samples of some polynomial PQ of degree $\leq m-1$,

Find the coeficients of PQ

(9) Since we have used the FFT to evaluate P and Q we use the inverse FFT to interpolate.

The only change needed is to modify the values in the Omega table

~~X~~change $\omega^j \longrightarrow \frac{1}{\omega^j} = X[j]$

let $\omega^j = \cos\frac{2\pi j}{n} + i \sin\frac{2\pi j}{n}$

$$\omega^j = a + bi$$

$$\frac{1}{\omega^j} = \frac{1}{a+bi} \times \frac{(a-bi)}{(a-bi)}$$

$$= \frac{a-bi}{a^2+b^2}$$

Therefore

$$X[j] = \cos\frac{2\pi j}{n} - i \sin\frac{2\pi j}{n}$$

since $\left(\cos\frac{2\pi j}{n}\right)^2 + \left(\sin\frac{2\pi j}{n}\right)^2 = \underline{1}$

(10) Complete the code for Poly multiply

(a) Pad with zeros

(b) call FFT

(c) multiply the values

(d) Call inverse FFT

+ Note must divide by $\frac{1}{2n}$