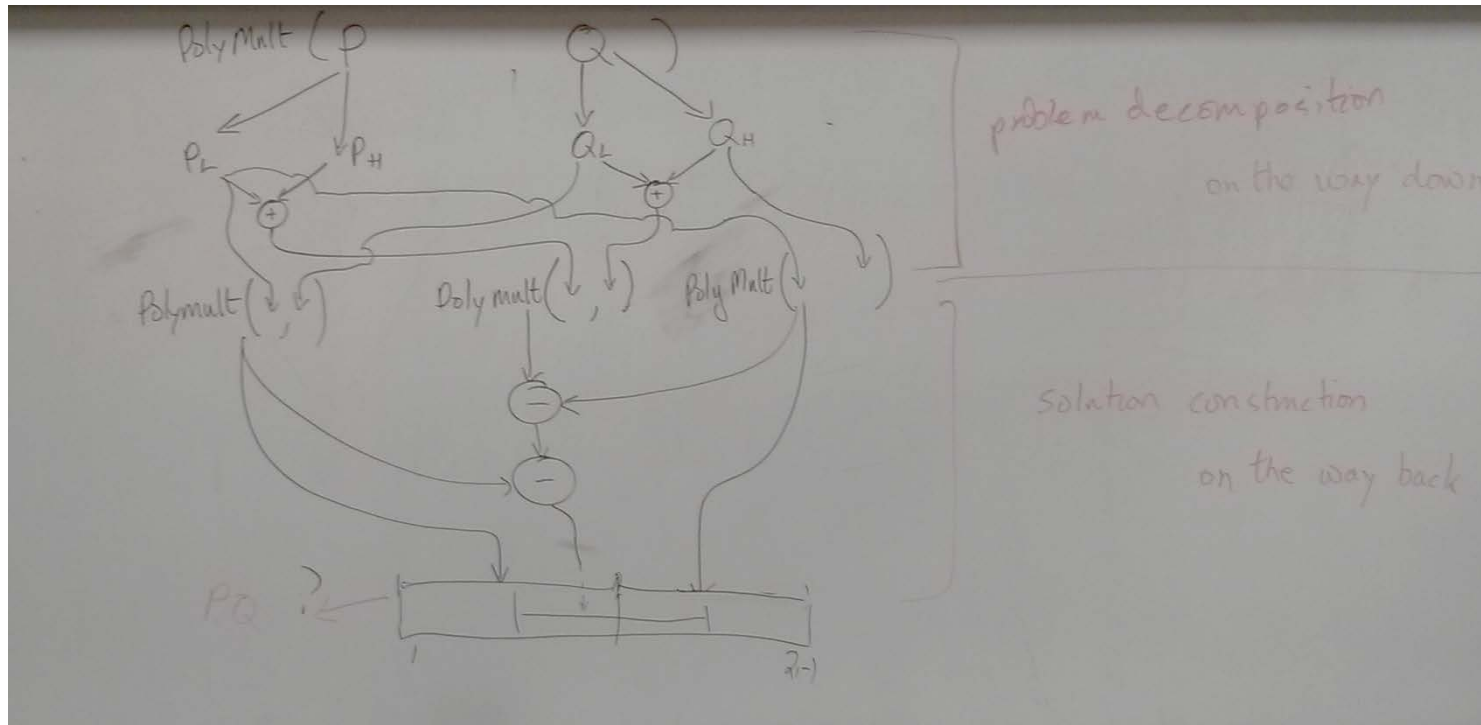


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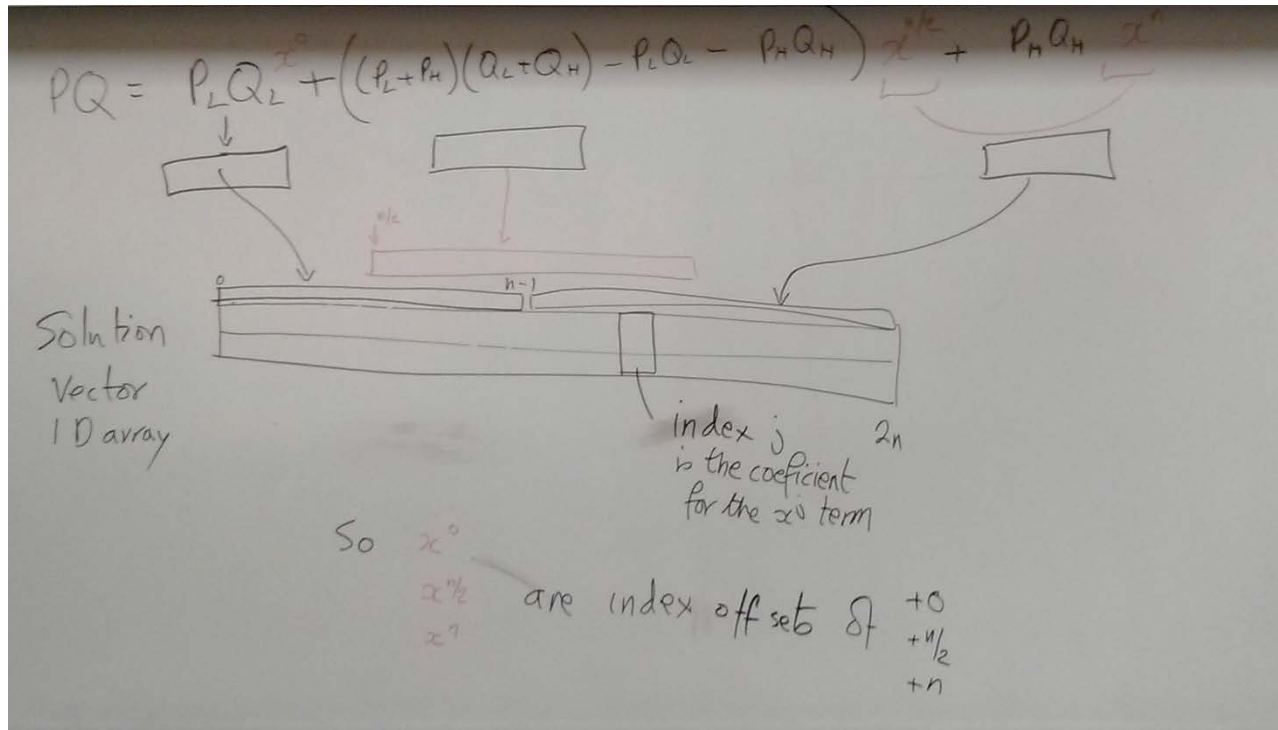


```
//Split P and Q into low and high
for i = 0 to n/2-1
  PL[i] = P[i]
  PH[i] = P[i+n/2]
end
```

Generating the sub problems by scanning through the input arrays

Need to understand that the  $i^{\text{th}}$  coefficient of the polynomial P is stored at P[i]

```
//Generate PL plus PH
for i = 0 to n/2-1
  PLandPH = PL[i] + PH[i]
end
```



PLQL = polyMult(PL, QL)

PHQH = polyMult(PH, QH)

PQSum = polyMult(PLandPH, QLandQH)

For  $i = 0$  to  $n-1$

    PQ[i] += PLQL[i]

    PQ[i+n/2] += PQSum[i] - PLQL[i] - PHQH[i]

    PQ[i+n] += PHQH[i]

end

PLQL, PHQH, PQSum are solutions to the subproblems, each is of size  $n$

PQ is the whole answer of size  $2n$

We put the subsolutions into the whole solution array at the correct offsets based on  $x^{n/2}$  (offset by  $n/2$ ) and  $x^n$  (offset by  $n$ )

③

Complex numbers

$$a + bi$$

$$i = \sqrt{-1}$$

$$i^2 = -1$$

$$(a+bi)(c+di) = \underbrace{ac}_{\text{1 multiply}} + \underbrace{(bc+ad)}_{\text{2 multiplies}}i - \underbrace{bd}_{\text{3 multiplies}}$$

$$\begin{pmatrix} 5 & 2i \\ a & b \end{pmatrix} \begin{pmatrix} 7 & 3i \\ c & d \end{pmatrix}$$

$$= 35 + i(14+15) - 6$$

$$= 29 + 29i \quad 29(1+i)$$

$$35 + (70 - 35 - 6)i - 6$$

$$29 + 29i$$

$$(a+b)(c+d) = ac + \boxed{ad+bc} + bd$$

$$ad+bc = (a+b)(c+d) - ac - bd$$

3 multiplies

$$= ac + ((a+b)(c+d) - ac - bd)i - bd$$

Complex numbers are two dimensional numbers, One real dimension and imaginary dimension

Multiplying two complex numbers using the "high school" algorithm requires 4 real multiplies

We can use the same trick that we use in polynomial multiplies:

$$(a+bi)(c+di) = a*c - b*d + ((a+b)*(c+d) - a*c - b*d)i$$

Only three real multiplies are needed

Complex Mult (C1, C2)

$$ac = C1.R * C2.R$$

$$bd = C1.I * C2.I$$

$$mid = (C1.R + C1.I) * (C2.R + C2.I) - ac - bd$$

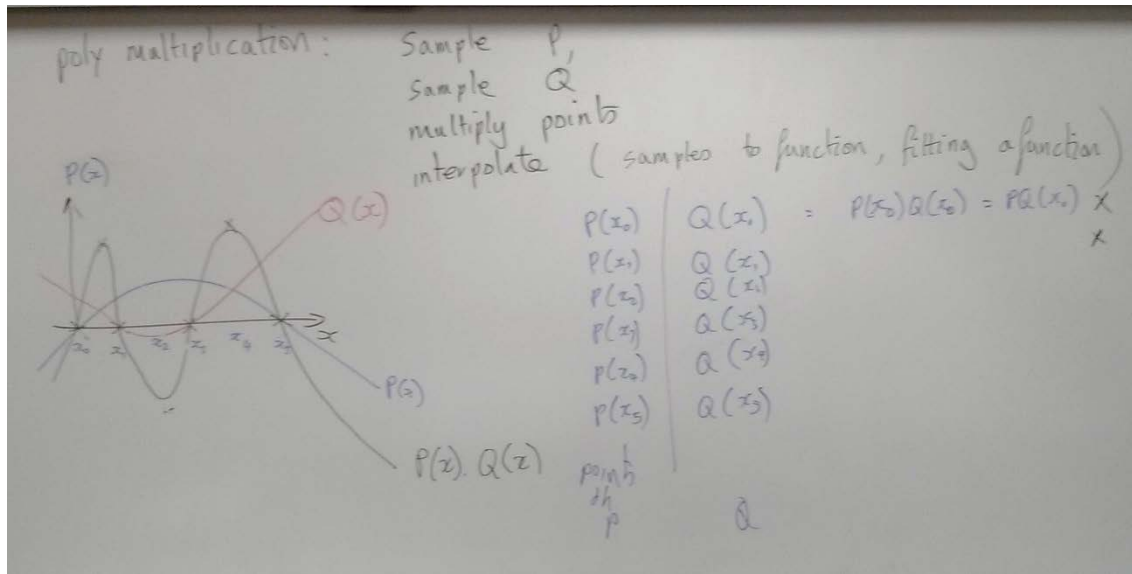
Cans ← new . . . .

$$Cans.R = ac - bd$$

$$Cans.I = mid$$

I	R	local
.	.	

Simple method for multiplying two complex numbers using three real multiplies



Problem:

Given two polynomials  $P = p_0 + p_1x + p_2x^2 \dots p_{n-1}x^{(n-1)}$  each represented as a 1D array of reals, so  $P[i]$  is the coefficient for the  $x$  to the  $i$  power

Find the polynomial  $P$  times  $Q$  as a 1D array of coefficients

New method that will turn out to be  $n \log n$

- 1) Generate  $2n$  real numbers and store in  $X[i]$
- 2) Evaluate the polynomial  $P$  and  $Q$  at  $2n$  values  $X[0], x[1] \dots x[2n-1]$ 
  - 1)  $Pvalues[i] = \text{evalPolynomial}(X[i])$
  - 2)  $Qvalues[i] = \text{evalPolynomial}(X[i])$
  - 3)  $PQvalues[i] = Pvalues[i] * Qvalues[i]$
- 3) We know that the polynomial formed from  $P$  times  $Q$  will pass through all  $PQvalues$ .
- 4) Last step is to use a special algorithm called interpolation that takes a list of points of a polynomial and finds the coefficients of that function.