ignment # 1 221

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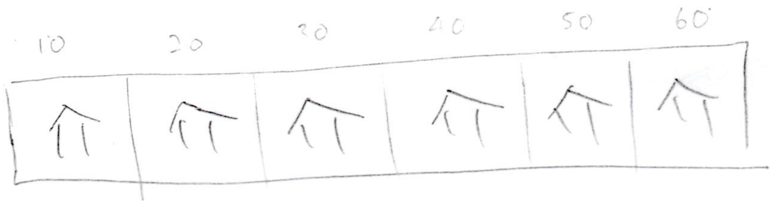
1)

```
Node *holding = Nodeb -> Next;
Nodeb -> Next = NodeA;

while ( NodeA -> Next ) {
        NodeA = NodeA -> Next;
}

NodeA -> Next = holding;
```

2) Suppose structure is a array, then it wood work lik this



Find house # thats > than the one I want to put in, I shift all the # that are greater down by 1 and then I insert the house I want to the # before it (aka the same index to the one thats greater.) Next-door time is O(n)

b)

for next-door it would take n times.
I can have a pointer pointing at the house I want from there I can find the previous and next.

B) I would use map. I think map is good since it has key and value. To insert a house it takes constant time and ~~to~~ to check the hoses taken O(n) time.

a) $\lg 32^n$

$\lg_2 32^n \qquad \lg_2 2 = 1$

$n \lg_2 2^5$

$5n \lg_2 2$

$5n$

b) $= 2^{\lg(n^2m^2) - \lg(m^2)}$

$= \frac{2^{\lg(n^2m^2)}}{2^{\lg(m^2)}} = \frac{n^2 \cancel{m^2}}{\cancel{m^2}} = n^2$

c) $-\log_2\left(\frac{1}{8}\right) = -\left(\log_2 1 - \log_2 8\right)$

$= -\left(0 - \log_2 2^3\right)$

$= -\left(0 - 3\cancel{\log_2 2}\right)$

$= -(0-3)$

$= 3$

d) $\log_p(1/p) = \frac{\log \frac{1}{p}}{\log p} = 1$

e) $64^{\lg(n^2)} = 64^{2\lg n}$

$= 2^{6(2(\lg n))}$

$\lg_2 : \log_{..}$

$= 2^{12 \lg_2 n} = a^{\log_a n} = n^{12}$

3.)

	n=10	n= 1000000
$n \log n$	10	
$n^{2\lg n}$		
$\log n$	1	
2^{2^n}	big	
$\sqrt{n}$	3.16277	
n^n	1 000 000 000	
n	10	
2^n	1024	
$\sum_{i=1}^{n} i^3$		
$\lg(n!)$	21.791061	

Ignore

Answer:

$$\sqrt{n} \leq n \leq \log n \leq n\log n \leq \lg(n!) \leq \sum_{i=1}^{n} i^3 \leq n^{2\lg n} \leq 2^n \leq 2^{2^n} \leq n^n$$

Fast ... Slow

a) $24n^2 + 8n + 72n + 24$

$T(n) = \underline{24}\, n^2 + 80n + 24$

$\Theta(n) = n^2$

b) $T(n) = \sum_{i=0}^{n} 2^{i+c} = 2^c + 2^{1+c} + 2^{2+c} + \dots 2^{n+c}$

$= 2^c(1+2+2^2+ \dots 2^n)$

$\Theta =$ $= 2^c\left(\frac{1-2^{n+1}}{1-2}\right)$

$= 2^c(-1+2^{n+1})$

$\Theta n \doteq 2^{n+1}$

d) $T(n) = \sum_{i=1}^{n} \sum_{j=i^2}^{n^2} c + c(i^2) + c(i^2+1) + c(i^2+2) \dots (n^3-n)^2$

$n \cdot n^2 \cdot c$

$\Theta(n) = n^3$

e) $T(0) = 1$

$T(1) = 1$

$T(n) = 2T(n-2) + 4$ for $n \geq 2$.

$\Theta(n) = n^2$

$T(n/2) \leq T(n/4) + k$

$T(n) \leq T(n/2) + k$

$\leq T(n/4) + 2k$

$\leq T(0) + \log n \, k$

constant.

5) $T(1) = 1$

$T(n) = T\lceil n/2 \rceil + 3$ for $n \geq 2$

$n=2.$ $\lceil \frac{2}{2} \rceil + 3$

$1 + 3 = 4$

$n3$ $T(\lceil \frac{3}{2} \rceil) + 3 = 7$

$n=4$ $T(\lceil 4/2 \rceil) + 3 = 7$

3

$n=5$ $T(\lceil 5/2 \rceil) + 3 = 10$

$T(3) + 3 = 10$

$n=6$ $T(\lceil 6/3 \rceil) + 3 = 10$

$T(3) + 3$

$\therefore \Theta(n) = \log n$

Prove $54n^3 + 17$ is $\Theta(n^3)$

f(x) is of order g, written $f(x)$ is $\Theta(g(x))$, if and only if there exist a positive real # A, B and non negative real number k such that

Prove $T(n) \in \Theta(f(n))$

$T(54n^3 + 17) \in \Theta(n^3)$

a). By definition $\exists\ c, d, > 0,\ \exists\ n_0 \in N^+$

$d \cdot f(n) \leq T(n) \leq c \cdot f(n)$ for all $n \geq n_0$

Let $d = 1$ $c = 600$ $n_0 = 1$ when $n \geq 1$

LHS: $1(1^3) \leq 54(1)^3 + 17$

$T(n) \in \Omega(f(n))$, $T(n)$ "$\geq$" $f(n)$

RHS: $54(1)^3 + 17 \leq 100(1)^3$

$T(n) \in O\, f(n)$, $T(n) \leq cf(n)$

QED

Prove $54n^3 + 17$ is not $\Theta(n^2)$

By definition $\exists\, c \cdot d > 0 \;\exists\, n_0 \in N^+$

$d \cdot f(n) \geq T(n) \geq c \cdot f(n)$ for all $n \geq n_0$

b) $d\,(n^2) \geq 54n^3 + 17 \geq c \cdot n^2$

$54n^3 \, (\Theta)$

$\lim_{n \to \infty} \frac{54n^3}{dn^2} = n = \infty = 0$

LHS: for $d\,(n^2)$, there does not exit a d such that

$d(n^2) \geq 54n^3 + 17$

Becaus as n^* aporach to ∞, n^3 will always be bigger than n^2.

Since LHS will never be satify, $53n^3 + 17$ is not $\Theta(n^2)$

QED

c) Let $T(n) = a_d n^d + a_{d-1} n^{d-1} + \ldots a_1 n + a_0$

By definition $\exists\, c \cdot d > 0 \;\exists\, n_0 \in N^+$ $\quad n_0$

$d \cdot f(n) \leq T(n) \leq c \cdot f(n)$

LHS: $d \cdot f(n) \geq a_d n^d + a_{d-1} n^{d-1} + \ldots + a_1 n + a_0$

$\leq 1n^d + 1n^{d-1} + \ldots 1n$

$\leq n^d + n^{d-1} + \ldots n$

$\leq n^d \qquad \therefore T(n) \in \Omega(n^d)$

$\therefore T(n) \in \Theta(n^d)$

QED

1)

a) $\Theta = \log n + 3$

b) $\Theta = \frac{1}{2} n^2$