CPSC 221: Assignment 3

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November 23, 2016

Problem 1.

a)

0	$\rightarrow$ 110
1	$\rightarrow$
2	$\rightarrow$
3	$\rightarrow$
4	$\rightarrow$ 70
5	$\rightarrow$ 93 $\rightarrow$ 60 $\rightarrow \emptyset$
6	$\rightarrow$ 50
7	$\rightarrow$ 150 $\rightarrow$ 40 $\rightarrow \emptyset$
8	$\rightarrow$ 96 $\rightarrow$ 140 $\rightarrow$ 30 $\rightarrow \emptyset$
9	$\rightarrow$ 97 $\rightarrow$ 20 $\rightarrow$ 130 $\rightarrow \emptyset$
10	$\rightarrow$ 10 $\rightarrow$ 120 $\rightarrow \emptyset$

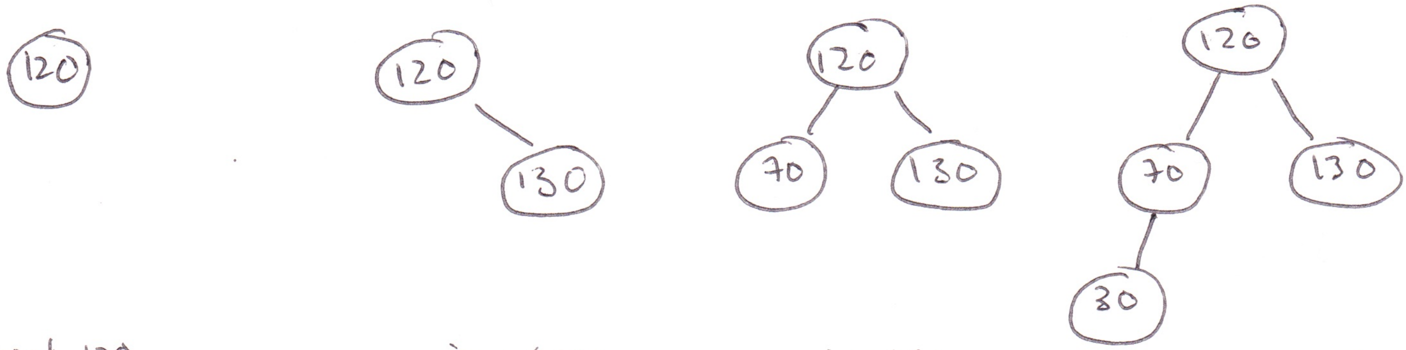
b)

0	$\rightarrow$ 93	12	$\rightarrow$ 150
1	$\rightarrow$ 70	13	$\rightarrow$ 96
2	$\rightarrow$ 140	14	$\rightarrow$ 60
3	$\rightarrow$	15	$\rightarrow$ 130
4	$\rightarrow$ 50	16	$\rightarrow$
5	$\rightarrow$ 120	17	$\rightarrow$ 40
6	$\rightarrow$	18	$\rightarrow$ 110
7	$\rightarrow$ 30	19	$\rightarrow$ 97
8	$\rightarrow$	20	$\rightarrow$ 20
9	$\rightarrow$	21	$\rightarrow$
10	$\rightarrow$ 10	22	$\rightarrow$
11	$\rightarrow$		

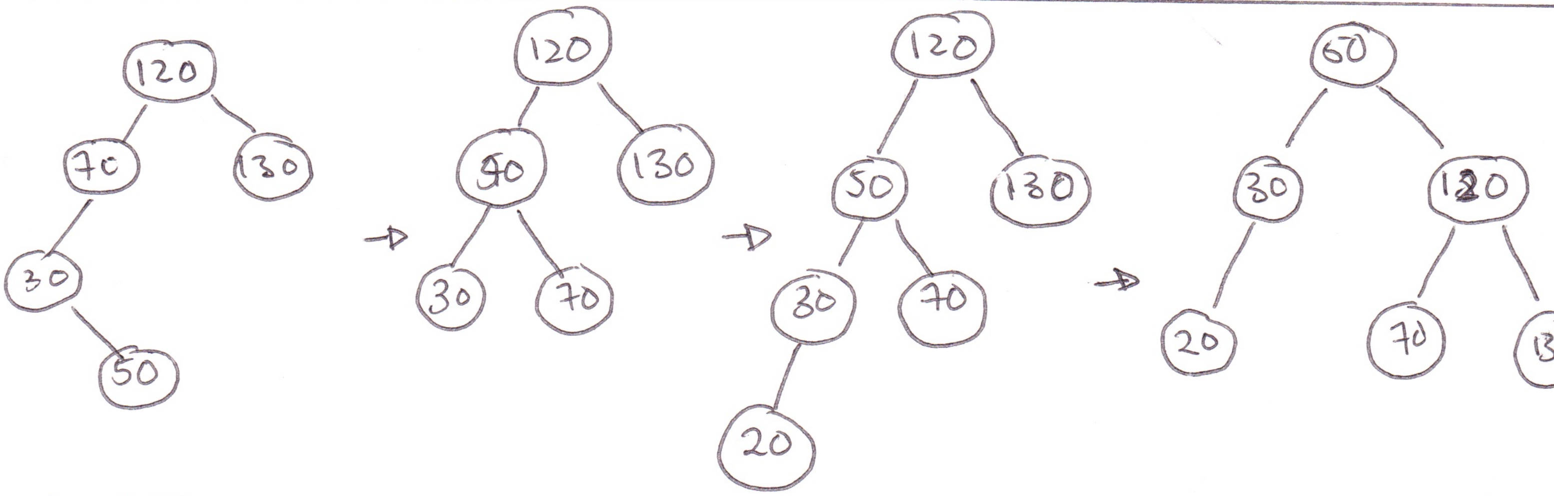
For 97 probed: 5, 12, 19
For 96 probed: 4, 12, 20, 5, 13
For 93, probed 1, 12, 0

c) On page 2 and 3

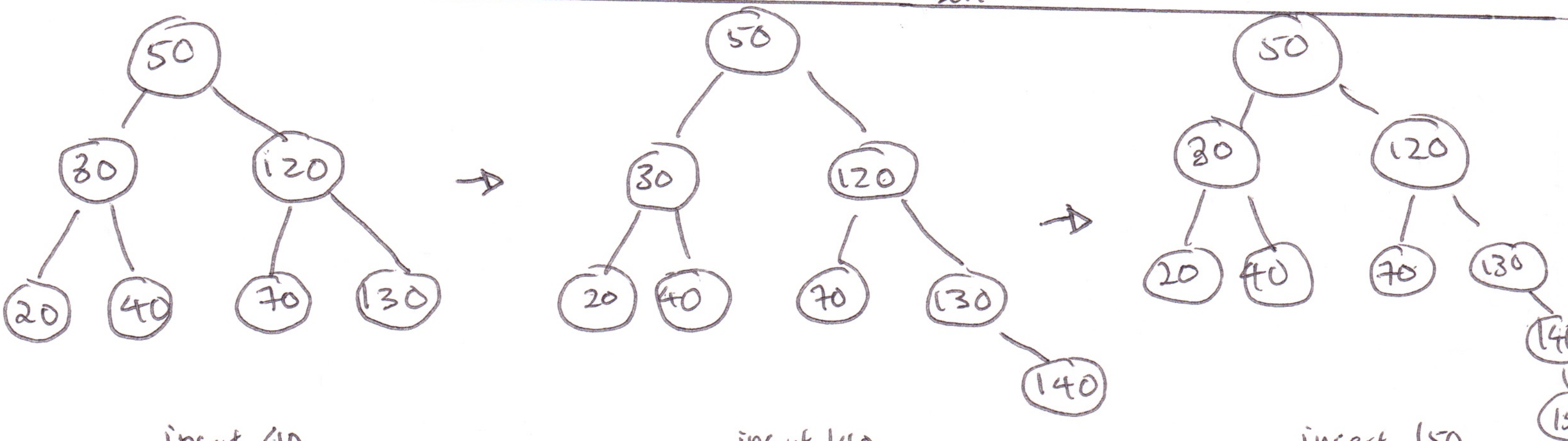
d) On page 4 and 5



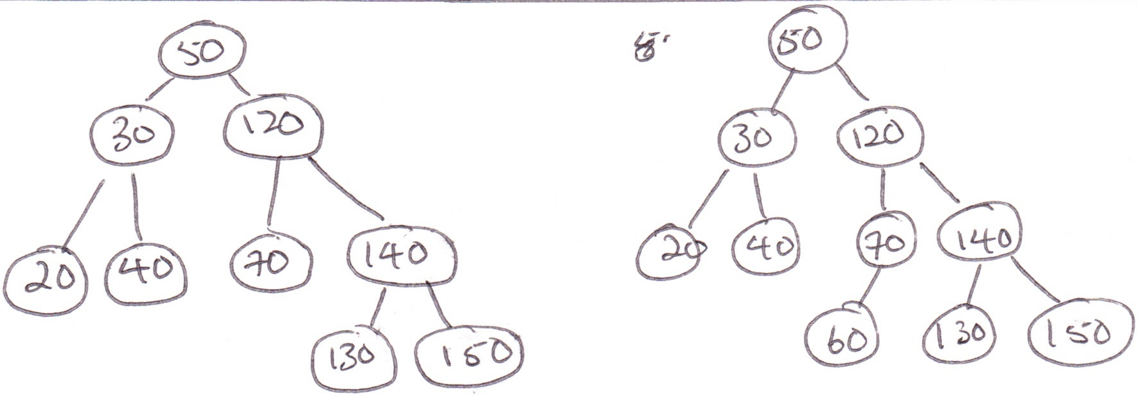
insert 120 insert 130 insert 70 insert 30



insert 30 insert 20



insert 40 insert 140 insert 150



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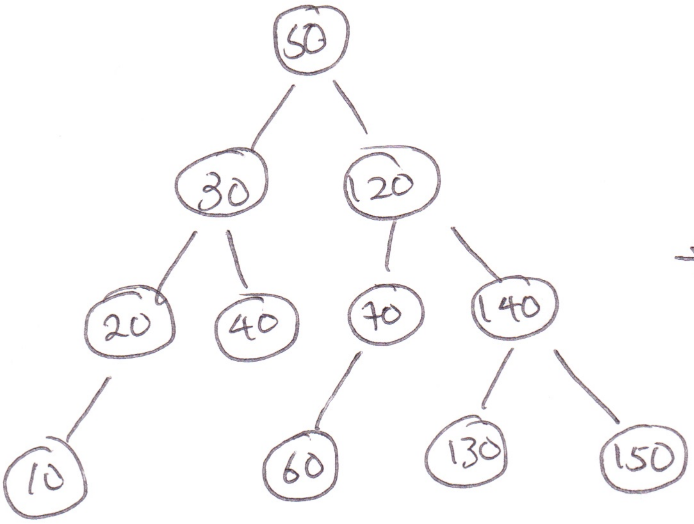
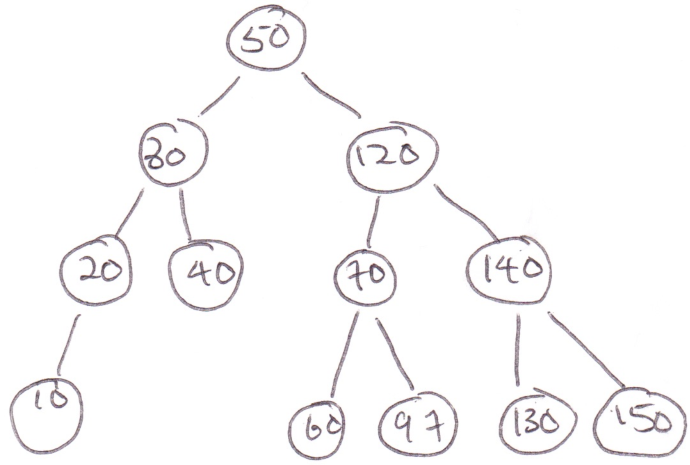
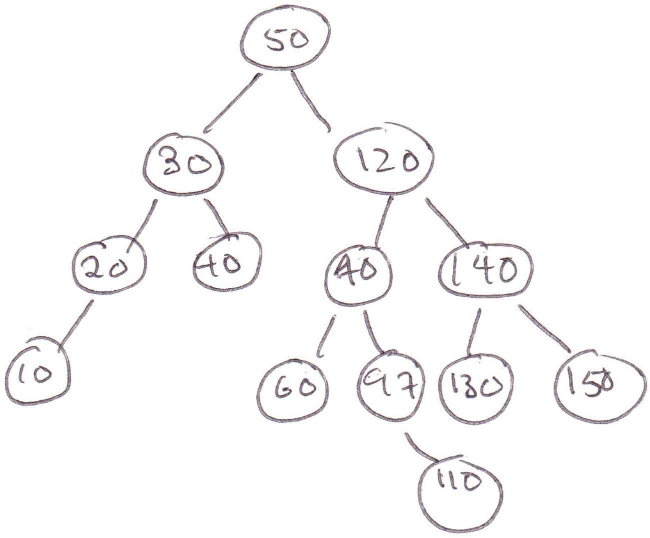
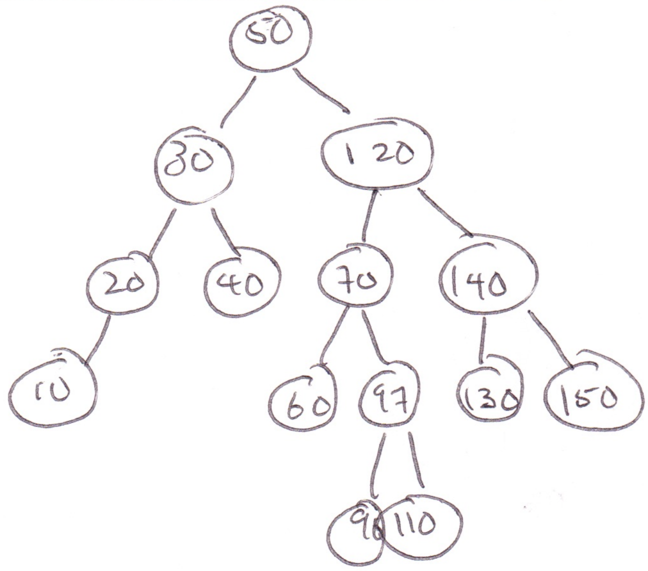




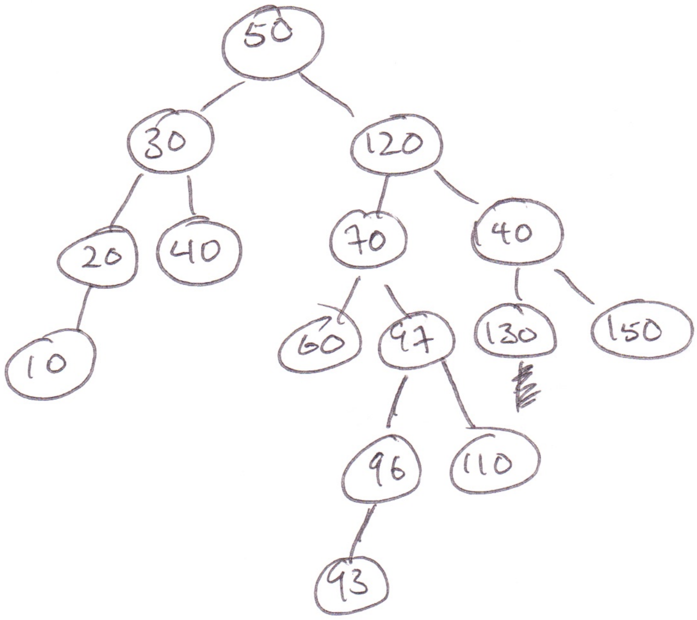
insert 10

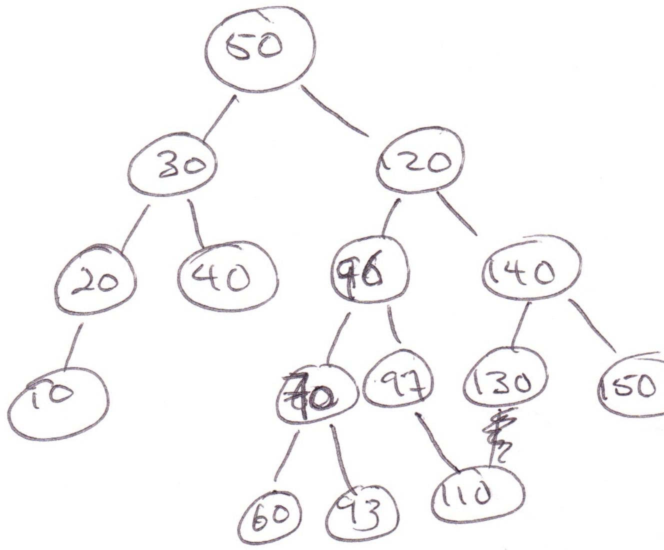
insert 97





insert 110



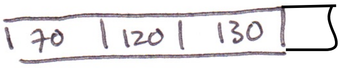


insert 93

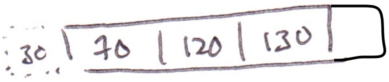
insert 120

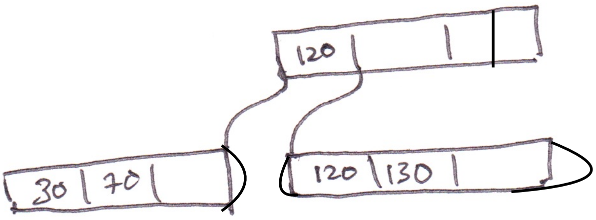
insert 130

insert 70



insert 30





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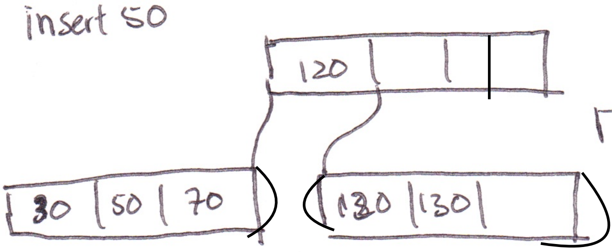

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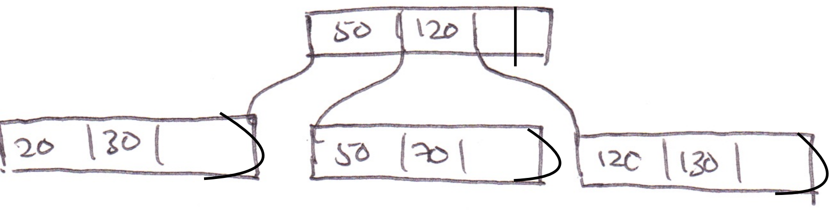
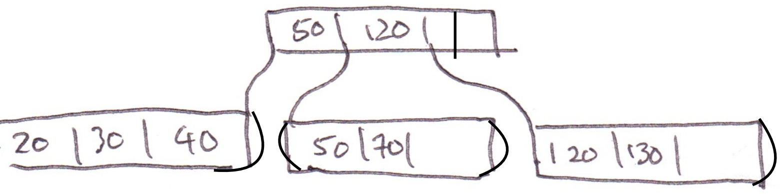
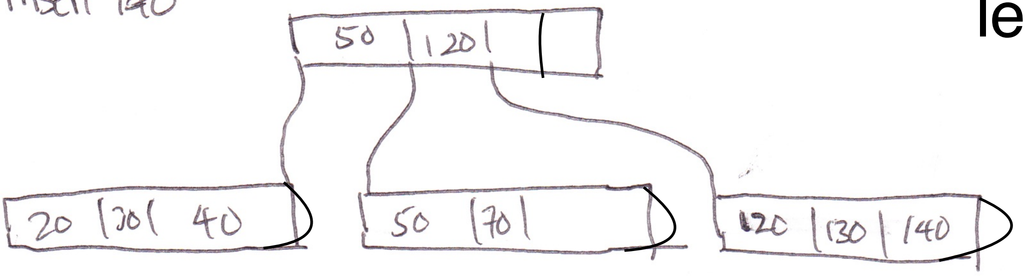


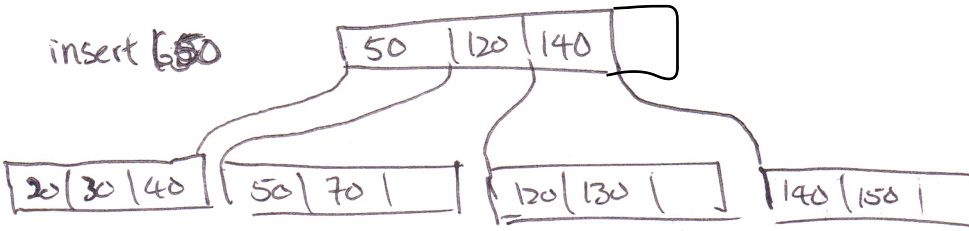
insert 40



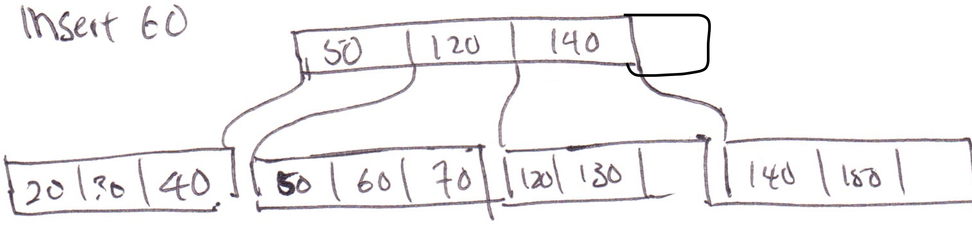
insert 140



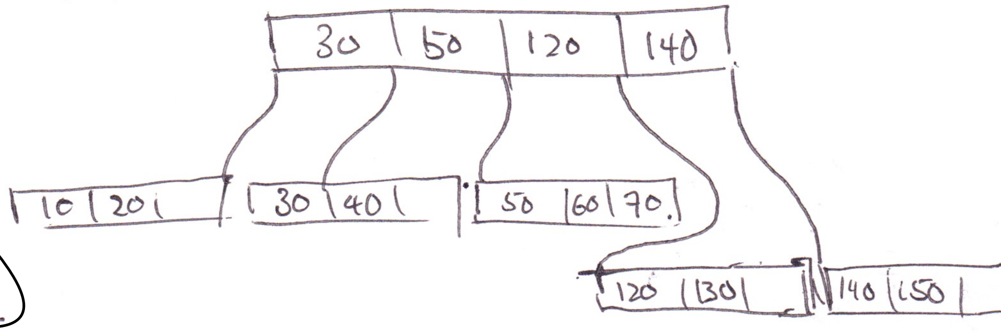
insert ~~650~~ 150



insert 60



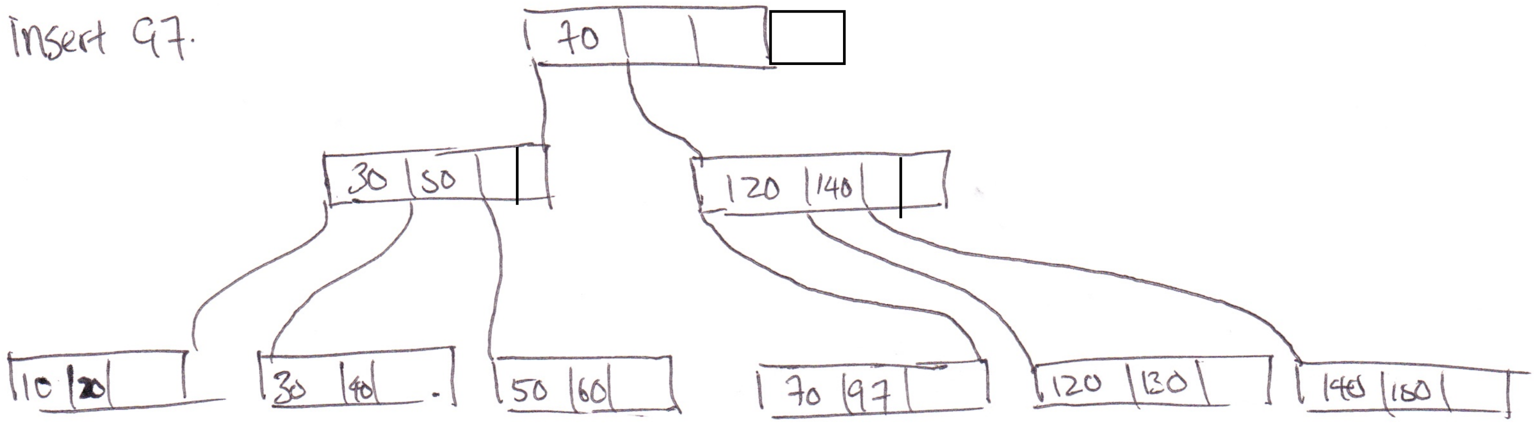
insert 10



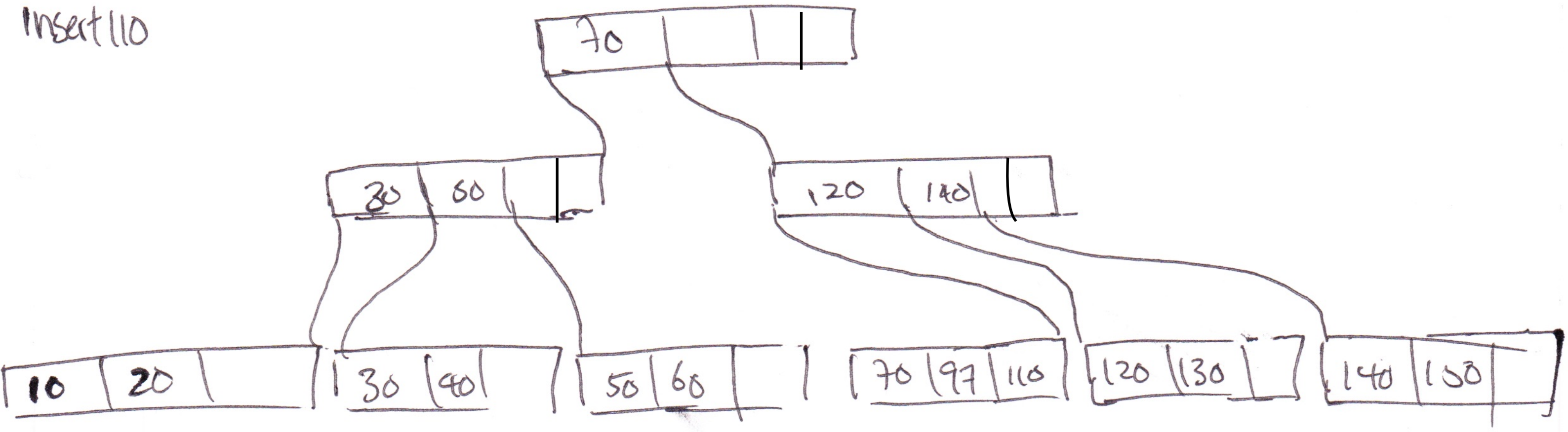
next page →

I understand that the leaf needs to be round but I cannot change it and re scan it. So lets assume all leafs look like a capsule.

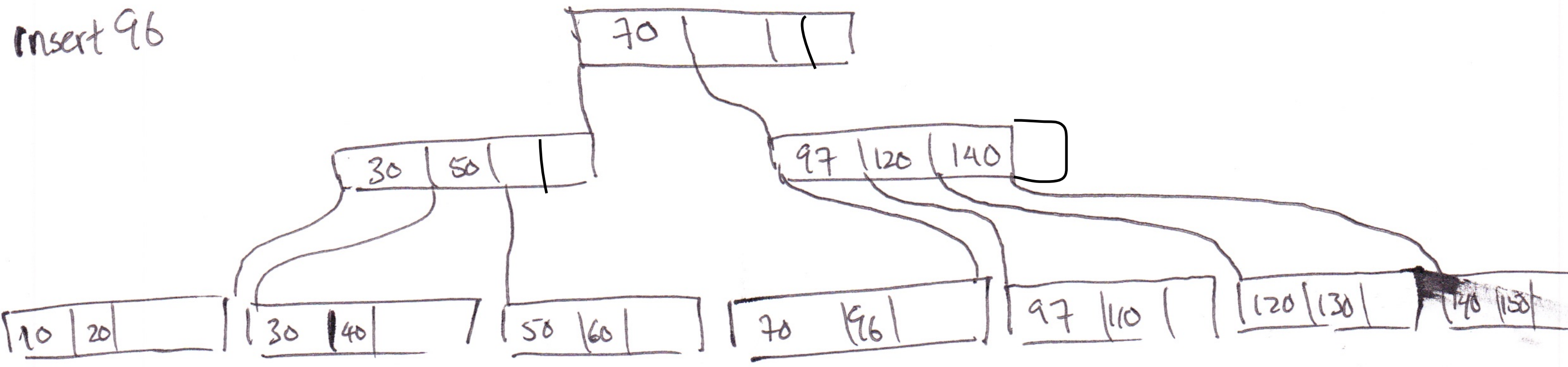
Insert 97.



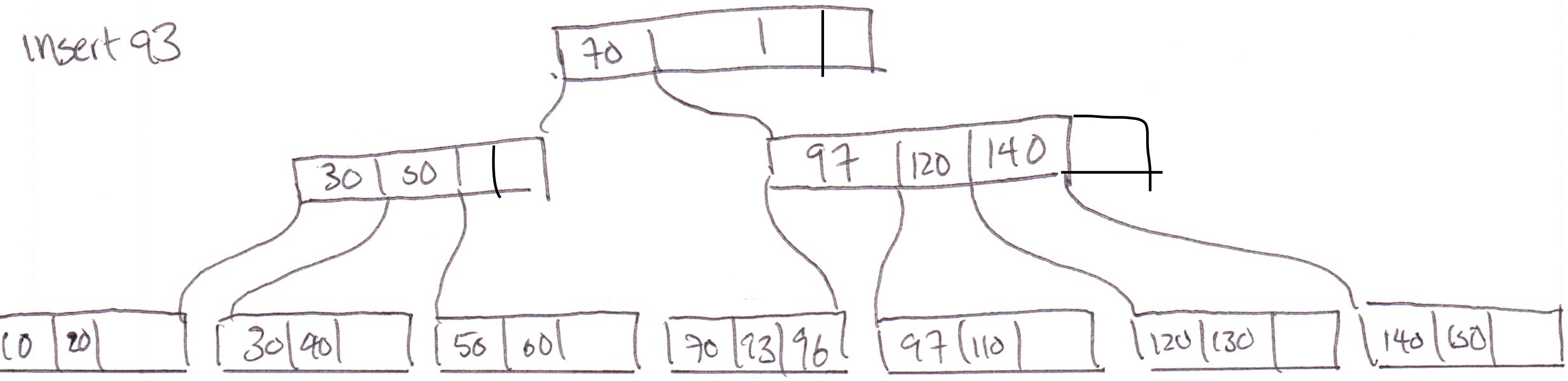
Insert 110



Insert 96



Insert 93



Problem 2. This problem is modeled by the pigeon hole principle. In this case the connections between computers are the pigeons the number of possible connections are the holes.
Since each computer can be connected to 0 or more other computers, and there are only 6 computers, the number of possible connections a computer can have is either: 0, 1, 2, 3, 4, or 5, 6 choices. But in reality there are only 5 choices since if one of the computers connects to 0 other computers, then no computer can be connected to 5 others. 6 computers and 5 choices, therefore at least two of the computers must have the same number of connections.

Problem 3. The smallest number of Nodes in an AVL tree is the number of nodes in its children plus the root.

$$N(h) = N_{children} + 1$$

Since we are looking for the smallest possible number of nodes, one of the children must be shorter than the other (since AVL tree's can vary by at most 1). We therefore have

$$N_{children} = N(h-1) + N(h-2)$$

Putting this together:

$$N(h) = N(h-1) + N(h-2) + 1$$

This could be read as left tree + right tree + root.

Now to prove $N(h) = F(h+3) + 1$:

Base cases:

$$N(0) = 1, F(3) = 2. Therefore N(0) = F(3) - 1$$
$$N(1) = 2, F(4) = 3. Therefore N(1) = F(4) - 1$$

Induction Hypothesis:

Let's assume that N(h) = F(h + 3) - 1 is true for h = 1....j

Inductive Step:

Show that is also true for h = j + 1

$$\begin{aligned} N(j+1) &= N(j) + N(j-1) + 1 && (1)\\ &= F(j+3) - 1 + F(J+2) - 1 + 1 && (2)\\ &= F(j+4) - 1 && (3) \end{aligned}$$

As we can see, the hypothesis holds for h = j+1 and therefore it is true for all h.