Subject:

Year ____ Month ____ Date ____ ( )

$$X_t = \begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix}, \quad u_t = \begin{bmatrix} v_x \\ \\ \omega_z \end{bmatrix}$$

$$v_R = r\omega_R, \; v_L = r\omega_L \rightsquigarrow v_x = \frac{v_R + v_L}{2}, \; \omega_z = \frac{v_R - v_L}{\omega}$$

$$\rightsquigarrow \begin{aligned} v_R &= v_x + \tfrac{\omega}{2}\omega_z \\ v_L &= v_x - \tfrac{\omega}{2}\omega_z \end{aligned}$$

Motion Model

از طرفی اگر motion model را بسته در نظر بگیریم:

$$\frac{dx}{dt} = v_x \cos\theta, \quad \frac{dy}{dt} = v_x \sin\theta$$

$$\frac{d\theta}{dt} = \omega_z$$

حال:

$$x_t = x_{t-1} + v_x \cos\theta_{t-1}\Delta t, \; y_t = y_{t-1} + v_x \sin\theta_{t-1}\Delta t$$

$$\theta_t = \theta_{t-1} + \omega_z \Delta t$$

$$\rightsquigarrow X_t = f(X_{t-1}, u_{t-1}) + \varepsilon_t \quad , \varepsilon_{t-1} \sim N(0, R_t)$$

Measurement Model

Visual Odometry $\rightsquigarrow (\Delta x, \Delta y)$
IMU $\rightsquigarrow \theta$

$$\Big\} \; Y_t = \begin{bmatrix} \Delta x_t \\ \Delta y_t \\ \theta_t \end{bmatrix}$$

$$\Delta x_t = x_t - x_{t-1}, \; \Delta y_t = y_t - y_{t-1}, \; \theta_t = \theta_t$$

بنابراین

$$Y_t = h(X_t) + \delta_t \quad , \quad \delta_t \sim N(0, Q_t)$$

$$h(X_t) = \begin{bmatrix} x_t - x_{t-1} \\ y_t - y_{t-1} \\ \theta_t \end{bmatrix}$$

: EKF

$$F_t = \frac{\partial f}{\partial x} = \begin{bmatrix} 1 & 0 & -v_k \sin\theta \Delta t \\ 0 & 1 & v_k \cos\theta \Delta t \\ 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\Sigma}_t = F_t \Sigma_{t-1} F_t^T + Q_t$$

measurement Model $\rightarrow$ $h(\hat{x}_{t|t-1})$

$$\hat{y}_t = \begin{bmatrix} \hat{x}_{t|t-1} - u_{t-1} \\ \hat{y}_{t|t-1} - y_{t-1} \\ \hat{\theta}_{t|t-1} \end{bmatrix} \rightsquigarrow \frac{\partial h}{\partial x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = H_t = I$$

$$z_t = y_t - \hat{y}_t \rightsquigarrow K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H^T + Q_t)^{-1}$$

$$= \bar{\Sigma}_t (\bar{\Sigma} + Q_t)^{-1}$$

$$\rightsquigarrow \hat{x}_{t|t} = \hat{x}_{t|t-1} + K_t z_t$$

$$\rightsquigarrow \Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$$