Research Report
Rice University
Department of Civil and Environmental Engineering

1. Project Information

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Project Title: Reduced-order model for dynamical snap-through simulations

Project duration: 05/15/2016 – 07/22/2016

Hours per week: 30

2. Project Description

2.1 Introduction:

New generation aircraft design usually requires the structure to be lightweight and consequently, the use of slender structural components is necessary. Such components, when loaded in extreme conditions, have great risk of loss of stability. One type of loss of stability that can happen is the snap-through, where the structure jumps from the initial equilibrium state to a remote one.

Snap-through is particularly dangerous when the load is dynamical. For certain load frequencies, the critical load for snap-through can be much lower than the static limit, and the structure can also present a persistent snap-through behavior, where it oscillates between the two equilibrium states continuously, which can increase the risk of fatigue damage. Therefore, precise numerical simulation of snap-through is important for the assessment of safety in aircraft design.

Finite element models (FEM) have shown good accuracy for this class of problems. However, since snap-through responses can have very high sensitivity in initial conditions and also bifurcations, extensive parametric studies have to be made in order to fully understand the behavior of the structure. Unfortunately, the computational cost of the FEMs is too large for these purposes. Therefore, a reduced order model (ROM) can be an important tool to make those extensive studies possible.

The reduced order model consists in extending the method of modal analysis, which is usually efficient in the study of linear dynamical systems, to nonlinear systems, by using a set of basis functions that can be obtained by many different methods. One of the most popular methods is the proper orthogonal decomposition (POD), which has been applied successfully in the field of structural analysis.

2.2 Objectives:

The objective of this project is to develop a reduced order model to simulate the dynamics of a curved arch, under dynamical loads, one of the simplest problems where snap-through happens, however, it is still challenging to be solved numerically and, in addition, it shows most of the nonlinear dynamical phenomena that are relevant for these type of problems, such as bifurcations, chaotic responses and high sensitivity to initial conditions.

The reduced order model should be general enough so that it could simulate arches of arbitrary geometries, arbitrary boundary and initial conditions and arbitrary loads. Also, it should be at least one order of magnitude faster than FEM for the same problem. Accuracy is of course required, but it is not expected to be more accurate than the FEM simulations.

2.2 Methodology:

In order to develop a ROM for snap-through simulations, we decided to start by approaching a simple structural problem that is capable of showing this behavior, a curved arch (more specific details about the arch are given in section 2.3). Results from [1] and [2] showed that sinusoidal modes can be used for the analysis of pinned-pinned arches, however, their approach can't be extended to other type of arches. So, we had to find a different way to construct a set of basis functions for our ROM, the numerical approach based on the proper orthogonal decomposition (POD) method seemed to be applicable. References [3] and [4] provides a good description of this method.

In order to build a set of basis function with POD, one needs to have some sample responses already calculated. To get these responses, we performed some FEM simulations for a specific arch and load condition using the Finite Element Analysis Program (FEAP).

After obtaining the basis functions, we implemented in MATLAB, a solver that discretizes the equations of motion using these functions and solve the discretized differential equation using a time integration method. The equation of motion used was a simplified 1-D equation based on the Bernoulli-Euler beam theory, which can be found in [1]. This part presented some interesting challenges. The first one was dealing with numerical derivatives, because we needed the derivatives of the basis functions that were calculated numerically. Our initial idea was to use finite differences to approximate the derivatives, but this method was not accurate enough. Therefore, we implemented a method for computing derivatives that consisted of fitting piece-wise polynomial functions to our basis functions and then taking the analytical derivatives of these fits.

A second challenge was the numerical stability of the method. In the first version of the solver, we used a time integrator from the Newmark family of methods, because this method performed well in a study done with sinusoidal modes. However for our problem, we had many issues with numerical stability, which required us to implement a time integration method with some numerical damping, which would damp the oscillations in the higher frequency modes but preserve the low frequency modes unchanged, this would prevent instability issues and also preserve the accuracy. The Hilber-Hughes-Taylor

(HHT)[5] family of methods is well known for having these proprieties this reason was chosen and implemented in the second version of our solver.

After developing the second version of the solver, we performed some simulations to compare the results with FEM data in order to validate the method.

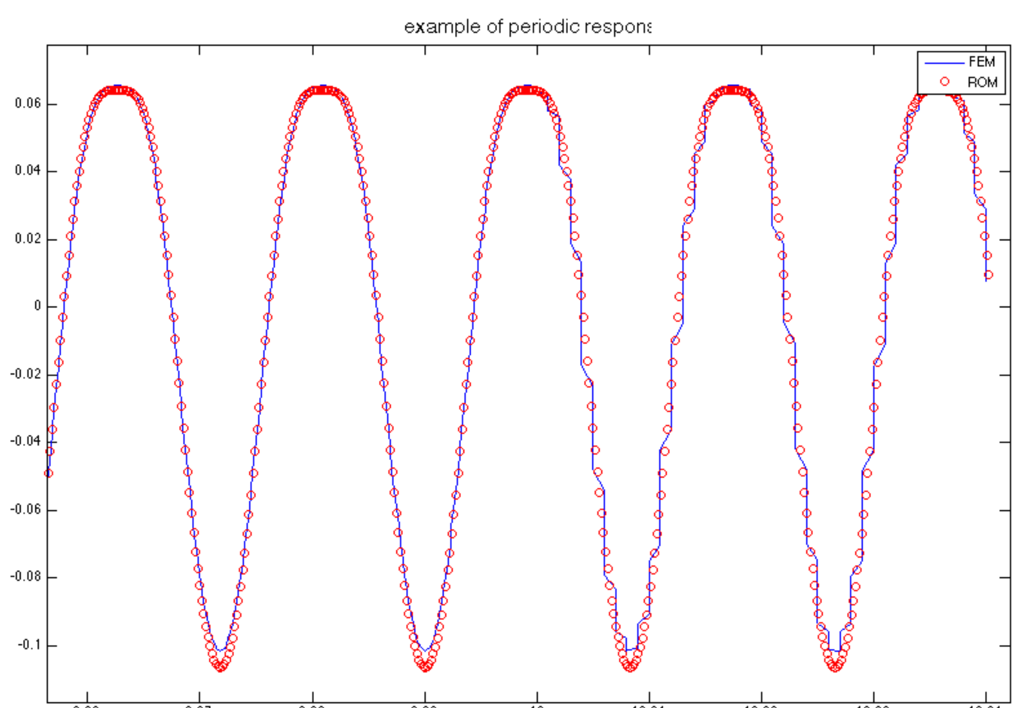
2.3 Results:

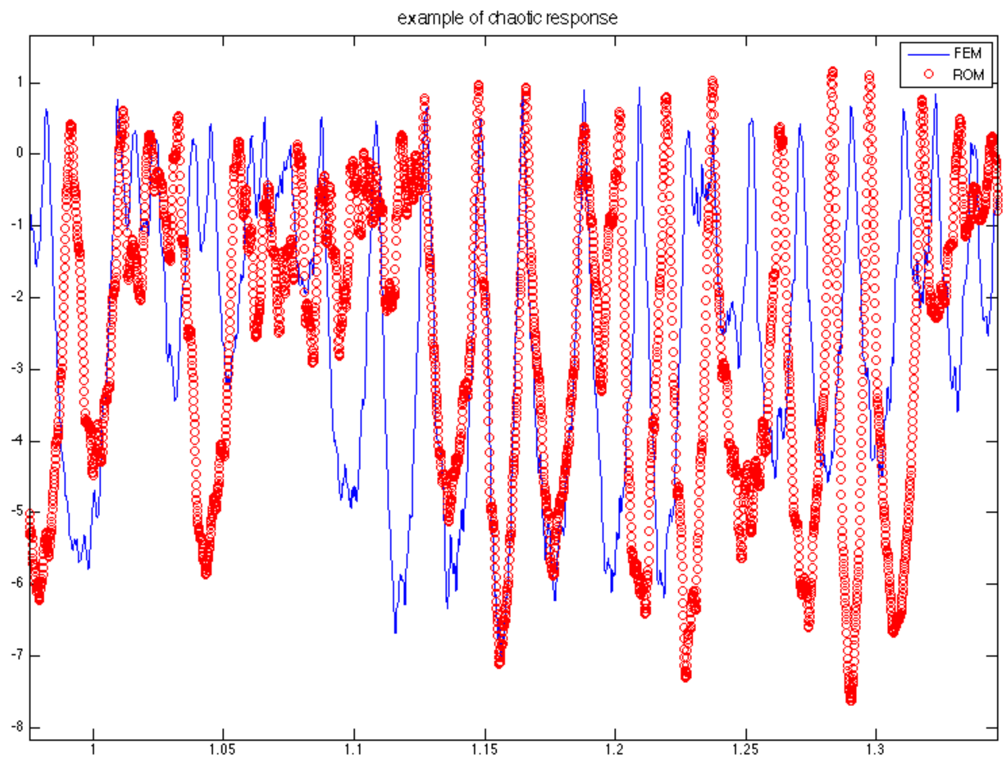
The simulations were done for a clamped-clamped circular arch. The arch radius is 3048 mm and the project length is 304.8 mm. The cross section and material parameters are listed below:

Cross Section Area	6.452 mm^2
Cross Section Moment of Inertia	0.1387 mm^4
Poisson‘s ratio	0.28
Density	7.834e-9 $N.s^2/mm^4$

The results obtained with the ROM were very accurate for periodic solutions, however, in many cases, we found chaotic responses, hence, a match between the FEM and ROM results was impossible, nevertheless, some response’s features as number of snaps per cycle and snap amplitude showed good agreement between the two results even in the presence of chaos. The time required for ROM and FEM simulations was also compared. The ROM was approximately 10 times faster than the FEM for the same problem.

Below, we have examples that show how the ROM compares to FEM solutions for periodic and chaotic responses. Both plots are the time series for the midpoint displacement of the arch.





2.4 Conclusions:

The computational time reduction obtaining using the ROM might make the intended extensive parametric studies possible, since the accuracy of the method was very good. The situations where the ROM solutions didn't match the FEM solutions were just in the cases of chaotic responses, which is expected, and also in case of bifurcated responses where the ROM and FEM would find different, but co-existent solutions.

It is also important to state that even though all the tests were performed for the same structure, just with different load parameters, this procedure for obtaining the basis functions can be used for many different problems. Some tuning in the solver's parameters might be required though, for example, in the time step and the HHT alpha parameter.

Finally, these first results definitely show that POD based ROMs are capable of simulating dynamical snap-through behavior of structures, even POD being a method that is based on fundamentals of linear dynamics (modal analysis). Therefore, the development of more complex ROMs for more complicated problems where thermal and acoustic loads are presented is an interesting next step that could be taken. Also, an extension of this procedure for 2-D geometries is also possible and would be very important.

3. References:

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